ADULT GENERAL EDUCATION
Program of Study

Mathematics

Subject Area: Mathematics, Science and Technology

Diversified Basic Education, DBE

Québec

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Chapter 1

Introduction

1.1 Contribution of the Subject to the Education of Adult Learners

Mathematics is a science and a universal language. On the one hand, it makes it possible to acquire basic skills that are considered indispensable for living in society. On the other hand, it allows individuals to contribute to the development of society. Discoveries in the field of mathematics sometimes occur incrementally and sometimes by leaps and bounds. Mathematics provides the tools for analyzing the world around us, and it is used to describe and explain certain aspects of this reality. Consisting of a set of theorems, knowledge and observations, it mainly involves devising models to account for the complexity of our world. These models are refined as mathematical discoveries are made. It is important to encourage adult learners to develop their sense of mathematics and to make them aware of the role that mathematics can play in their ability to make informed decisions. The program of study promotes a teaching approach that emphasizes the four fundamental aims that have always guided mathematicians in their quest to understand the universe. These aims are: to interpret reality, to predict, to generalize and to make decisions.

To interpret reality, societies have developed different resources related to spatial sense, such as representation, position and movement, the order of magnitude, the location of objects, scales and measurement. They use graphs, probability, statistics and proportional reasoning to describe their environment.

People have learned how to predict outcomes by relying on human intuition, experience and the ability to compare situations. The capacity to predict enables human beings to plan, visualize effects, and determine actions to be taken. Mathematics contributes to the development of this aptitude by teaching individuals how to approximate a result. By learning to compare models, to optimize situations in a planning and organizational context, and to use deductive reasoning, adults are also able to develop the aptitude to predict outcomes.

Throughout history, people have used observation and reasoning to identify models, which they reapply, develop or modify so as to be able to use them in other situations. Mathematics is a valuable tool for generalizing a situation and requires that adult learners use their reasoning skills. It also makes it possible to observe and identify patterns and trends by analyzing data and relationships among variables or by attempting to find measurements.

Mathematics has enabled human beings to hone their ability to make decisions and convince themselves that the chosen solution is the optimal one and that it reflects the constraints of a situation. In statistics and probability, in addition to experimentation, the concepts of odds and chance are used to guide the decision-making process and to assess risk or the margin of error.

The classroom therefore provides an ideal opportunity for adult learners to understand and analyze situations involving mathematics and to learn to make informed decisions. Exposed to a variety of mathematical problems related to everyday situations, adult learners represent these problems in order to plan the steps involved in finding a solution. They use their mathematical knowledge and their ability to employ strategies and reasoning skills. They are careful to validate and convey the different steps in their solution in accordance with the established rules and conventions. By studying mathematics, adult learners enrich their vision of the world and become empowered.

This program is aimed at helping adult learners develop three closely related and equally important subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

1.2 Approach to the Subject

One of the main objectives of mathematics education is to help adult learners solve different types of situational problems by teaching them how to approach a problem with intellectual rigour and self-confidence, among other things. Mathematics is not reserved for a select few and can be useful to all; thus, it has a distinct cultural dimension. Mathematics helps to develop skills that involve using various methods to solve simple and complex problems. The program of study is centred on the competencies needed to be able to do this. It is also critical that adult learners acquire the necessary knowledge in the branches of mathematics covered in the program (i.e. algebra, geometry, statistics and probability) to be able to choose and implement the solutions appropriate to the given situational problems. To enable adult learners to appreciate mathematics and grasp its scope, the program ensures that the solutions they implement can be applied in many other situations.

1.3 Connections Between the Subject and the Other Elements of the Diversified Basic Education Program

The components of the Diversified Basic Education Program, such as the broad areas of learning, the cross-curricular competencies and the other subject areas enrich the Mathematics program through the many connections they allow adult learners to make.

1.3.1 Connections With the Broad Areas of Learning

The educational aims and focuses of development of the five broad areas of learning provide the context for developing learning situations that promote the development of subject-specific and cross-curricular competencies. These in turn make it possible to relate academic learning to the concerns of adult learners. Because of the diversity and ubiquity of mathematics in everyday life, all of the broad areas of learning are addressed, and connections are made with each one of them.

Health and Well-Being

The adoption of healthy lifestyle habits has a definite impact on health and well-being. Adult learners can make the most of their subject-specific competencies by developing models and processing data related to health. In this way, they can predict the impact of certain decisions when examining issues relating to proper nutrition, sexuality, behaviours or lifestyle habits.

In the *Data Collection* course, adult learners who are concerned with nutrition could make connections or establish correlations between cholesterol levels and cardiovascular diseases, fats and calories, and so on. Knowing about such relationships may encourage them to make wiser nutritional choices.

Career Planning and Entrepreneurship

In Secondary III, adult learners are required to reflect on their preferences, interests and aptitudes. The program of study encourages teachers to propose learning situations in which adult learners can become aware of their talents, qualities and aspirations. The decisions that adult learners make at the end of this first year may help them choose the Secondary IV and Secondary V option best suited to their future career path. Mathematics education provides adult learners with the tools they need to explore various career possibilities.

For example, in drawing up plans according to different perspectives in a Geometric Representation class, adult learners will realize on their own whether or not they have good spatial perception. Ability in geometry will provide them with attractive career prospects in occupations that involve spatial representation or the construction of objects (e.g. mechanical drafting or machining).

Environmental Awareness and Consumer Rights and Responsibilities

Mathematics provides adult learners with a number of resources for gaining some perspective on their relationship to the environment and consumer society. By developing their ability to generalize and to establish various relationships between variables (e.g. cause and effect, dependency, chance), adult learners will have a better grasp of interactions between the environment and human activity. The work they do in learning situations that involve personal finances and business plans equips them to make informed choices when it comes to making purchases and balancing their budget. Such situations also help them understand the economic repercussions of their actions and choices.

In the Algebraic Modelling course, for instance, adult learners could use a graphical or algebraic model to evaluate the efficiency of a water purification system, an exercise that would help heighten their awareness of certain environmental practices. They could use another graphical or algebraic model to represent and analyze different car leasing options, which would help them recognize the importance of the quality-price ratio.

Media Literacy

Mathematics can contribute to the development of critical, ethical and aesthetic judgment with respect to the media. When they analyze messages that use materials and codes specific to media communication, adult learners recognize the different registers of representation used, distinguish among them and gauge their appropriateness. They make sure that the mathematical information in the message is plausible. They use their number sense and their ability to analyze data to determine the intention of those conveying the message and the sources of bias that can influence their judgment. They use mathematical reasoning to determine the extent of the discrepancy between fact and opinion. Similarly, their spatial sense and knowledge of shapes, geometric figures and proportions can help them develop criteria for assessing media representations in terms of image and movement.

For example, adult learners could be asked to work with poll results published in the media during an election campaign. This would allow them to become familiar with this type of language so that they can better understand its subtleties. They could even be asked to critically assess and analyze the sampling method used by certain firms, which will help them realize that some sources of information are more reliable than others and that the media are not always careful to choose credible polling firms.

Citizenship and Community Life

Some of the activities adult learners carry out in mathematics class introduce them to ways of ensuring harmonious community life or the requirements of the democratic process. Work that calls for cooperation with their peers or teacher also teaches them to observe certain principles and rules of teamwork.

In the Geometric Representation course, adult learners could be asked to organize living space in order to accommodate a parent who can no longer live on his or her own or to provide their child with a place to do homework. Such projects carried out in Mathematics give adult learners some useful tools for managing their relationships with others.

1.3.2 Connections With the Cross-Curricular Competencies

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems related to the broad areas of learning and the subject-specific competencies. To varying degrees, the subject-specific competencies contribute to the development of the cross-curricular competencies, and vice versa.

Diversified basic education provides an opportunity for acquiring each of the cross-curricular competencies prescribed in the program of study. Described below are examples of connections that can be made.

Intellectual Competencies

Faced with a situation that can be processed mathematically, adult learners use the competency *Uses information* when they collect data, identify the relevant elements of the situation, summarize information and synthesize their knowledge. In so doing, they evaluate the validity of information according to certain criteria. It is not simply a question of finding information, but of being able to determine its value, organize it, and distinguish between essential and nonessential items.

All three subject-specific competencies (*Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*) are used in the problem-solving process. These subject-specific competencies share several strategic elements with the cross-curricular competency *Solves problems* and reflect a similar approach to asking questions and examining solutions.

Adult learners must *exercise their critical judgment* to make connections between different statistical analyses concerning a specific subject. Was the question biased? Was an appropriate type of representation used? They can study the sampling method and determine whether or not there is a source of bias. They analyze the way the situation was approached and assess the method used to draw conclusions.

Adult learners *use creativity* whenever they exercise the subject-specific competencies. In looking for strategies to solve situational problems, they consider several possible solutions, explore various models, use their intuition, juggle ideas, try out new approaches and express their perceptions in new ways. Creativity has less to do with the quantity of new resources or knowledge acquired than with the manner in which these are used.

Methodological Competencies

Being able to use the subject-specific competencies in a learning situation involves *adopting effective work methods* that combine rigour and flexibility. Whether they are using strategies to solve a situational problem or using mathematical reasoning, adult learners must structure their thinking and organize their approach. When outlining their solution and explaining their reasoning, they must use appropriate registers of representation and observe the rules associated with the different ways they may choose to present their work. By mastering mathematical processes and strategies, adult learners can make connections between the work methods they need to develop in mathematics and certain aspects of this cross-curricular competency. In addition, they learn to recognize the most appropriate methods for dealing with a situation. It is also important that they become aware of their personal learning style so that they may adopt effective work methods that correspond to their needs and way of doing things.

Mathematical competencies play an important part in developing the cross-curricular competency *Uses information and communications technologies*. They help adult learners create simulations and models, make conjectures, manipulate large amounts of data and a multitude of geometric figures, and represent them in different forms. Technology is a valuable validation instrument. After making connections and formulating conjectures, adult learners use technological tools to validate their solutions.

Personal and Social Competencies

If adult learners are to be able to choose the option in which they wish to enroll in Secondary IV and V to continue their mathematics education, it is important that they know themselves and strive to make full use of their abilities. To the extent that they are able to define their aspirations and aptitudes, they will expand their knowledge within meaningful contexts that meet their expectations. This will allow them to build self-confidence and become more autonomous, thus maximizing their chances of *achieving their potential*, while recognizing the influence of others.

The scope and complexity of learning situations in mathematics require teamwork. Adult learners interact with their classmates and teacher when they express themselves and make choices or decisions. Whether they are attempting to inform, explain or convince, they must compare their perceptions with those of others. The various mathematical activities that call for cooperation give adult learners the opportunity to exchange points of view, share problem-solving strategies, explain their ideas, defend their opinions, justify their choices or convince others of the effectiveness of an approach in a given situation. This helps them develop the competency *Cooperates with others.*

Communication-Related Competency

When adult learners communicate using mathematical language, they decode, interpret or produce messages. The development of this mathematical competency is closely linked to the competency *Communicates appropriately* since adult learners are required to determine the purpose of a message, define their intentions, express their viewpoint, compare their ideas with those of others, explain a line of reasoning, discuss their problem-solving strategy, adjust their communication style according to the reactions of their audience, write proofs, summaries and syntheses, and so on. All of these situations are opportunities for adult learners to express themselves by using language accurately.

1.3.3 Connections With the Other Subject Areas

Making connections between Mathematics and other subjects enriches and contextualizes the learning situations in which adult learners will be developing their subject-specific competencies. The examples below, grouped by subject area, reflect the many different connections that can be made between mathematical knowledge and some of the other subjects.

Languages

The mastery of language and the acquisition of certain strategies related to its use help adult learners develop and use mathematical competencies. These strategies make it possible to understand the elements of a learning situation, to work out a solution and to communicate it. Furthermore, language is necessary for developing networks of mathematical cognitive resources, and for formulating and validating conjectures. Lastly, the ability to switch from one register of representation to another is dependent on language ability. These abilities reinforce each other.

Learning English and learning mathematics can be combined to develop the capacity for reasoning, the ability to present an argument and a concern for using exact language. These two subjects encourage adult learners to determine what rule applies when the observation of patterns leads to generalizations. They help to form citizens capable of abstract reasoning, critical judgment and logical expression. It is therefore crucial to ensure the quality of the language used.

Science and Technology

Science and technology deal with a variety of problems that call for the development of mathematical models. Conversely, these mathematical models help to promote an understanding of scientific phenomena. When processing observed or collected data, students use mathematical reasoning while exercising their scientific and technological competencies. Solving a situational problem in mathematics and seeking answers or solutions to scientific or technological problems involve similar processes (e.g. decoding problems, creating models, and developing and validating solutions).

Social Sciences

In History of Québec and Canada, adult learners use statistics and financial mathematics to analyze situational problems and provide a solid basis for their judgments. The use of cultural references with a historical connotation may help students make connections between the two subjects. For example, in situating mathematical concepts and processes in their historical context and identifying the needs that they addressed, students become aware of the social phenomena associated with different eras and come to understand the social dimension of mathematical knowledge.

Chapter 2

Pedagogical Context

2.1 Learning Situations

Many factors have an impact on the quality of learning and make the classroom a place where adult learners are encouraged to participate actively in the learning process and to make use of their curiosity, creativity, intellectual skills and autonomy.

A learning situation is a set of conditions proposed by the teacher with a view to helping adults learn. It must involve an issue of interest to adult learners, one or more complex tasks or learning activities aimed at helping adults acquire mathematical knowledge associated with each of the courses, and the development or application of mathematical and cross-curricular competencies. Learning situations must be rich in mathematical content, while equipping adults to adapt to everyday situations: the closer a learning situation is to real life, the more likely it is to interest adult learners, who will recognize that the learning serves a purpose beyond the school walls.

Because they can be used as a vehicle for a wealth and diversity of learning, situational problems are an excellent tool for mathematics teachers. A situational problem is a *complex task* that cannot be effectively completed without the acquisition of certain specific learning. Its purpose is to guide adult learners in their progress between a situation in which they are having trouble adapting and one they can resolve because of the learning they have acquired. If the proposed learning situation is really a problem, adult learners must be incapable of solving it without learning new concepts or developing problem-solving strategies. The structure of the situation takes into account adult learners' motivations, and its level of difficulty is adapted to their competencies and skills.

A situational problem can be complex and open-ended, but it must first and foremost be meaningful. A situational problem is meaningful if it deals with the concerns of adult learners, stimulates their curiosity and makes them think and learn. It is complex if it involves using more than one subject-specific and cross-curricular competency, poses an intellectual challenge, gives rise to a cognitive conflict and requires the mobilization of a variety of resources. Lastly, it is open-ended if it has more than one solution. It could have no solution, one solution or several solutions.

While situational problems challenge adult learners, the teacher helps them face that challenge. Adult learners examine their actions, a process that sheds light on their strengths and difficulties, and that encourages them to make adjustments. This can only be done in an atmosphere of trust, and the teacher must adapt to adult learners' needs in order to be able to provide the help they need to achieve tangible results.

This type of personalized support is consistent with the *Policy on the Evaluation of Learning*, and is aimed at helping adults progress in their learning. Thus, the teacher encourages adult learners to evaluate their work and to explain their process based on the tools and evaluation criteria provided. By evaluating the work of their peers and comparing their judgment with that of their classmates and teacher, they can hone their judgment and actively seek ways to improve their performance.

Adult learners are encouraged to draw on their experiences and to build on them through the acquisition of new mathematical knowledge so that they can deal effectively with situational problems. They are required to explore, construct, broaden, deepen, apply and integrate their knowledge of the concepts and processes covered in the different courses. They will then mobilize the resources associated with one of the subject-specific competencies. An example is provided for each course to support teachers in the development of learning situations.

2.2 Families of Learning Situations

Families of learning situations consist of groups of situations applicable to a given course that share the same characteristics, deal with the same issues, or have other points in common. Adult learners are invited to solve real-life situational problems, while constructing their mathematical knowledge and developing the subject-specific competencies.

Throughout their Secondary III, IV and V mathematics education, adult learners will explore the prescribed families of learning situations briefly explained and illustrated in the table below.

MEASUREMENT AND SPATIAL REPRESENTATION	
• This family consists of situational problems that involve providing a geometric representation of an object, a physical space, a transformation or a geometric locus. • These situations are aimed at helping adult learners develop spatial representation skills.	
For example:	Adult learners draw up a list of constraints related to the situation by consulting Web sites as needed.
RELATIONSHIP BETWEEN QUANTITIES	
• This family consists of situational problems that can be represented using a graphical or algebraic model that expresses a relation or a dependency relationship between quantities. • Representation sometimes involves a functional model.	
For example:	Adult learners select relevant information to determine the relationship between the amount set aside to pay back a debt (dependent variable) and the interest rate (independent variable).
PROCESSING DATA	
• This family consists of situational problems in which adult learners develop the ability to collect data or compare collections of data. • They may be required to interpret the data derived from a statistical study or a random experiment. • They may deal with contexts that involve making a social choice, meaning that they must take a position on the basis of statistical or probability data.	
For example:	Adult learners identify relevant information that can be presented verbally, graphically or in a table of values or a diagram

OPTIMIZING SOLUTIONS	
• This family consists of situational problems that require that adult learners optimize situations by means of linear programming or graphs. • These problems may involve maximizing a profit, a process, or a number of objects or people, or they may involve minimizing costs or losses. • Adult learners may also be required to minimize distances travelled, find a critical path or draw a diagram of an optimal network.	
For example:	Adult learners compare their solution and results with those of others in order to identify the strengths and weaknesses of the various models.

2.3 Educational Resources

The Mathematics program has adult learners rely on their experience and on various resources to help them develop the different competencies. “Internal,” or personal, resources include knowledge, prior experience (concepts, cultural references, competencies), attitudes and learning strategies, all of which constitute a person’s mathematical literacy. Adult learners also have access to “external,” or human and material, resources.

The situational problems proposed by the teacher encourage adult learners to use a variety of processes and learning strategies, to discover and learn more about new resources, and to choose and effectively mobilize the necessary internal and external resources.

The following are some examples of human and material resources adult learners may find useful.

Human resources

- *Academic*: teacher, peers, resource people, etc.
- *Community*: resource people, etc.

Material resources

- *Documentary*: library, data sheets, textbooks, etc.
- *Media*: television shows, scientific journals, etc.
- *Objects*: geometry blocks, algebra tiles, graph or dot paper, geometry set, calculator, objects related to the fields of health care, the arts, construction, etc.
- *Technology*: geometry or other software, stopwatch, sensors and probes, electrical circuits, Web sites, etc.

A hands-on approach is of great importance in the construction of mathematical knowledge. The frequent use of concrete materials constitutes an important tool for learning mathematics. These materials can promote or facilitate exploration, or lead to the formulation of conjectures or a flash of intuition.

Although technological tools are extremely useful, they cannot replace intellectual activity. Adult learners can nonetheless use these valuable tools to validate the steps in a solution and to acquire learning. In this way, they can devote themselves to meaningful activities, use their mental computation skills to make approximations and expand their knowledge of mathematics.

Chapter 3

Subject-Specific Competencies

3.1 How the Subject-Specific Competencies Work Together

The aim of this program is to develop three subject-specific competencies that complement one another. These competencies are compulsory and foster the ability to transfer learning, which involves developing the capacity to effectively reuse the resources constructed in previous situations by adapting them to new contexts. To develop these competencies, adult learners must mobilize and combine learning strategies, concepts and cultural references in different contexts.

Competency 1: *Uses strategies to solve situational problems*

Competency 2: *Uses mathematical reasoning*

Competency 3: *Communicates by using mathematical language*

The Mathematics program fosters the development of each of these competencies by taking into account three aspects: acting in context, the mobilization of resources and reflection.

Acting in context

A competency is demonstrated through appropriate action in a given context and first and foremost through adult learners' ability to correctly represent the problems assigned to them. They must take into account any constraints that exist in the context, adjust their actions so as to consider different solutions to the problem at hand, and then propose the one that seems the best suited to the situation.

Mobilization of resources

Applying a competency involves mobilizing a set of diversified and organized resources. Some of these resources, such as knowledge and strategies, are specific to the individual. Development of a competency centres on the effective acquisition and organization of resources and the ability to arrange and combine them in different ways. Other resources include knowledge from other sources, instruments and technologies, as well as the possibility of calling on expert advice.

Reflection

A competency involves adult learners' ability to call into question their ways of doing things and to find the most effective way to proceed. They can, of course, change their approach along the way. Adult learners make sure that they are consistent in the way they carry out each step of the problem-solving process. Lastly, reflection refers to adult learners' ability to explain the solution they have chosen to use. They improve their ability to show how they went about solving a problem so that they themselves and others are convinced of the value of what they are doing. Throughout the competency development process, adult learners become increasingly adept at regulating their actions by examining how they deal with different situations.

The three subject-specific competencies of the Mathematics program are used in each of the Secondary III, IV and V courses. It should be noted that competency development is nonlinear because adult learners must go back and forth among the concepts and processes they have acquired. Each of the three subject-specific competencies is developed in relation to the other two, in the same way that the development of mathematical knowledge is closely related to the development of all three competencies. It is preferable to alternate between simple and complex situational problems. Regardless of the competency targeted, the situational problems must take into account the following factors:

- adult learners' familiarity with the context
- the scope of the mathematical concepts and processes involved
- translations from one register of representation to another
- the existence of intradisciplinary or interdisciplinary links
- the degree of autonomy required of adult learners to carry out a task

3.2 Competency 1: Uses strategies to solve situational problems

3.2.1 Focus of the Competency

To deal effectively with a situational problem, it is essential that adult learners adopt strategic behaviours. Such behaviours involve making informed choices associated with a set of processes in order to achieve a specific goal. Adult learners who use strategies competently to solve situational problems go beyond the automatic or routine application of rules and procedures. For example, they could decide to estimate a result in order to verify the consistency of the different steps in their solution, not because they are required to do so, but because this seems to be the appropriate thing to do at that point.

In dealing with a situational problem, adult learners use different strategies to properly define the problem. After they represent the problem, they search for possible solutions and can then use tables, diagrams, concrete materials or brainstorming techniques. Then, by determining the best relationship between the constraints and the possible consequences, adult learners choose the most suitable solution. They then implement the solution by using various strategies such as referring to the solutions of similar problems or breaking down a problem into simpler subproblems. Adult learners then choose the validation method they consider the most appropriate. They can check their results by using examples or counterexamples or by comparing their results with those of others.

Together with the competencies *Uses mathematical reasoning* and *Communicates by using mathematical language*, the competency *Uses strategies to solve situational problems* helps adult learners solve situational problems, which is the very essence of all mathematical activity.

3.2.2 Key Features and Manifestations of the Competency

Uses strategies to solve situational problems

Key Features	Manifestations
1. Defines the problem	• Reformulates the situational problem in his/her own words • Identifies the task to be carried out • Represents the situational problem mentally or in writing • Determines the key elements to be considered and the obstacles to be overcome • Selects observation techniques or tools
2. Searches for possible solutions	• Makes connections • Uses lists, tables, diagrams, concrete materials or drawings • Refers to the solution of a similar situational problem • Uses brainstorming techniques
3. Chooses a solution	• Takes the constraints into account • Takes the consequences into account • Takes his/her aptitudes into account • Determines the best relationship between the constraints and the consequences
4. Implements the solution	• Proceeds by trial and error • Reviews his/her work • Refers to the solutions of a similar situational problem • Breaks down a complex situational problem into subproblems • Simplifies the situational problem • Establishes a plan of action • Carries out the plan of action
5. Validates the solution	• Checks his/her solution by using examples or counterexamples • Compares his/her results to the expected results • Compares his/her solution and results to those of others • Ensures that his/her solution makes sense

3.2.3 Development of the Competency

The five key features of the first subject-specific competency, *Uses strategies to solve situational problems*, represent the steps that adult learners carry out when they exercise this competency. Each key feature is defined in terms of manifestations or actions.

This subject-specific competency is associated with certain parameters specific to learning situations, namely:

- the strategies to be used in drawing up, implementing and validating a solution plan
- the number of constraints, quantity of data or number of variables involved
- the level of abstraction required in order to understand the situational problem
- the nature and form of the expected or potential result
- the number and type of steps involved in working out the solution
- the nature of the connections that must be made between the different branches of mathematics or between the concepts and processes within the same branch
- the specific aspects of the required models

By the end of Secondary V, adult learners in all three options are able to use various strategies to solve situational problems. They are able to:

- use various strategies to represent a situational problem
- use more refined techniques to find possible solutions
- choose and work out a solution
- validate the solution

While adult learners may work with one or more branches of mathematics, their main task is to solve situational problems. Their focus is choosing appropriate strategies to deal with the problem.

3.3 Competency 2: Uses mathematical reasoning

3.3.1 Focus of the Competency

Reasoning is an intellectual activity that helps us decide how specifically to deal with a situation. When they use mathematical reasoning, adult learners apprehend a situation, orient their actions and organize their thinking using inductive and deductive reasoning. Inductive reasoning involves deriving laws, rules and properties on the basis of observations. Deductive reasoning involves concluding that a proposition is true based on accepted or probable premises.

Adult learners construct a line of reasoning by first exploring a situational problem. This process enables them to determine what they are looking for in order to be able to make a conjecture. They then draw a conclusion: they validate their conjecture and try to generalize it. In applying their line of reasoning, they must make relevant connections between various items of knowledge in order to build a network of mathematical cognitive resources. They can do this by referring to similar situational problems and by using analogy, comparison, classification, refutation by means of counterexamples, and different registers of representation. These connections form the foundation that supports each step of their line of reasoning.

Logical reasoning helps adult learners organize their thinking. The competency *Uses mathematical reasoning* can be applied to several areas of life. It enables adult learners to analyze and better understand their environment or a theoretical model, to construct an argument, to further develop certain ideas or to make informed decisions.

Generally speaking, the process that leads to a conclusion is not strictly linear: it involves doubts, dead ends, contradictions and a certain amount of backtracking. Whether or not they are conscious of it, adult learners use different types of reasoning and adjust their process as needed. The verbal or written expression of a solution is the most accessible aspect of their reasoning and provides the necessary evidence of their work.

Together with the competencies *Uses strategies to solve situational problems* and *Communicates by using mathematical language*, the competency *Uses mathematical reasoning* helps adult learners solve situational problems, which is the very essence of all mathematical activity.

3.3.2 Key Features and Manifestations of the Competency

Uses mathematical reasoning

Key Features	Manifestations
1. Explores the situational problem	• Examines the situational problem • Describes the characteristics of the situational problem • Asks questions about the situational problem • Gathers information about the situational problem
2. Makes a conjecture	• Proposes probable or plausible ideas • Predicts the implications of the ideas proposed • Uses examples to find invariants • Makes a conjecture
3. Constructs and uses networks of mathematical cognitive resources	• Establishes organized and functional relationships between different types of knowledge (by associating, classifying, ordering, etc.) • Uses different registers of representation • Selects relevant information • Refers to similar situational problems • Finds additional information
4. Draws a conclusion	• Finds examples to verify the conjecture • Finds counterexamples to clarify, adjust or refute the conjecture • Generalizes by deriving laws, rules or properties • Deduces a proposition

3.3.3 Development of the Competency

The four key features of the second subject-specific competency, *Uses mathematical reasoning*, represent the steps that adult learners carry out when they exercise this competency to correctly solve situational problems. Each key feature is defined in terms of manifestations or actions.

This subject-specific competency is associated with certain parameters specific to learning situations, namely:

- adult learners' familiarity with the types of reasoning they must use
- the nature of the data (explicit, implicit or unknown) on the basis of which adult learners must identify the essential, necessary or sufficient information
- the scope of the conjecture formulated or to be formulated
- the type of proof required

- the number and type of steps involved in validating a conjecture, coming to a conclusion or making a decision
- the nature of the connections or relations that link the various branches of mathematics or the different networks of mathematical cognitive resources specific to a given branch
- the level of abstraction involved in the mental and operational representation of the mathematical cognitive resources used and in translations between different registers of representation

By the end of Secondary V, adult learners in all three options are able to use mathematical reasoning. They are able to:

- understand situational problems
- use the appropriate mathematical concepts and processes
- make conjectures
- confirm or refute their conjectures by using various types of reasoning
- draw conclusions in order to make a generalization, establish a rule or a law, or deduce a proposition

While adult learners may work with one or more branches of mathematics, their main task is to solve situational problems. They emphasize mathematical reasoning throughout the task to be carried out.

3.4 Competency 3: Communicates by using mathematical language

3.4.1 Focus of the Competency

Communicating using mathematical language requires that adult learners understand its specific elements and be able to organize them appropriately in order to explain, produce and convey messages, and regulate mathematical communication. In addition to the clarity and concision expected in any message, precision and rigour are also required to develop this competency.

Communication that involves mathematical language plays an increasingly important role in our society. It is characterized by the fact that it employs everyday language to convey very specific ideas. In fact, many words in everyday language do not have the same meaning in a mathematical context. Symbols, notation, rules and registers of representation are all indispensable elements for interpreting or producing a mathematical message. This is why adult learners must learn certain basic principles if they want to use mathematical language effectively.

In order to use mathematical language to communicate, adult learners must first decode the symbols, terms, notation, knowledge, rules and codes that give a message its specific meaning. These elements are essential for interpreting the message correctly. By identifying the connections between its various elements, adult learners are able to identify the overall meaning of the message. In turn, they organize a message on the basis of their interpretation and observe the conventions of the language involved. The message thus produced or interpreted can take the form of an equation, a graph, a table, a proof or the solution to a problem. To interpret and produce this message, adult learners may use rulers or compasses or different technological tools such as calculators and computers.

Among other things, this competency is useful for interpreting statistical tables in newspapers, reading nutritional data on labels, understanding the information that goes into completing a tax form, presenting conclusions drawn after reading a document or justifying an opinion about a scientific article. The competency increases adult learners' ability to make the effort to correctly understand and convey these messages, while giving them a better comprehension of mathematical knowledge and conventions. Consequently, it helps adult learners better deal with situational problems in their personal lives, at school or at work.

Together with the competencies *Uses strategies to solve situational problems* and *Uses mathematical reasoning*, the competency *Communicates by using mathematical language* helps adult learners solve situational problems, which is the very essence of all mathematical activity.

3.4.2 Key Features and Manifestations of the Competency

Communicates by using mathematical language

Key Features	Manifestations
1. Decodes the elements of mathematical language	• Recognizes codes and rules • Recognizes the meaning of symbols, terms and notation • Distinguishes between the mathematical and everyday meaning of various terms • Consults different sources of information
2. Interprets a mathematical message	• Makes connections between the elements of the message • Distinguishes between elements that are relevant and those that are not • Identifies the key elements • Identifies the subject of the message • Determines the overall meaning of the situational problem • Associates images, objects or knowledge with mathematical terms and symbols • Switches from one register of representation to another • Verifies his/her understanding of the message
3. Produces a mathematical message	• Determines the subject of the message • Observes codes and rules • Uses the symbols, terms and notation in accordance with their meaning • Uses a register of representation • Organizes the message • Consults different sources of information

3.4.3 Development of the Competency

The three key features of the third subject-specific competency, *Communicates by using mathematical language*, represent the steps that adult learners carry out when they exercise this competency. Each key feature is defined in terms of manifestations or actions.

The subject-specific competency is associated with certain parameters specific to learning situations, namely:

- the presentation of the information to be explored, decoded and interpreted
- the terminology, symbolism and knowledge used in the wording of the situational problem

- the scope of the rules of conformity, transformation or conversion in the registers of representation used in the situational problem
- the type of work involved
- the expected level of quality of the message to be produced
- the purpose of the message (to describe, inform, explain, convince)
- the number and type of steps involved in producing, organizing and structuring the message

By the end of Secondary V, adult learners in all three options are able to communicate using mathematical language. They are able to:

- produce oral and written messages that are clear, coherent and adapted to the situational problem and the audience
- interpret and analyze a mathematical message
- critically assess and improve the mathematical message so that it meets the requirements of the situational problem
- use their ability to decode, describe, translate, transpose, represent and schematize, taking into account the subject and purpose of the message

While adult learners may work with one or more branches of mathematics, their main task is to solve situational problems. They use appropriate mathematical language, drawing on different registers of representation to demonstrate their understanding of a concept or message.

3.5 Process and Strategies

When solving a problem, adult learners naturally draw on what they know about the problem, even though this may not be sufficient. Among other things, they need effective strategies to guide them, strategies that they adapt to the situations presented. Strategies enable them to work out a solution to the situational problem. The ability to use effective strategies empowers adult learners. Effective strategies enable them to use rigorous mathematical reasoning and to communicate using the language of this discipline, observing its codes and conventions. Adult learners succeed in solving situational problems through an integrated use of the three subject-specific competencies, the related concepts and processes, as well as the associated cross-curricular competencies.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

Problem solving is a dynamic process that involves often going back and forth among these phases, which are not necessarily carried out in the order indicated above. It also calls for anticipation, discernment and critical judgment. Adult learners may have to go back to any of these phases at any time in order to validate their approach or overcome an obstacle.

In the courses in the Mathematics program, adult learners are encouraged to apply strategies that have already proved effective. They select and organize those strategies they consider relevant to the task at hand and determine ways to apply them. The table below gives an overview of each of the phases involved in solving a situational problem, as well as a few examples of possible strategies,[1] with a view to helping adult learners understand the scope of these strategies.

[1] Examples of general and specific strategies are given in each course in the program.

<table>
<tr><th colspan="2">PROCESS AND STRATEGIES</th></tr>
<tr><th colspan="2">REPRESENTATION</th></tr>
<tr><td colspan="2">- Adult learners examine the situational problem.
- They use observation strategies to properly represent it.</td></tr>
<tr><td>Examples of strategies</td><td>• Exploring the situation carefully and methodically
• Gathering all the relevant information
• Noting, comparing, classifying and retaining essential information
• Eliminating superfluous information
• Asking questions about the situation</td></tr>
<tr><th colspan="2">PLANNING</th></tr>
<tr><td colspan="2">- Adult learners set priorities and identify the relevant resources.
- They look for ways of approaching the problem and choose those that seem the most efficient.
- They take the time to clarify their ideas and then develop a plan.</td></tr>
<tr><td>Examples of strategies</td><td>• Using brainstorming techniques to find possible solutions
• Dividing the situational problem into subproblems
• Finding a rule that reflects the best relationship between the constraints and possible consequences of the situational problem</td></tr>
<tr><th colspan="2">ACTIVATION</th></tr>
<tr><td colspan="2">- Adult learners follow the plan they have worked out and take into account the constraints involved.
- They mobilize a variety of resources to verify, clarify, adjust or refute their conjectures.</td></tr>
<tr><td>Examples of strategies</td><td>• Referring to previously solved situational problems
• Using technology to analyze a graph or a figure
• Comparing representations in order to identify patterns and differences
• Using approximation to predict a result</td></tr>
<tr><th colspan="2">REFLECTION</th></tr>
<tr><td colspan="2">- Adult learners use a reflective approach when solving situational problems.
- They make adjustments at the different phases of the problem-solving process.
- Reflection helps them hone their ability to use exact mathematical language.</td></tr>
<tr><td>Examples of strategies</td><td>• Checking their solution
• Determining whether there is a more effective approach
• Comparing a result with the anticipated result
• Comparing their solution with that of others
• Using a calculator to validate a conjecture</td></tr>
</table>

Chapter 4

Subject-Specific Content

4.1 Knowledge

The subject-specific content of the Mathematics program in Diversified Basic Education groups together resources that are essential to using and developing the competencies associated with this discipline. The mathematics courses in this program can be divided into four groups:

1. *Algebraic and Graphical Modelling*
2. *Data Collection* or *Vote Distribution Models and Random Experiments*
3. *Geometric Representation*
4. *Optimization*

These groups are related to the various branches of mathematics. To give a more specific focus to how adult learners must deal mathematically with the various learning situations in a course, the program prescribes the acquisition of integrative processes. These processes foster the integration of the mathematical concepts and processes covered in the course, as well as the three subject-specific competencies, and are listed under Subject-Specific Content in the Knowledge section. For each group of courses, the processes are as follows.

1. *Algebraic and Graphical Modelling*
   - Using an algebraic or graphical model (a functional model in some courses) to represent a situation
   - Interpolating or extrapolating from an algebraic or graphical model
   - Using an algebraic or graphical model (a functional model in some courses) to generalize a set of situations

2. *Data Collection* or *Vote Distribution Models and Random Experiments*

   Data Collection
   - Collecting data
   - Comparing collections of data
   - Interpreting data resulting from an experiment

   Vote Distribution Models and Random Experiments
   - Interpreting data resulting from a random experiment
   - Making decisions concerning social choices

3. *Geometric Representation*

Secondary III and IV

- Describing an object or a physical space and representing it in two or three dimensions
- Organizing a physical space

Secondary V

- Describing an object or a physical space and representing it in two or three dimensions
- Describing geometric loci and representing them algebraically and graphically
- Using vectors to generalize geometry principles

4. *Optimization*

- Optimizing a situation using linear programming
- Optimizing a situation using graph theory
- Optimizing space when designing or using three-dimensional objects

The prescribed mathematical knowledge for each course consists of the concepts under study and the actions needed to construct, develop and use them. The tables outline only the new concepts and processes to be introduced in the courses. However, it goes without saying that the learning process does not simply involve this new content, since the use of previously acquired knowledge is essential.

The table of mathematical knowledge for each course can be viewed in a linear fashion owing to the sequence of prerequisites or as a network of connections among the four groups of courses that make up the Mathematics program. The elements of the learning content should be regarded as symbiotic because they are interconnected.

Chapter 6 of the program of study outlines and clarifies the mathematical knowledge specific to each course. This prescribed knowledge makes up the resources to be mastered and used at the appropriate moment to solve situational problems.

The following tables provide an overview of the concepts and processes covered in each group of courses.

4.1.1 Prescribed Knowledge in the *Algebraic and Graphical Modelling* Courses

Secondary III Course

MTH-3051-2
Algebraic and Graphical Modelling

Inequality

- Inequality relation
- Solving first-degree equations and inequalities in one variable

Relation

- Observing, describing, interpreting and representing the dependency relationship between the variables of a situation
- Functions and inverse functions (constant, linear, rational, piecewise)
- Drawing a scatter plot representing an experiment or a statistical study
- Representing and interpreting the inverse of a linear or reciprocal function
- Determining the rule of correspondence
- Describing the properties of a function in context
- Providing a qualitative description of how the graph is affected by a change in the value of a parameter of a linear function

System

- Solving systems of first-degree equations in two variables

Secondary IV Course **MTH-4151-1** **Algebraic and Graphical Modelling in a General Context**	**Secondary IV Course** **MTH-4261-2** **Algebraic and Graphical Modelling in an Applied Context 1**	**Secondary IV Course** **MTH-4271-2** **Algebraic and Graphical Modelling in a Fundamental Context 1**
	Numerical and algebraic expressions • Solving equations and inequalities in one variable: second-degree, square root, exponential, logarithmic (including the properties of radicals, exponents and logarithms) • Operations on numerical and algebraic expressions (multiplying and dividing polynomials, simplifying rational expressions, numbers expressed using rational exponents, radicals and the powers of base 2 and 10) • Constructing and interpreting tables of values consisting of positive rational numbers written in base 2 and base 10 (exponential and logarithmic forms) • Expanding, factoring (factoring by grouping and using second-degree algebraic identities, including the perfect square trinomial and the difference of two squares)	**Algebraic expressions** • Solving first-degree equations and inequalities in one or two variables and second-degree equations and inequalities in one variable • Operations on algebraic expressions (multiplying and dividing polynomials, simplifying rational expressions) • Expanding, simplifying or substituting expressions using significant algebraic identities (perfect square trinomial and difference of two squares) • Factoring trinomials using roots • Completing the square

Secondary IV Course **MTH-4151-1** **Algebraic and Graphical Modelling in a General Context**	**Secondary IV Course** **MTH-4261-2** **Algebraic and Graphical Modelling in an Applied Context 1**	**Secondary IV Course** **MTH-4271-2** **Algebraic and Graphical Modelling in a Fundamental Context 1**
Relation, function and inverse • Experimenting with real functions as well as observing, interpreting, describing and representing them (second-degree polynomial, exponential, periodic, step, piecewise) • Describing and interpreting the properties of real functions using a graph	**Relation, function and inverse** • Experimenting with real functions as well as observing, interpreting, describing and representing them (second-degree polynomial, exponential, square root, periodic, step, logarithmic, greatest integer, piecewise) • Describing and interpreting the properties of real functions • Interpreting the multiplicative parameter • Solving and graphing first-degree inequalities in two variables	**Relation and function** • Experimenting with real functions as well as observing, interpreting, describing and representing them (second-degree polynomial, step, greatest integer) • Describing and interpreting the properties of real functions • Interpreting the multiplicative and additive parameters • Switching from one form to another in writing second-degree polynomial functions
System • Representing a situation using straight lines • Solving systems of first-degree equations in two variables	**System** • Representing a situation using straight lines or half-planes • Solving systems of first-degree equations in two variables	**System** • Representing a situation using straight lines or half-planes • Solving systems of first-degree equations in two variables • Solving systems composed of a first-degree equation and a second-degree equation in two variables

Secondary V Course MTH-5151-1 Algebraic and Graphical Modelling in a General Context 2	Secondary V Course MTH-5161-2 Algebraic and Graphical Modelling in an Applied Context 2	Secondary V Course MTH-5171-2 Algebraic and Graphical Modelling in a Fundamental Context 2
Financial mathematics • Calculating, interpreting and analyzing financial situations ○ Simple and compound interest ○ Interest period ○ Discounting ○ Compounding		
Numerical and algebraic expressions • Real numbers (definition and change of base): ○ powers ○ logarithms	**Numerical and algebraic expressions** • Completing the square • Dividing second-degree polynomials in one or two variables by a first-degree binomial	**Numerical and algebraic expressions** • Real numbers: ○ absolute values ○ radicals ○ exponents ○ logarithms • Manipulating arithmetic and algebraic expressions: ○ laws of exponents ○ properties of radicals ○ equivalence between logarithmic and exponential expressions

Secondary V Course MTH-5151-1 Algebraic and Graphical Modelling in a General Context 2	Secondary V Course MTH-5161-2 Algebraic and Graphical Modelling in an Applied Context 2	Secondary V Course MTH-5171-2 Algebraic and Graphical Modelling in a Fundamental Context 2
Relation, function and inverse • Solving exponential or logarithmic equations using a change of base, if necessary	**Relation, function and inverse** • Experimenting with real functions and their inverse as well as observing, interpreting, describing and representing them: ○ second-degree polynomial ○ exponential ○ logarithmic ○ rational ○ square root ○ sinusoidal ○ tangent ○ greatest integer • Describing and interpreting the properties of real functions • Operations on functions	**Relation, function and inverse** • Experimenting with real functions and their inverse as well as observing, interpreting, describing and representing them: ○ exponential ○ logarithmic ○ rational ○ square root ○ sinusoidal ○ tangent ○ piecewise ○ absolute value • Describing and interpreting the properties of real functions • Operations on functions • Finding the rule of a function or its inverse, depending on the context

Secondary V Course MTH-5151-1 Algebraic and Graphical Modelling in a General Context 2	Secondary V Course MTH-5161-2 Algebraic and Graphical Modelling in an Applied Context 2	Secondary V Course MTH-5171-2 Algebraic and Graphical Modelling in a Fundamental Context 2
	Relation, function and inverse (cont.) • Solving equations and inequalities in one variable ○ first-degree trigonometric containing either a sine, a cosine or a tangent ○ second-degree ○ square root ○ rational ○ exponential and logarithmic • Interpreting additive parameters in the different registers of representation	**Relation, function and inverse (cont.)** • Determining the type of dependency relationship using the curve of best fit, with or without the help of technology • Solving equations and inequalities in one variable ○ first-degree trigonometric containing either a sine, a cosine or a tangent ○ square root ○ rational ○ exponential and logarithmic ○ absolute value • Interpreting additive and multiplicative parameters in the different registers of representation
	System • Finding the graphical solution for situations consisting of systems of equations or inequalities involving different functional models	

4.1.2 Prescribed Knowledge in the *Data Collection* or *Vote Distribution Models and Random Experiments* Courses

Secondary III Course **MTH-3052-2** **Data Collection**
One-variable statistical distributions • Organizing and interpreting statistical data (sampling methods: stratified and cluster) • Constructing and interpreting distributions (condensed data, data grouped by classes) • Representing and interpreting graphs (histogram and box-and-whisker plot) • Calculating measures of central tendency (mode, median, weighted mean) and measures of dispersion (range of each part of a box-and-whisker plot and interquartile range) **Probability** • Enumerating possibilities and calculating probabilities (discrete and continuous random variables) • Representing events (tables, tree diagrams, schematic drawings, geometric figures)

Secondary IV Course MTH-4152-1 Data Collection in a General Context	Secondary IV Course MTH-4262-2 Data Collection in an Applied Context	Secondary IV Course MTH-4272-2 Data Collection in a Fundamental Context
One-variable distribution • Determining and interpreting measures of position and dispersion (percentile, mean deviation) • Representing statistical data related to a population or a sample (stem-and-leaf diagram)	**One-variable distribution** • Determining and interpreting measures of position and dispersion (mean deviation and standard deviation)	
Two-variable distribution • Constructing and interpreting two-variable distributions • Drawing a scatter plot • Representing the regression line by means of a rule or graph • Interpolating or extrapolating using the regression line • Approximating and interpreting the correlation coefficient	**Two-variable distribution** • Constructing and interpreting two-variable distributions • Drawing a scatter plot • Representing and determining the equation of the regression line or curves related to the functional models under study • Interpolating or extrapolating using the regression line • Approximating and interpreting the correlation coefficient	**Two-variable distribution** • Constructing and interpreting two-variable distributions • Drawing a scatter plot • Representing and determining the equation of the regression line • Interpolating or extrapolating using the regression line

Secondary IV Course MTH-4152-1 Data Collection in a General Context	Secondary IV Course MTH-4262-2 Data Collection in an Applied Context	Secondary IV Course MTH-4272-2 Data Collection in a Fundamental Context
Two-variable distribution (cont.) • Interpreting a correlation qualitatively and quantitatively	**Two-variable distribution (cont.)** • Interpreting a correlation qualitatively and quantitatively • Interpolating and extrapolating using the functional model best suited to the situational problem	**Two-variable distribution (cont.)** • Interpreting a correlation qualitatively and quantitatively • Interpolating and extrapolating using the functional model best suited to the situational problem
	Probability • Calculating and interpreting mathematical expectation • Calculating probabilities using statistical reports • Representing and determining conditional probability • Determining the *odds for* or the *odds against* • Changing the value of parameters or conditions • Distinguishing between mutually exclusive, nonmutually exclusive, independent and dependent events	

Secondary V Course MTH-5152-1 Vote Distribution Models and Random Experiments		
Probability • Distinguishing among theoretical, experimental and subjective probability • Distinguishing between probability and odds • Approximating and predicting results • Calculating and interpreting mathematical expectation • Calculating and interpreting conditional probability • Distinguishing between mutually exclusive and nonmutually exclusive events • Distinguishing between dependent and independent events • Representing random events • Counting and enumerating possibilities		

Secondary V Course MTH-5152-1 Vote Distribution Models and Random Experiments		
Fair distribution model • Weighted mean • Comparing and interpreting different voting procedures: ○ majority rule ○ plurality voting ○ Borda count ○ Condorcet method ○ approval voting ○ runoff method ○ proportional representation		

4.1.3 Prescribed Knowledge in the *Geometric Representation* Courses

Secondary III Course MTH-3053-2 Geometric Representation
Numerical and algebraic expressions • Manipulating rational and irrational numbers (square, square root, cube, cube root, exponential notation, radicals) • Manipulating numerical and algebraic expressions (four operations on algebraic expressions, finding the common factor, and operations on numbers expressed exponentially or in scientific notation) **Solids** • Describing, constructing and representing objects (orthogonal, parallel and central projections) • Net, projection and perspective (cavalier and axonometric perspectives) • Converting various units of measure (length, area, volume, capacity) • Finding measurements (length, lateral or total area, volume, appropriate choice of units of measure)

Secondary IV Course MTH-4153-2 **Geometric Representation in a General Context 1**	Secondary IV Course MTH-4263-2 **Geometric Representation in an Applied Context 1**	Secondary IV Course MTH-4273-2 **Geometric Representation in a Fundamental Context 1**
Metric and trigonometric relations in triangles	**Metric and trigonometric relations in triangles**	**Metric and trigonometric relations in triangles**
• Determining the slope, measurements and positions using metric and trigonometric relations in triangles:	• Determining the slope, measurements and positions using metric and trigonometric relations in triangles:	• Determining the slope, measurements and positions using metric and trigonometric relations in triangles:
○ angles of a triangle	○ angles of a triangle	○ angles in triangles or in figures that can be split into triangles
○ altitude to the hypotenuse (orthogonal projection of the legs on the hypotenuse)	○ altitude to the hypotenuse (orthogonal projection of the legs on the hypotenuse)	○ altitude to the hypotenuse (orthogonal projection of the legs on the hypotenuse)
○ sides of a triangle	○ sides of a triangle	○ sides of a triangle
○ area of a triangle and a quadrilateral	○ area of a triangle	○ area and volume of figures
○ coordinates of a point of division	○ coordinates of a point of division	
○ length of a segment	○ length of a segment	○ length of a segment resulting from an isometry or a similarity transformation
	○ perpendicular bisector of a segment	
○ distance	○ distance	○ distance
	○ areas of triangles, given the measure of an angle and the lengths of two sides or given the measures of two angles and the length of one side	

Secondary IV Course **MTH-4153-2** **Geometric Representation in a General Context 1**	**Secondary IV Course** **MTH-4263-2** **Geometric Representation in an Applied Context 1**	**Secondary IV Course** **MTH-4273-2** **Geometric Representation in a Fundamental Context 1**
Metric and trigonometric relations in triangles (cont.) • Representing and interpreting situations using triangles: ○ trigonometric ratios (sine, cosine and tangent) ○ sine law ○ Hero's formula ○ other relations in triangles, specified in the course's list of principles • Describing the properties of trigonometric ratios	**Metric and trigonometric relations in triangles (cont.)** • Representing and interpreting situations using triangles: ○ trigonometric ratios (sine, cosine and tangent) ○ other relations in triangles, specified in the course's list of principles • Describing the properties of trigonometric ratios	**Metric and trigonometric relations in triangles (cont.)** • Representing and interpreting situations using triangles: ○ trigonometric ratios (sine, cosine and tangent) ○ sine law ○ cosine law ○ other relations in triangles, specified in the course's list of principles • Justifying their solution using the properties of trigonometric ratios
Similar and congruent triangles • Determining the minimum conditions required to conclude that triangles are congruent or similar	**Similar and congruent triangles** • Determining the minimum conditions required to conclude that triangles are congruent or similar	**Similar and congruent triangles** • Determining the minimum conditions required to conclude that triangles are congruent or similar

		Secondary IV Course MTH-4273-2 Geometric Representation in a Fundamental Context 1
		Equivalent figures (plane figures or solids) • Finding measurements: ○ lengths of segments ○ areas ○ volumes ○ capacities *These unknown measurements are found by applying the properties of congruent, similar or equivalent figures.*

	Secondary V Course MTH-5163-2 Geometric Representation in an Applied Context 2	Secondary V Course MTH-5173-2 Geometric Representation in a Fundamental Context 2
	Geometric transformations • Representing (using algebraic rules and matrices) and describing geometric transformations	

	Trigonometric relations in triangles
	
	• Representing and interpreting situations using triangles
	  ○ sine law
	  ○ cosine law
	  ○ other relations in triangles, specified in the course's list of principles	

	Secondary V Course **MTH-5163-2** **Geometric Representation in an Applied Context 2**	**Secondary V Course** **MTH-5173-2** **Geometric Representation in a Fundamental Context 2**
	Equivalent figures (plane figures or solids) • Finding measurements: ○ lengths of segments ○ areas ○ volumes ○ capacities *These unknown measurements are found by applying the properties of congruent, similar or equivalent figures.*	
	Metric relations in circles • Finding measurements: ○ arcs or angles (degrees or radians) ○ lengths (segments, chords)	

	Secondary V Course **MTH-5163-2** **Geometric Representation in an Applied Context 2**	**Secondary V Course** **MTH-5173-2** **Geometric Representation in a Fundamental Context 2**
	Standard unit circle • Finding measurements: ○ arcs or angles (radians) • Finding the coordinates of points associated with important angles $(\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{2})$	**Standard unit circle** • Finding measurements: ○ arcs or angles (radians) • Finding the coordinates of points associated with important angles $(\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{2})$
	Trigonometric identities • Manipulating simple trigonometric expressions using definitions	**Trigonometric identities** • Manipulating simple trigonometric expressions using definitions

	Secondary V Course MTH-5163-2 Geometric Representation in an Applied Context 2	Secondary V Course MTH-5173-2 Geometric Representation in a Fundamental Context 2
	Geometric loci and relative positions • Describing, representing and constructing geometric loci ○ plane loci ○ conics studied: ▪ parabola (centred at the origin and translated) ▪ circle (centred at the origin and translated) ▪ ellipse (centred at the origin and translated) ▪ hyperbole (centred at the origin and translated)	**Geometric loci:** • Describing, representing and constructing geometric geometric loci: ○ conics studied: ▪ parabola (centred at the origin and translated) ▪ circle (centred at the origin) ▪ ellipse (centred at the origin) ▪ hyperbola (centred at the origin) • Solving a system of second-degree equations with respect to conics • Determining the coordinates of points of intersection between a line and a conic or between a parabola and another conic

<table>
<tr><th></th><th>Secondary V Course
MTH-5163-2
Geometric Representation in an Applied Context 2</th><th>Secondary V Course
MTH-5173-2
Geometric Representation in a Fundamental Context 2</th></tr>
<tr><td></td><td>Vectors
Resultant and projection
Operations on vectors:
vector addition and subtraction
multiplication of a vector by a scalar
scalar product of two vectors</td><td>Vectors
Resultant and projection
Operations on vectors:
vector addition and subtraction
multiplication of a vector by a scalar
scalar product of two vectors
properties of the scalar product of two vectors
linear combination
properties of vectors
Determining the coordinates of a point of division</td></tr>
</table>

4.1.4 Prescribed Knowledge in the *Optimization* Courses

Secondary V Course MTH-5150-2 Optimization in a General Context	Secondary V Course MTH-5160-2 Optimization in an Applied Context	Secondary V Course MTH-5170-2 Optimization in a Fundamental Context
Algebraic expressions • Solving first-degree inequalities in two variables		
Linear programming • System of first-degree inequalities in two variables • Representing the constraints and the function to be optimized (objective or economic function) • Determining and interpreting the vertices and the feasible region (bounded or unbounded) • Changing the conditions associated with the situation to provide a more optimal solution	**Linear programming** • System of first-degree inequalities in two variables • Representing the constraints and the function to be optimized (objective or economic function) • Determining and interpreting the vertices and the feasible region (bounded or unbounded) • Changing the conditions associated with the situation to provide a more optimal solution	**Linear programming** • System of first-degree inequalities in two variables • Representing the constraints and the function to be optimized (objective or economic function) • Determining and interpreting the vertices and the feasible region (bounded or unbounded) • Changing the conditions associated with the situation to provide a more optimal solution
Graph • Representing and modelling a situation using a graph • Comparing different graphs		

Secondary V Course MTH-5150-2 Optimization in a General Context	Secondary V Course MTH-5160-2 Optimization in an Applied Context	Secondary V Course MTH-5170-2 Optimization in a Fundamental Context
Graph (cont.) • Finding Euler and Hamiltonian paths and circuits, a critical path, the shortest path, a tree of minimum or maximum values or the chromatic number		

Finding measurements
 	
• Equivalent figures
• Finding measurements of:
  ○ positions
  ○ angles
  ○ lengths (segments, chords)
  ○ areas
  ○ volumes
• Relations in triangles: cosine law		

4.1.5 Prescribed Knowledge in the *Optional* Courses

Secondary V Course **MTH-5154-2** **Financial Mathematics in a General Context**	**Secondary V Course** **MTH-5164-2** **Sequences and Series in an Applied Context**	
Performing financial calculations related to an investment or a loan • Determining the interest period, future value and current value • Determining the interest rate (simple, compound or equivalent)	**Arithmetic and geometric sequences** • Determining the general term, the convergence and the limits of a sequence • Experimenting, observing, interpreting, describing and representing situations using number sequences	
Analyzing a series of equal or unequal payments • Producing and analyzing a statement outlining the current situation regarding of an investment • Interpreting calculations in a financial plan	**Series** • Determining the formula, convergence and limits of a series • Experimenting, observing, interpreting, describing and representing situations using number series	
Analyzing annuities • Interpreting annuity calculations • Interpreting calculations of current value and future value		
Analyzing amortization • Interpreting amortization tables or calculations		

4.2 Cultural References

Different issues have given rise to different ideas through the ages. History and current events are replete with anecdotes that adult learners can use to develop their knowledge. To satisfy their curiosity, they may find inspiration by consulting an encyclopedia, a newspaper, a scientific magazine or an article on the Internet, and share their thoughts on the topics that concern or captivate them. They could also analyze current events from a mathematical point of view. The Mathematics program sheds new light on the past and the present and opens up possibilities in all fields of interest.

The Subject-Specific Content section found in each course described in Chapter 6 suggests various cultural references that could be used to help adult learners identify the issues that have contributed to the advancement of mathematics and situate mathematical concepts in their historical, social, economic or scientific context. It should be noted that other cultural references could prove to be just as relevant. These points of reference should enable adult learners to better appreciate the influence of mathematics in their everyday lives, recognize the needs it fulfills in society and see that an event in society can trigger a discovery in mathematics. The program of study highlights the contribution of mathematics in this regard.

Chapter 5

Organization of the Courses in the Program

The Mathematics program for Diversified Basic Education is organized in a way that takes into account the specific needs of adult learners.

5.1 A Diversified Path

In Secondary III, the program of study suggests placing adult learners in a variety of learning situations that will help them develop a thorough understanding of the characteristics and orientations of each option offered in Secondary IV and V. Adult learners choose their option for Secondary IV at the end of Secondary III. The three options for Secondary IV and V are geared to a variety of needs and are as follows: the Cultural, Social and Technical (CST) option, the Technical and Scientific (TS) option and the Science (S) option.

Adult learners must choose the option that best suits their aspirations, interests and aptitudes. In general, they remain in the same option throughout their studies. However, adult learners whose aspirations or interests change have the opportunity to choose another option at the end of any Secondary IV or V course. There are special accommodations in this regard; they are defined in an appendix at the end of this program of study. The appendix also lists the subject matter to be studied if students must upgrade their knowledge in order to enroll in a new option.

Each of the three options prepares adult learners for postsecondary studies and can also lead to trades, occupations or technical fields that can be studied in secondary school or CEGEP. The following table profiles the three options offered in the program of study.

Table of the Three Options

Cultural, Social and Technical	Technical and Scientific	Science
The *Cultural, Social and Technical* option is intended for adult learners who like to design objects and activities, develop projects or participate in making them or carrying them out. It stimulates the interest of adult learners in social causes and helps them develop their sense of initiative.	The *Technical and Scientific* option is intended for adult learners who wish to explore situations that sometimes involve both manual and intellectual work.	The *Science* option is intended for adult learners who seek to understand the origin of different phenomena and how they work, as well as to explain them and make decisions that pertain to them. Adults learn to develop formal proofs in learning situations where there is a need to confirm a truth.
The emphasis is on situations that adult learners will encounter in their personal and professional lives.	The emphasis is on case studies as well as the development of adult learners' ability to identify errors and anomalies in solutions with a view to defining the problem and taking appropriate corrective action.	The emphasis is on finding, developing and analyzing models within the context of experiments mainly related to different scientific fields.
It brings together aspects of mathematics that will help adult learners become autonomous citizens who are active and thoughtful members of society. The learning content for this option allows adult learners to build on their knowledge of basic mathematics.	It also requires that adult learners identify the mathematical concepts and processes associated with the design, operation or use of certain technical instruments.	By focusing on the properties of mathematical objects, this option places greater emphasis on adult learners' capacity for abstract thinking in that they are required to perform more complex algebraic operations. This option fosters the development of intellectual skills.
Specifically, it prepares adult learners for studies in the arts, communications, the humanities and the social sciences.	This option is especially designed to equip adult learners to work effectively in technical fields related to nutrition, biology, physics, business administration and graphic arts.	It prepares adult learners to pursue their studies in the hard sciences or perhaps to eventually specialize in research.

DISTRIBUTION OF COURSES

<table>
<tr><th colspan="2">Level</th><th>Number of courses</th><th>Total hours of instruction</th></tr>
<tr><td colspan="2">Secondary III</td><td>Three 50-hour courses</td><td>150 hours</td></tr>
<tr><td rowspan="3">Secondary IV</td><td>CST option</td><td>Two 25-hour courses
One 50-hour course</td><td>100 hours</td></tr>
<tr><td>TS option</td><td>Three 50-hour courses</td><td>150 hours</td></tr>
<tr><td>S option</td><td>Three 50-hour courses</td><td>150 hours</td></tr>
<tr><td rowspan="3">Secondary V</td><td>CST option</td><td>Two 25-hour courses
One 50-hour course</td><td>100 hours</td></tr>
<tr><td>TS option</td><td>Three 50-hour courses</td><td>150 hours</td></tr>
<tr><td>S option</td><td>Three 50-hour courses</td><td>150 hours</td></tr>
<tr><td colspan="2">Optional Secondary V subjects</td><td>Two 50-hour courses</td><td>100 hours</td></tr>
</table>

5.2 Overview of the Courses in the Program

The subject-specific content for Secondary III, IV and V has been broken down into groups of 25- and 50-hour courses. The following tables show the overall organization of the courses.

Course	Course title	Number of hours	Number of credits
MTH-3051-2	*Algebraic and Graphical Modelling*	50 hours	2 credits
MTH-3052-2	*Data Collection*	50 hours	2 credits
MTH-3053-2	*Geometric Representation*	50 hours	2 credits

Course	Course title	Number of hours	Number of credits
MTH-4151-1	*Algebraic and Graphical Modelling in a General Context 1*	25 hours	1 credit
MTH-4152-1	*Data Collection in a General Context*	25 hours	1 credit
MTH-4153-2	*Geometric Representation in a General Context 1*	50 hours	2 credits

Course	Course title	Number of hours	Number of credits
MTH-5150-2	*Optimization in a General Context*	50 hours	2 credits
MTH-5151-1	*Algebraic and Graphical Modelling in a General Context 2*	25 hours	1 credit
MTH-5152-1	*Vote Distribution Models and Random Experiments*	25 hours	1 credit

Course	Course title	Number of hours	Number of credits
MTH-4261-2	*Algebraic and Graphical Modelling in an Applied Context 1*	50 hours	2 credits
MTH-4262-2	*Data Collection in an Applied Context*	50 hours	2 credits
MTH-4263-2	*Geometric Representation in an Applied Context 1*	50 hours	2 credits

Course	Course title	Number of hours	Number of credits
MTH-5160-2	*Optimization in an Applied Context*	50 hours	2 credits
MTH-5161-2	*Algebraic and Graphical Modelling in an Applied Context 2*	50 hours	2 credits
MTH-5163-2	*Geometric Representation in an Applied Context 2*	50 hours	2 credits

Course	Course title	Number of hours	Number of credits
MTH-4271-2	*Algebraic and Graphical Modelling in a Fundamental Context 1*	50 hours	2 credits
MTH-4272-2	*Data Collection in a Fundamental Context*	50 hours	2 credits
MTH-4273-2	*Geometric Representation in a Fundamental Context 1*	50 hours	2 credits

Course	Course title	Number of hours	Number of credits
MTH-5170-2	*Optimization in a Fundamental Context*	50 hours	2 credits
MTH-5171-2	*Algebraic and Graphical Modelling in a Fundamental Context 2*	50 hours	2 credits
MTH-5173-2	*Geometric Representation in a Fundamental Context 2*	50 hours	2 credits

Course	Course title	Number of hours	Number of credits
MTH-5154-2	*Financial Mathematics in a General Context*	50 hours	2 credits
MTH-5164-2	*Sequences and Series in an Applied Context*	50 hours	2 credits

The subject-specific content of the courses in this program of study may be similar from one option to another. The difference lies mainly in the thrust of the option to which each course belongs. For this reason, each option has its own set of courses.

For Secondary V, the program also offers two optional courses.

Chapter 6

Courses

The Mathematics program for Diversified Basic Education takes various factors into account. It meets the educational needs of adult learners, promotes the acquisition of knowledge and fosters the development of the subject-specific and cross-curricular competencies. Each course allows for the exploration of the broad areas of learning in a way that reflects the spirit of the option to which it belongs. The following information is given for each course: level (Secondary III, IV or V), option, if any, course code and course title. The components of each course are presented in the following order.

Headings
1. Introduction
2. Subject-Specific Competencies
3. Process and Strategies
4. Cross-Curricular Competencies
5. Subject-Specific Content
6. Family of Learning Situations
7. Broad Areas of Learning
8. Example of a Learning Situation
9. End-of-Course Outcomes
10. Evaluation Criteria

A specific effort has been made to contextualize learning in order to foster the development of the subject-specific competencies and the acquisition of knowledge. The restrictions and clarifications outlined in the subject-specific content for each course, as well as its description, make it possible to define the scope of the mathematical concepts and processes covered.

A detailed presentation of the different courses in the Mathematics program for Diversified Basic Education follows.

6.1 Secondary III Courses (Common Core)

Course

MTH-3051-2

Algebraic and Graphical Modelling

Mathematics

INTRODUCTION

The goal of the *Algebraic and Graphical Modelling* course is to enable adult learners to deal with situations that involve using an algebraic or graphical model to represent a relationship between quantities.

In this course, adult learners deal with various situational problems to expand their knowledge of algebra. By solving problems, they become familiar with algebra as a generalization tool used to represent relationships between quantities on the basis of observed patterns. In these situational problems, adult learners must identify relevant information given verbally, algebraically, graphically or in a table of values. They must also interpolate or extrapolate from a model, using the functions covered in the course. Interpreting and representing a situation sometimes involves producing inverse models. In the case of first-degree polynomial functions and rational functions, adult learners compare the rules, graphs and verbal descriptions of the dependency relationship in question. Studying functions is an important part of the modelling process. In examining the graphic representation of an experiment, adult learners come to realize that, because of handling or measurement errors or the level of precision of the instrument used, the resulting data do not always form a curve that corresponds exactly to a given mathematical model.

By the end of this course, adult learners will be able to use algebra to represent concrete situations in accordance with the rules and conventions of mathematics. By algebraically or graphically representing a situation using a first-degree function or a rational function, they will be able to deduce results through interpolation or extrapolation. In addition, they will use different registers of representation to generalize a model so it can be applied to a range of situations.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly while observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

<table>
<tr><th colspan="2">PROCESS AND STRATEGIES</th></tr>
<tr><th colspan="2">REPRESENTATION</th></tr>
<tr><td colspan="2">– Adult learners examine the situational problem and represent it adequately by switching from one register to another.
– In attempting to understand the context and the problem, they use deductive reasoning, particularly in situations that involve implicit data.</td></tr>
<tr><td>Examples of strategies</td><td>• Identifying relevant information given verbally, algebraically, graphically or in a table of values
• Determining the nature of the task involved (e.g. instructions, expected results, goals, time allotted)
• Describing the situation in their own words and comparing their understanding of the problem with that of their classmates and teacher
• Determining the mathematical characteristics of the relationship described in the situation (e.g. x-intercept, y-intercept)</td></tr>
<tr><th colspan="2">PLANNING</th></tr>
<tr><td colspan="2">– Adult learners set priorities and identify the relevant resources.
– They look for ways of approaching the problem and choose those that seem the most efficient.
– They develop a plan, taking into account the elements of mathematical language (key elements, subject of the message, overall meaning of the situation).</td></tr>
<tr><td>Examples of strategies</td><td>• Using brainstorming techniques
• Dividing the situational problem into subproblems
• Systematically determining the functional model best suited to the situation, while taking into account the limitations of the model
• Finding an algebraic rule that reflects the best relationship between the constraints and possible consequences of the situational problem</td></tr>
<tr><th colspan="2">ACTIVATION</th></tr>
<tr><td colspan="2">– Adult learners follow the plan they have worked out and take into account the constraints involved.
– In developing their reasoning, adult learners propose probable or plausible ideas, anticipate the implications of these ideas and use examples to find invariants.</td></tr>
<tr><td>Examples of strategies</td><td>• Proceeding by trial and error to determine certain properties of the relation
• Dividing the situational problem into subproblems to work out a solution
• Making connections between the algebraic and graphical representation of a situation
• Illustrating a correlation graphically in order to confirm their intuition</td></tr>
<tr><th colspan="2">REFLECTION</th></tr>
<tr><td colspan="2">– Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
– Reflection helps them hone their ability to use exact mathematical language.</td></tr>
<tr><td>Examples of strategies</td><td>• Comparing their results with the expected results and those of others
• Finding examples and counterexamples
• Checking their solution by making sure that the resulting values satisfy the range of the function, and by substituting the values of the variables in the algebraic expression in order to validate a graphical interpolation or extrapolation</td></tr>
</table>

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Relationship between quantities*. Two cross-curricular competencies are considered particularly relevant to this course: *Adopts effective work methods* and *Communicates appropriately*.

Methodological Competency

The competency *Adopts effective work methods* is an essential tool for adult learners when they use an algebraic model to represent a situation. For example, being organized and using a structured approach can help them compare the costs of purchasing or leasing a car. In this regard, adult learners represent the costs of each option as accurately as possible and adapt their work methods to the type of information collected.

Communication-Related Competency

The competency *Communicates appropriately* is often used in establishing models. Modelling involves using mathematical language to represent a phenomenon or experiment and, by virtue of that fact, is a valuable tool for helping adult learners illustrate their ideas on a specific topic.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of arithmetic and algebra. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following three integrative processes:

- **using an algebraic or graphical model to represent a situation**
- **interpolating or extrapolating from an algebraic or graphical model**
- **using an algebraic or graphical model to generalize a set of situations**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover all three processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Inequality	
• Inequality relation	The relations studied are: • $a \leq b$, $a \geq b$, $a > b$ and $a < b$ such that a and b belong to the set of real numbers.
• Solving first-degree equations and inequalities in one variable	The inequalities studied are of the form: • $ax + b \leq cx + d$ • $ax + b \geq cx + d$ • $ax + b > cx + d$ • $ax + b < cx + d$ such that a and b belong to the set of real numbers.
Relation	
• Observing, describing, interpreting and representing the dependency relationship between the variables of a situation	The dependency relationship between the variables can be described and interpreted using the following registers of representation: • literal or verbal expression • algebraic rule • graph • table of values

Mathematical Knowledge	Restrictions and Clarifications
Relation (cont.)	
• Functions and inverse functions	Only polynomial functions of degrees 0 or 1 and rational functions are studied in this course: • constant function $f(x) = b$ • linear functions $f(x) = ax$ $f(x) = ax + b$ • rational function of the form $f(x)=\frac{k}{x}$, where $k \in \mathbb{Q}_+$ • piecewise function (*In Secondary III, adult learners are informally introduced to this function.*)
• Drawing a scatter plot representing an experiment or a statistical study	*It should be noted that a scatter plot is used only to illustrate the relationship between the variables and that adult learners could use a linear or rational function to represent any dependency relationships. The scatter plot is only an approximation, since in this course adult learners are not required to determine the correlation coefficient or the linear regression line.*
• Representing and interpreting the inverse of a function	The inverse of a function (linear or reciprocal) can be represented or interpreted using the following registers of representation: • literal or verbal expression • algebraic rule • graph • table of values
▪ Determining the rule of correspondence	The following information could be used to derive the rule: • an ordered pair and the rate of change • two ordered pairs Certain values will be determined graphically or by using the rule, with the degree of precision required by the context.

Mathematical Knowledge	Restrictions and Clarifications
Relation (cont.)	
▪ Describing the properties of a function in context	The properties of functions studied are: • domain and codomain (range) • increasing and decreasing intervals • extrema • sign • x- and y-intercepts Adult learners derive the properties in an informal manner and always in relation to the context.
• Providing a qualitative description of how the graph is affected by a change in the value of a parameter of a linear function	*Parameters* a *and* b *are never changed at the same time. In this course, adult learners analyze how a change in parameter* a *or parameter* b *of the linear function affects the graph, but each parameter is studied separately.* *In Secondary III, adult learners are informally introduced to the study of properties.*
System	
• Solving systems of first-degree equations in two variables	The equations must be of the form $y = ax + b$ and may be solved: • using a table of values • graphically • algebraically (using the comparison method)

Cultural References

Algebra is an ancient discipline that dates back to the Babylonians, but it only came into its own in the West during the Renaissance. Adult learners could discover that the word “algebra” originated in the ninth century and comes from the Arab mathematician Al-Khwarizmi who used the term “al-djabr” to refer to a calculation that involved adding the same number to both members of an equality. His work on the decimal number system and on solving first- and second-degree equations led to the development of the algebraic processes used today.

Algebra is widely used in the modern world. Adult learners could learn that simple yet ancient algebraic processes like proportional reasoning are commonly used in fields such as health care and construction. With time, algebraic symbolism became standardized and more complex. In addition, algebra has become an intrinsic part of such branches of mathematics as geometry, function theory and logic. When observing and analyzing different phenomena before making decisions, financial experts and demographers use algebra, among other tools. In the modern world, algebra has become indispensable in a number of fields.

Professionals in the field of pharmacology have long applied proportional reasoning to determine the dosage for different types of medication on the basis of a patient’s age or weight. Adult learners interested in public health issues and, more specifically, in the safe consumption of medication, could study how to apply proportional reasoning to determine, for example, the number of tablets that a person may take to treat certain symptoms.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Relationship between quantities* involve problems that can be solved in part by using an algebraic or graphical model to represent a relationship between quantities. The *Algebraic and Graphical Modelling* course provides adult learners with an opportunity to establish connections or dependency relationships between quantities.

In the situational problems in this course, adult learners select relevant information in order to establish relationships between two elements; they represent, graphically, algebraically or using a table of values, the inverse of a previously determined function; and they describe how a change in one of the parameters will affect this graph.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Environmental Awareness and Consumer Rights and Responsibilities, and Health and Well-Being.

Environmental Awareness and Consumer Rights and Responsibilities

Integrating the broad area of learning Environmental Awareness and Consumer Rights and Responsibilities can prove useful, especially in situations in which adult learners must compare types of investments and loans, as well as purchase or lease options for goods and services. Using a model to represent their finances, adult learners could make certain extrapolations based on their model in order to make informed consumer choices.

Health and Well-Being

The algebraic concepts studied in this course could help adult learners reflect on healthy lifestyle habits. In analyzing situational problems that focus on specific relationships between various aspects of health, adult learners can become aware of certain factors that will harm or benefit their health. They can then anticipate the health impact of certain decisions regarding nutrition and physical activity. This awareness is consistent with the educational aim of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Environmental Awareness and Consumer Rights and Responsibilities
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Relationships between quantities
Targeted cross-curricular competencies • Are developed at the same time and in the same context as the subject-specific competencies.	• Adopts effective work methods • Communicates appropriately
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

<table>
<tr><th>Situational problem</th><th colspan="2">Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the Relationship between quantities family of learning situations</th></tr>
<tr><td rowspan="2">Upon completing her education, an adult learner must reimburse a student loan. She wants to estimate the monthly amount to be set aside to pay back that debt, taking into account possible interest rates and the term over which the loan will be reimbursed.

Using a linear function, she may have to provide a graphical or algebraic justification for the amortization term chosen.</td><td colspan="2">Integrative process: Using an algebraic or graphical model to represent a situation
In carrying out the four phases in the problem-solving process, adult learners could:</td></tr>
<tr><td>Representation</td><td>• Select relevant information to determine the relationship between the amount set aside to pay back the debt (dependent variable) and the interest rate (independent variable)
• Describe the characteristics of the situational problem orally or in writing in order to determine the independent variable (interest rate) and the dependent variable (amount set aside each month to pay back the debt)</td></tr>
</table>

<table>
<tr><th>Situational problem</th><th colspan="2">Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the Relationship between quantities family of learning situations</th></tr>
<tr><td rowspan="3">The situation could be modified by including constraints (e.g. limiting the amount of the monthly payment). In this case, the adult learner could be asked to use extrapolation to make a conjecture about the term over which the loan will be repaid.</td><td>Planning</td><td>Find out about current interest rates and determine the amount to be paid back each month by reading newspapers or using Web-based personal loan calculators</td></tr>
<tr><td>Activation</td><td>Illustrate graphically, through linear approximation, the dependency relationship between the variables; for example, if the interest rate increases, the monthly payments will also increase
Establish the rule of linear correspondence using two points on the graph
Determine the interest rate corresponding to a specific monthly amount; in other words determine, graphically, algebraically or using a table of values, the inverse of the previously determined function</td></tr>
<tr><td>Reflection</td><td>Describe how a change in one of the parameters (e.g. the initial debt amount) will affect this graph</td></tr>
</table>

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Relationships between quantities*, adult learners represent a situation, interpolate or extrapolate, and generalize a set of situations using an algebraic or graphical model. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To represent a situational problem using an algebraic or graphical model, adult learners adopt various strategies to identify the problem. They determine the mathematical characteristics of the relationship described in the situation: y-intercept, x-intercept, increasing or decreasing intervals, sign, etc. They choose the most accurate representation, aware that it does not necessarily reflect what they have observed, but that it is the best choice given the functions studied in the course. They systematically determine the functional model best suited to the situation, bearing in mind the limitations regarding the model's precision: $f(x) = b$ or $f(x) = ax$ or $f(x) = ax + b$. They produce mathematical messages, using recognized codes and conventions in order to effectively communicate their intention: rate, y-intercept, x-intercept, increasing interval, decreasing interval, etc. They choose the register of representation best suited to the situation (table of values, Cartesian coordinate graph or algebraic rule).

To interpolate or extrapolate results using an algebraic or graphical model with a view to making a decision, adult learners interpret the algebraic or graphical model presented and distinguish between the elements that are relevant and those that are not. In addition, they use mathematical reasoning to explore the situational problem and to determine questions about the issue involved, and gather relevant information in order to draw a conclusion. They deduce the rate of change of the relationship and determine the y-intercept in accordance with actual data: initial value, value of the function at time zero, quantity at the beginning of the experiment, etc.

To generalize results in order to obtain a family of linear functions or a system of linear relations, adult learners deduce similar properties by observing a variety of situations. They identify the parameters at play: rate of change, y-intercepts, increasing function, etc. They use inductive reasoning to determine the type of relationship between the variables (linear or rational function). Using graphical or algebraic representation, they confirm that the parameterized model $f(x) = ax + b$ does indeed correspond to a set of situations. In addition, when the situation involves a system of linear relations, they use a system of two first-degree relations in two variables to represent it and then solve the system algebraically (using the comparison method) or graphically, in accordance with the limitations imposed by the context. They validate their solution by substituting the values in the algebraic expression associated with the system in question.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (inequality, relation and system). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

  * The solution includes a procedure, strategies and a final answer.
  ** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*

Course

MTH-3052-2

Data Collection

Mathematics

INTRODUCTION

The goal of the *Data Collection* course is to enable adult learners to deal with situations that involve collecting or processing data pertaining to a one-variable distribution.

In this course, adult learners draw conclusions or make informed decisions based on the results of a statistical report. The data collected, whether discrete or continuous, are represented using various tools (tables, graphs, measures) that make it possible to synthesize information about a given population. The situational problems presented involve the use of statistical reasoning to produce, compare or critically examine different studies. They therefore provide an opportunity for adult learners to analyze data and justify their conclusions using tools such as statistical measures and graphs. Adult learners also interpret different statistical measures and information contained in drawings and geometric constructions. To process data resulting from random experiments, they represent, interpret and compare probability data by enumerating possibilities and calculating probabilities (discrete and continuous random variables). In some situational problems, adult learners organize data from a sample, whether or not they have collected the data themselves, in order to describe a population and draw conclusions. They also analyze distributions using the appropriate statistical measures or critically examine an existing study. In others, they compare measures to qualify and quantify probabilities and, depending on the case, use experimental or theoretical probabilities to predict and validate results.

By the end of this course, adult learners will be able to use data collection as a tool and compare the results of a statistical experiment using different instruments to validate their observations of a problem that they themselves have identified. They will present the results of their analysis in accordance with the rules and conventions of mathematics. They will use problem-solving strategies to determine the most appropriate solution. In addition, they will use mathematical reasoning to interpret probability data resulting from a random experiment and make decisions.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

<table>
<tr><th colspan="2">PROCESS AND STRATEGIES</th></tr>
<tr><th colspan="2">REPRESENTATION</th></tr>
<tr><td colspan="2">- Adult learners examine the situational problem to identify the context, the problem and the task to be performed, and suitably master the elements of mathematical language.
- In attempting to understand the context and the problem, they use deductive reasoning.</td></tr>
<tr><td>Examples of strategies</td><td>• Identifying relevant information given verbally, graphically, in a table of values or in a diagram
• Describing the situation in their own words and comparing their understanding of the problem with that of their classmates and teacher
• Organizing data from a sample in order to describe a population and make it easier to process the information
• Proceeding by analogy with games of chance (e.g. working with dice, cards or other objects to determine the constraints in a random experiment)</td></tr>
<tr><th colspan="2">PLANNING</th></tr>
<tr><td colspan="2">- Adult learners look for ways of approaching the problem and choose those that seem the most efficient.
- In planning the solution, they use different types of reasoning to work out the steps involved. They can refer to similar situations they have solved in the past.
- They develop a plan, taking into account the elements of mathematical language (symbols, terms and notation used, and the different registers of representation).</td></tr>
<tr><td>Examples of strategies</td><td>• Comparing the situation with other situations they have already studied in order to identify any similarities
• Dividing the situational problem into subproblems
• Determining the main steps in a plan aimed at developing an intuitive correlation model
• Developing an appropriate counting method when studying the concept of fair game</td></tr>
<tr><th colspan="2">ACTIVATION</th></tr>
<tr><td colspan="2">- In developing their reasoning, adult learners propose probable or plausible ideas, anticipate the implications of these ideas and use examples to find invariants.
- They make rigorous use of mathematical language and, to avoid confusion, they use the symbols terms and notation in accordance with their meaning.</td></tr>
<tr><td>Examples of strategies</td><td>• Referring to previously studied situational problems
• Using a table to connect the elements associated with the correlation: ordering the statistical data, finding the mode, the median or the weighted mean
• Drawing the line of best fit through a scatter plot</td></tr>
<tr><th colspan="2">REFLECTION</th></tr>
<tr><td colspan="2">- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution. This reflection helps them hone their ability to use exact mathematical language.</td></tr>
<tr><td>Examples of strategies</td><td>• Comparing their results with the expected results and those of others
• Checking their solution by, for example, comparing different measures of central tendency, or validating quartile measures using the corresponding graph
• Identifying the strategies used to deal with the situation
• Using a set of metacognitive questions such as Why did I proceed in this way? Is there a better way of doing this?
• Using a spreadsheet program to validate their work</td></tr>
</table>

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Processing data*. Two of these are considered particularly relevant to this course: *Exercises critical judgment* and *Communicates appropriately*.

Intellectual Competency

As the saying goes, there are lies, white lies and statistics. In collecting data, adult learners use the competency *Exercises critical judgment* before forming an opinion and drawing conclusions about their study. They are able to assess the extent to which reason and emotion influence their actions and to base their thinking on a logical and ethical foundation. This competency enables them to form, express and qualify their opinion by objectively and rigorously examining the facts, and by choosing an appropriate graph and scale to present the results as objectively as possible.

Communication-Related Competency

The competency *Communicates appropriately* is often used in presenting the results of a study. Graphs, histograms and distributions are generally used to convey information in an organized manner in order to convince or inform others. The situational problems in this course could foster the development of this competency by giving adult learners the tools to organize their thinking and share their opinions about current events, which enables them to use an appropriate mode of communication.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of statistics. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following three integrative processes:

- **collecting data**
- **comparing collections of data**
- **interpreting data resulting from an experiment**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover all three processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
One-variable statistical distributions	
• Organizing and interpreting statistical data	The sampling methods studied in this course are: • stratified • cluster
• Constructing and interpreting distributions	In this course, data is interpreted and tables are constructed using: • tables of condensed data • tables with data grouped into classes
• Representing and interpreting graphs	The graphs studied in this course are: • the histogram • the box-and-whisker plot
• Calculating measures of central tendency and dispersion	The measures of central tendency studied in this course are: • the mode • the median • the weighted mean The only measure of dispersion studied in this course is the range of each part of a box-and-whisker plot (including the interquartile range).

Mathematical Knowledge	Restrictions and Clarifications
Probability	
• Enumerating possibilities and calculating probabilities	Two types of random variables are studied in this course: • discrete • continuous The enumeration of possibilities and the calculation of probabilities is carried out in various situations, including those that deal with measurement (including geometric probabilities) Since reasoning is used to perform calculations (arrangement, permutation and combination), it is not necessary to use counting formulas.
• Representing events	Events are represented by means of: • tables • tree diagrams • graphs • geometric figures

Cultural References

The forecasting aspect of statistics dates back to the 18th century, with the advent of the first mortality tables. The ability to determine life expectancy led to the creation of the first life insurance companies. Since then, statistical data processing has spread to other fields. For example, some specialists can determine the risk of having your car stolen if you live in a certain area. Moreover, insurance companies establish their rates on the basis of this type of statistical data. This knowledge could help adult learners make more informed choices.

The world of sports is replete with statistics, and adult learners could draw on such material. If they are so inclined, they could track performances in a particular sport since the creation of the modern Olympic Games, and predict trends or possible limitations regarding human capabilities, for instance in swimming or running. They could also track the development of techniques and materials used to shave off the clock the precious thousandths of a second that go into making a world champion. Adult learners who are interested in hockey could make graphs or tables of statistics in order to make predictions.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Processing data* involve problems that can be solved in part by collecting or processing data. The *Data Collection* course provides adult learners with an opportunity to learn how to collect and compare data.

In the situational problems in this course, adult learners organize data from a sample in order to describe a population and draw conclusions, interpret data resulting from a statistical study or a random experiment, or take a position on the basis of statistical or probability data.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Three broad areas of learning are considered particularly relevant to this course: Media Literacy, Citizenship and Community Life, and Environmental Awareness and Consumer Rights and Responsibilities.

Media Literacy and Citizenship and Community Life

In fulfilling their duties as citizens, especially during an election campaign, adult learners are exposed to numerous polls and statistical studies. This course provides them with the tools to better understand, interpret and compare these data. A deeper understanding of data-collection methods, in particular sampling methods, fosters the development of a critical attitude toward statistics presented in the media and toward the media per se, which reflects the educational aim of the broad area of learning Media Literacy. In this way, adult learners can participate in a more informed way in the democratic life of their society, thus achieving the educational aim of the broad area of learning Citizenship and Community Life.

Environmental Awareness and Consumer Rights and Responsibilities

Different situations that involve analyzing statistical studies may prove useful in helping adult learners take a critical look at consumer behaviour in a society. Among other things, this might mean examining the origin of various consumer products; the impact of globalization on cultures, ways of life and the distribution of wealth; the working conditions of those who produce goods or provide services and the fair distribution of resources. Developing an awareness of the social, economic and ethical aspects of consumption is consistent with the educational aim of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Media Literacy • Citizenship and Community Life
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Processing data
Targeted cross-curricular competencies • Are developed at the same time and in the same context as the subject-specific competencies.	• Exercises critical judgment • Communicates appropriately
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Processing data* family of learning situations
As citizens, adult learners are called upon to exercise their right to vote on numerous occasions—in federal, provincial, municipal or other elections. From a list of candidates, they must choose the person they feel is the most suitable for the position in question. Before deciding who to vote for, adult learners must find out more about the various political platforms and check certain polls that track voting intentions. They realize that two polling firms have obtained different results regarding the candidate who is thought to be the likely winner. The method used by each polling firm must then be examined to explain the difference in the results.	**Integrative process:** *Comparing collections of data* In carrying out the four phases in the problem-solving process, adult learners could: Representation • Consult newspapers to find information that is produced by two different polling firms and that could be expressed using different registers of representation (data tables, graphs, etc.) Planning • List the steps to be carried out in order to properly compare these two polls Activation • Appropriately present the data provided by each of the firms (e.g. in the form of a table) in order to better compare them • Compare the statistical data provided, after using the same register of representation Reflection • Analyze the way in which the firms dealt with non-respondents • Analyze the way in which the firms presented their results to see if this promotes a particular point of view. Would another method of representing the data change the interpretation of the results?

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Processing data*, adult learners collect, compare and interpret data resulting from an experiment. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To collect data pertaining to a one-variable distribution with a view to solving a situational problem, adult learners identify relevant information that can be presented verbally, graphically, or in a table of values or diagram. They work out their solution by following the main steps involved in statistical work, namely collecting and processing (interpreting and analyzing) data. They make conjectures and draw conclusions from their analysis of the results in order to make informed decisions. They decide whether a histogram or a box-and-whisker plot is the most appropriate register of representation for communicating the results of their analysis.

To compare collections of data, adult learners use tables, tree diagrams or geometric figures to conduct an effective comparative analysis. When data are graphed, the adults decode and interpret the elements of mathematical language and then use inductive or deductive reasoning to formulate the necessary propositions. They organize data from a sample in order to describe a population and make it easier to process the information. They check their solution by comparing different measures of central tendency and by validating the quartile measures using the corresponding graph.

Interpreting data resulting from a random experiment helps adult learners develop and use networks of mathematical cognitive resources which, in turn, help them derive through inductive reasoning properties and laws of probability using fractions and ratios. In solving problems, they use different strategies to represent the relationship between the results of the experiment and different concepts such as probable, certain or impossible events. They often proceed by analogy with games of chance, working with dice, cards or other objects to determine the constraints in a random experiment. Lastly, they check their solution by ensuring, for example, that the sum of the probabilities of an event and its complementary event is always equal to 1.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (one-variable statistical distributions, theoretical and geometric probabilities). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

  * The solution includes a procedure, strategies and a final answer.

  ** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*

Course

MTH-3053-2

Geometric Representation

Mathematics

INTRODUCTION

The goal of the *Geometric Representation* course is to enable adult learners to use metric relations, figures and solids to deal with situations that involve the geometric representation of an object or a physical space.

In this course, adult learners develop their spatial sense by visualizing, manipulating and representing different objects. In representing three-dimensional figures and drawing the nets of solids, they are able to explore various tools such as orthogonal projections with different views, parallel projections (cavalier and axonometric perspectives) and central projections (with one or two vanishing points). In situational problems, they construct or represent geometric figures using different methods. To describe and interpret contexts related to geometric figures or the concept of similarity, they use their spatial sense, as well as their knowledge of measurement and proportionality. In other situational problems, they use their spatial and measurement sense, as well as the different relationships associated with geometric figures and the determination of unknown measurements (length, area and volume). Lastly, in all of the situational problems, they illustrate their reasoning using different types of representations (words, symbols, graphs, tables of values, drawings), depending on the context and the branch of mathematics in question.

By the end of this course, adult learners will be able to use different types of solids or planes to represent and describe an object or a physical space in accordance with the rules and conventions of mathematics. They will also be able to use different strategies and types of reasoning in planning the organization of a physical space, taking into account the different constraints of the situational problem.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

PROCESS AND STRATEGIES	
REPRESENTATION	
- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. The strategies they use clarify the different patterns and invariants and help them conceive of probable or plausible relationships. - In attempting to understand the context and the problem, they use deductive reasoning.	
Examples of strategies	• Constructing, drawing or making a diagram of geometric figures using different methods • In a table, determining the nature of the task involved (e.g. instructions, expected results, goal, time allotted) • Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to find unknown measurements • Describing the problem in their own words in order to show that they have understood the situation, for example, when using projection to represent an object
PLANNING	
- Adult learners look for ways of approaching the problem and choose those that seem most efficient. - They develop a plan, taking into account the elements of mathematical language (key elements, subject of the message, overall meaning of the situation). - After solving a number of situational problems, they are able to make conjectures about the sign of a number for which they have found the square root, the cube root or any other root.	
Examples of strategies	• Using brainstorming techniques • Dividing the situational problem into subproblems • Using diagrams, drawings or sketches to plan their solution
ACTIVATION	
- In developing their reasoning, adult learners propose probable or plausible ideas, anticipate the implications of these ideas and use examples to find invariants. - In implementing their solution, they make rigorous use of mathematical language and, in order to avoid confusion, they use the symbols, terms and notation in accordance with their meaning. - They use different strategies, associating images, objects or concepts with mathematical terms and symbols, and switching from one register of representation to another.	
Examples of strategies	• Referring to previously studied situational problems, distinguishing between representation problems and those that involve finding an unknown measurement • Drawing the vanishing lines or reference points needed to produce a projection • Analyzing the dimensions of a three-dimensional figure to be able to understand the possible connections between them and the formula for calculating capacity, for example
REFLECTION	
- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution. This reflection helps them hone their ability to use exact mathematical language.	
Examples of strategies	• Comparing their results with the expected results and those of others • Checking their solution by, for example, comparing the dimensions of a three-dimensional figure • Determining the strategies for dealing with situational problems in geometry (applying a rule, referring to a theorem, etc.) • Using a calculator or geometric modelling software to validate their work

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Measurement and spatial representation*. Two of these are considered particularly relevant to this course: *Uses information and communications technologies* and *Uses creativity*.

Methodological Competency

The cross-curricular competency *Uses information and communications technologies* makes it easier to perform a variety of tasks. It enables adult learners to use tools for manipulating two- or three-dimensional geometric shapes. Architectural or landscaping software can be useful for producing plans according to different perspectives or for calculating certain measurements. Interactive geometry software can be used to show different relationships and thereby help adult learners develop the ability to derive rules through inductive reasoning.

Intellectual Competency

The competency *Uses creativity* is developed through situational problems that involve the organization of physical space. By being original and creative, adult learners draw on personal and material resources. This ability is used to solve the problem, whether it involves building a skateboard ramp, setting up an office space or any other project. The preparation and flexibility required in such projects go hand in hand with being open to new ideas and exploring creative strategies, when necessary.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of geometry. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following two integrative processes:

- **describing an object or a physical space and representing it in two or three dimensions**
- **organizing a physical space**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover both processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Numerical and algebraic expressions	
• Manipulating rational and irrational numbers	The rational and irrational numbers studied are: • square and square root • cube and cube root *Making connections between exponential notation and radicals (Examples:* $9^{1/2} = \sqrt{9}$ *and* $8^{1/3} = \sqrt[3]{8}$*). Radicals should be kept if it is not appropriate to convert them.*
• Manipulating numerical and algebraic expressions	Adult learners must be able to perform operations on numbers expressed: • exponentially (rational base; integral or fractional exponent) • in scientific notation

<table>
<tr><th>Mathematical Knowledge</th><th>Restrictions and Clarifications</th></tr>
<tr><td>Numerical and algebraic expressions (cont.)</td><td></td></tr>
<tr><td>Manipulating numerical and algebraic expressions (cont.)</td><td>Operations on numerical or algebraic expressions are limited to:
adding and subtracting algebraic expressions
multiplying algebraic expressions of degree 0, 1 or 2
dividing algebraic expressions by a monomial
finding the common factor
Adult learners are introduced to algebraic operations so that they can understand how to manipulate or simplify units of measure and compare different representations of the same formula, among other things. For example, the perimeter of a rectangle is represented by the following formula:
$P = 2h + 2b = 2(h + b)$</td></tr>
<tr><td>Solids</td><td></td></tr>
<tr><td>Describing, constructing and representing objects</td><td></td></tr>
<tr><td>Net, projection and perspective</td><td>Three-dimensional figures are represented in the plane by using such projections as:
orthogonal projections with different views
parallel projections (cavalier and axonometric perspectives)
central projections (one or two vanishing points)
For constructions using cavalier perspective, adult learners will be asked to draw the receding edges at 30^{o} or 45^{o} angles.</td></tr>
</table>

Mathematical Knowledge	Restrictions and Clarifications
Solids (cont.)	
• Converting various units of measure	Conversions involve units of: • length • area • volume • capacity Conversions can be done within the same system of measurement or from one system to another.
• Finding measurements	The unknown measurements pertain to the following elements: • length – side of a right triangle (Pythagorean theorem) – segments resulting from an isometry or a similarity transformation and segments in a plane figure or a solid • lateral or total area – sphere, right cone and decomposable figure – plane figure resulting from a similarity transformation • volume – solid that can be split into a right prism, a right cylinder, a right pyramid, a right cone and a sphere – solid resulting from a similarity transformation • appropriate choice of units of measure – conversions between various units of measure (length, area, volume, capacity)

Principles
Adult learners must master the following compulsory principles, which may be used in a proof: **P1.** In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other sides (Pythagorean theorem). **P2.** A triangle is right-angled if the square of the length of one of its sides is equal to the sum of the squares of the lengths of the other sides.

Cultural References

Humans have always sought to represent the world in various ways: the study of perspective has provided some solutions. During the Renaissance, the introduction of perspective revolutionized the arts. Today, perspective is used in various fields: geography, the media, computer graphics, design, engineering, architecture, photography, film, theatre, painting and so on. Studying perspectives and projections could be a way of introducing adult learners to certain elements of the arts and architecture.

Adult learners can also become aware of the connection between the arts and mathematics, for example, by analyzing the graphic art of Oscar Reutersvärd (1915-2002). They could observe the perspectives that this artist applied to the figures he drew and determine at which point the perspective becomes distorted. The work entitled the “impossible triangle” (renamed the Penrose triangle, after the mathematician who popularized it in the 1950s), which Reutersvärd completed in 1934, is ideal for this exercise. In this way, adult learners could be encouraged to see mathematics differently.

In addition, adult learners’ interest in astronomy could lead them to study and compare the volumes, masses and densities of the different heavenly bodies in our solar system, which would require them to learn how to work with very large numbers.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Measurement and spatial representation* involve problems that can be solved in part through the geometric description or representation of an object or a physical space. The *Geometric Representation* course gives adult learners an opportunity to develop their spatial representation skills.

In the situational problems in this course, adult learners describe the characteristics of the situation, identify the constraints and patterns involved by exploring different figures, and use new symbols to describe an organization or representation of their physical environment.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Citizenship and Community Life, and Environmental Awareness and Consumer Rights and Responsibilities.

Citizenship and Community Life

The educational aim of this broad area of learning is to encourage adult learners to develop an open attitude toward others. A connection can be made between this broad area of learning and the *Geometric Representation* course when, for instance, adult learners build or renovate a space for different groups of people. For example, adult learners could set up a room so that a teenager can do homework without being disturbed by siblings, or a space to accommodate an ailing parent. They could also draw up the plans for a skateboard ramp at the local youth centre.

Environmental Awareness and Consumer Rights and Responsibilities

This course offers a variety of resources that enable adult learners to critically examine their relationship to their environment and consumerism. The work they do in learning situations that deal, for example, with the organization of a physical space can help them make sound consumer choices that are good for the environment. They could draw up the plans for an ecological landscaping project that takes into account biodiversity and the specific elements of a particular environment. In this way, they could be made aware of the active relationship they have with their environment. In selecting materials, they could become aware of the economic consequences of their choices. Such situations help them to become familiar with certain ethical or economic aspects of the environment and consumer behaviour.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Citizenship and Community Life • Environmental Awareness and Consumer Rights and Responsibilities
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Measurement and spatial representation
Targeted cross-curricular competencies • Are developed at the same time and in the same context as the subject-specific competencies.	• Uses information and communications technologies • Uses creativity
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

<table>
<tr><th>Situational problem</th><th colspan="2">Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the Measurement and spatial representation family of learning situations</th></tr>
<tr><td rowspan="2">An adult learner who wishes to take in a family member who can no longer manage on his/her own must determine the layout of a bedroom, taking into account the family member’s health and the available budget.</td><td colspan="2">Integrative process: Organizing a physical space
In carrying out the four phases in the problem-solving process, adult learners could:</td></tr>
<tr><td>Representation</td><td>Draw up a list of constraints related to the situation by consulting Web sites, if necessary. For example, they could make sure that:
– the door is at least 84 cm wide to accommodate the wheelchair
– there is at least 1.5 metres of free space on each of three sides of the bed
– there is enough room for a standard adapted bed (i.e. 2.3 m by 1.1 m)
– there is a low window 45 cm above the floor
– electrical switches are located 110 cm above the floor
– a wood floor will replace the carpet
– the walls will be painted
– a wheelchair will be purchased
– an air conditioner suited to the volume of the room will be purchased</td></tr>
</table>

<table>
<tr><th>Situational problem</th><th colspan="2">Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the Measurement and spatial representation family of learning situations</th></tr>
<tr><td rowspan="3">The adult learner must replace the carpet with a wood floor, purchase a wheelchair and install an air conditioner suitable for the volume of the room. He or she must also repaint the walls.

A detailed work plan must be established as well as a scale drawing of the bedroom, with and without any changes.</td><td>Planning</td><td>• Determine the task to be performed, establish a budget and set deadlines for the work to be done
• Draw up a logical work plan (e.g. move the switches before painting the walls)</td></tr>
<tr><td>Activation</td><td>• Draw a diagram of the situational problem using an orthogonal projection of the room on three planes, with a front, side and top view, taking into account the constraints
• Carry out the work plan: calculate the surface area of the floor to be covered, the surface area of the walls to be painted and the number of litres of paint required; determine the volume of air in the room before choosing an air conditioner
• Use an appropriate system of measurement and perform the necessary conversions</td></tr>
<tr><td>Reflection</td><td>• Make sure the goal has been achieved and that the physical and budget-related constraints have been taken into account
• Validate the work plan by showing it to classmates</td></tr>
</table>

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Measurement and spatial representation*, adult learners describe an object or a physical space and represent it in two or three dimensions, and organize a physical space. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To describe an object or a physical space and represent it in 2-D or 3-D, adult learners interpret and produce sketches, drawings or plans using orthogonal, parallel and central projections. In situational problems, they use networks of geometrical concepts and processes to determine unknown measurements by deductive reasoning or to validate conjectures. They illustrate their reasoning using different representations (words, symbols, graphs, tables of values, drawings), depending on the mathematical context. Thus, they use an inductive process or modus ponens (the law of detachment) in a process of proofs. They also distinguish between the key elements of mathematical language (e.g. scale, dimensions, perimeter, area, volume), and associate images, objects or knowledge with mathematical terms and symbols.

To organize a physical space, adult learners produce and interpret different measurements and information contained in drawings and geometric constructions. In situational problems, they represent three-dimensional figures in two dimensions using a projection, in order to describe and interpret contexts related to geometrical figures or the concept of similarity. They use their spatial sense as well as their knowledge of measurement and proportionality to find measurements. Because of the geometric nature of the situational problems, adult learners can use existing definitions, properties and principles. Lastly, the adult learners justify their choice of graphs, procedures and solutions.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (numerical and algebraic expressions and solids). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

  * The solution includes a procedure, strategies and a final answer.

  ** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*

6.2 Cultural, Social and Technical Option

The *Cultural, Social and Technical* option is aimed at helping adult learners develop mathematical literacy so that they will be equipped to meet the demands of their daily lives now and in the future. This option provides adult learners with tools that will help them increase their capacity for analysis, consider different possibilities, make informed decisions, support their reasoning and take a position with respect to various issues. It also allows them to build on their basic education and to continue to develop their sense of citizenship. It helps them integrate into society and prepares them for higher education in the arts, the humanities or the social sciences. It also opens the door to various types of vocational and technical training. The context associated with the different courses in this option involves general mathematics.

Besides continuing to develop their mathematical competencies and familiarizing themselves with new concepts and processes, adult learners who choose this option will further their understanding of previously learned concepts. It is important to allow them to use what they already know and to approach the learning content in a way that illustrates how mathematical ideas build on one another. Emphasis is placed on consolidating and integrating knowledge in a variety of activities: hands-on activities, exploration activities, simulations, games, presentations, meetings with resource persons, and so on. In this option, adult learners have an opportunity to use or develop observational, design, managerial, optimization, decision-making, argumentative and other skills. They are generally required to carry out concrete and practical activities. Nevertheless, switching from the concrete to the abstract and using mathematical objects in concrete situations will help adult learners see the usefulness of these objects and encourage them to view certain situations mathematically. Learners will also use technology to represent or process large amounts of data and to simplify tedious calculations.

Adult learners use their competencies and mathematical knowledge in different contexts. They examine the social, economic, artistic, technical or, on occasion, scientific situations that they will encounter in their personal and working lives. The problems may involve:

- lifestyle habits, nutrition, how the body works, health care, physical activity and sports
- planning physical spaces (organization, plan, structure), resource management, population growth or decline, personal finances, constraints related to production, costs related to consumption, design and advertising
- drawing up plans, event organization, market studies
- presentation of information, comparison of presentations on the same topic, appreciation or creation of different works of art and media images
- social choices, equity and justice, cultural diversity, opinion polls, etc.

These situations are conducive to the use of spatial sense, measurement sense, proportional reasoning, number sense, dependency relationships, probability models, statistical tools, and processes associated with graphs.

In addition, the cultural references suggested for this option involve historical and social aspects of the development of mathematics. They also provide examples of contexts in which adult learners can develop their mathematical knowledge.

Adult learners who choose this option are given many opportunities to learn about their world and are provided with an education that makes them aware of the many different skills and attitudes needed in our society. As a result, they develop competencies that will equip them to work effectively in a changing world and to act as informed citizens.

6.2.1 Secondary IV Courses

Course

MTH-4151-1

Algebraic and Graphical Modelling in a General Context 1

Mathematics

INTRODUCTION

The goal of the *Algebraic and Graphical Modelling in a General Context* course is to enable adult learners to deal with situations that involve using an algebraic or graphical model to represent a dependency relationship between quantities in a general context.

In this course, adult learners deal with various situational problems to expand their knowledge of algebra. In studying real functions, they learn to characterize the different types of dependency relationships between two quantities. They explore situations that do not necessarily involve linear relationships, such as exponential, periodic, quadratic and step models. They observe patterns and distinguish between linear growth (arithmetic progression) and exponential growth (geometric progression) in situations involving population growth, for example. In addition, they make conjectures based on functions, and validate them using different types of reasoning involving direct or indirect proofs, or refute them using counterexamples. In situational problems, adult learners identify and use relevant information. For example, in a situation that involves planning the purchase of goods and services, this information can be presented verbally, algebraically, graphically or in a table of values. Adult learners must analyze different possibilities and make decisions. They must also use their number and operation sense and proportional reasoning to validate their solutions. In addition, they must solve systems of linear equations to compare and analyze phenomena in order to make choices. They can interpolate or extrapolate from models, using real functions expressed in different forms. In terms of communication, they analyze graphs and tables of values in order to identify specific information and draw conclusions. In other situations, they transpose a verbal or written description using one or more algebraic expressions (equation, inequality, system of linear equations or function). They use numbers written in different notation, taking the units into account where applicable. They may have to describe a situation expressed as a graph or table of values.

By the end of this course, adult learners will be able to use algebra to represent concrete situations. They will produce clear and accurate work in accordance with the rules and conventions of mathematics. By algebraically or graphically representing a situation using real functions and their inverse, they will be able to deduce results through interpolation or extrapolation. In addition, adult learners will use different registers of representation to generalize a model so it can be applied to a range of situations.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

PROCESS AND STRATEGIES	
REPRESENTATION	
- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. - They use observational and representational strategies that are essential to inductive reasoning. - In attempting to understand the context and the problem, they use deductive reasoning, particularly in situations that involve implicit data.	
Examples of strategies	• Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to identify a dependency relationship and determine the variables in the situation • Using examples involving numbers, determining the types of relationship that exist between the variables in the situation • Listing their algebraic strategies and knowledge pertaining to the situation • Describing the characteristics of the situation
PLANNING	
- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient. - They develop a plan, taking into account the elements of mathematical language (symbols, terms and notation used, and the different registers of representation). - Through reasoning, they establish organized and functional relationships among different aspects of their knowledge, thus expanding their networks of cognitive resources.	
Examples of strategies	• Systematically determining the functional model best suited to the situation, bearing in mind the limitations regarding the model's precision • Finding an algebraic rule that reflects the best relationship between the constraints and possible consequences of the situational problem
ACTIVATION	
- By making connections between algebraic and graphical representations in studying a system of equations, adult learners can derive rules and conditions that determine the number of solutions for the system and make generalizations. - By drawing on their knowledge of the properties of functions in dealing with a situation, they are able to deduce certain relationships (e.g. between the extrema and the optimal value, between the increasing interval of the function and the growth of the company).	
Examples of strategies	• Carrying out a simulation using concrete objects or technology to determine a relationship • Using technology (e.g. spreadsheet program, graphing calculator) to analyze the role of the different parameters of a function • Using the parameters of a function to make a sketch in order to predict results
REFLECTION	
- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution. - Reasoning can be used to reject extrapolations that would yield nonsensical results.	
Examples of strategies	• Comparing their results with the expected results and those of others • Checking their solution by, for example, making sure that the resulting values satisfy the range of the function • Using a set of metacognitive questions such as, Why did I proceed in this way? What would I change and why? • Using a calculator to validate their work.

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Relationship between quantities*. Two of these are considered particularly relevant to this course: *Exercises critical judgment* and *Uses information and communications technologies*.

Intellectual Competency

Many companies seek to attract customers and develop their loyalty by offering plans based on a contract that lasts one, two, three or more years. These offers sometimes seem unbeatable, and adult learners could rush to benefit from these plans for fear of missing out on a unique opportunity. By learning to use an algebraic model to extrapolate in this situation, they would be better able to critically assess the choices offered. In rigorously analyzing and comparing the available options, they could become more objective and more realistic about the long-term costs involved. By developing the competency *Exercises critical judgment*, adult learners could take a more thoughtful approach before signing a contract that will lock them in for many months.

Methodological Competency

Representing a functional model involving any kind of complexity could be made easier through the use of special software. In order to see their task through to completion, adult learners could develop the competency *Uses information and communications technologies* to create and manipulate graphs by modifying certain parameters. For example, it would be easier to compare different cooking methods, since simulation would save on the time involved in trial-and-error experimentation. Using technology, adult learners could find a model more quickly and focus on analyzing and justifying it rather than on performing algebraic calculations.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of arithmetic and algebra. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following three integrative processes:

- **using an algebraic or graphical model to represent a situation**
- **interpolating or extrapolating from an algebraic or graphical model**
- **using an algebraic or graphical model to generalize a set of situations**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover all three processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Relation, function and inverse	
• Experimenting with real functions as well as observing, interpreting, describing and representing them	The real functions studied are: • second-degree polynomial function $f(x) = ax^2$ • exponential function $f(x) = ab^x$, where $a \neq 0$ and $b > 0$ • periodic function • step function • piecewise function Functions can be represented using: • a table of values • an algebraic rule • a graph, with or without the use of technology *For periodic, piecewise and step functions, graphical representations of the context are emphasized even if, in some cases, the symbolic register could be used.* *In determining the value of the exponent in an exponential function, adult learners use a graph, a table of values or technological tools.*

<table>
<tr><th>Mathematical Knowledge</th><th>Restrictions and Clarifications</th></tr>
<tr><td>Relation, function and inverse (cont.)</td><td></td></tr>
<tr><td>• Describing and interpreting the properties of real functions using a graph</td><td>The properties of real functions studied are:
• domain and codomain (range)
• increasing and decreasing intervals
• extrema
• sign
• x- and y-intercepts
The properties of functions must be studied in context.</td></tr>
<tr><td>System</td><td></td></tr>
<tr><td>• Representing a situation using straight lines</td><td>The properties of the following lines are studied:
• parallel lines
• intersecting lines
• coincident lines
• perpendicular lines

The equation of the line in standard form:
• $f(x) = ax + b$

The symmetric and general forms of the equation of a line are not covered in the Cultural, Social and Technical option.</td></tr>
<tr><td>• Solving systems of first-degree equations in two variables</td><td>Systems of equations may be solved by means of:
• a table of values
• an algebraic method chosen by the adult learner
• a graphical method, with or without the use of technology</td></tr>
</table>

Cultural References

Proportional reasoning is used in everyday life and in different occupations in fields such as construction, the arts, health care, tourism and business administration. Observations of the dependency relationship between two quantities contributed to the development of the concept of function, which is used in such areas as navigation, astronomy and ballistics. Adult learners will have the opportunity to discover its importance and to appreciate how different mathematicians, such as Oresme, Descartes and Fermat, contributed to the development of this concept.

More recently, Thomas Malthus analyzed arithmetic and geometric progressions in his work on population growth and the growth of the food supply. With this idea in mind, adult learners could be asked to make observations, comparisons or decisions regarding different instances of growth and decline in fields such as demography and finance.

Today, scuba divers quickly learn the principle of decompression stops in order to prevent any nitrogen that accumulated in the body during the dive from forming bubbles in their blood if they come up to the surface too rapidly. Divers learn their decompression tables by heart. Adult learners who are interested in this sport and in safety could study the related principles through modelling and graph the duration of the decompression stops as a function of the depth of the dive.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Relationship between quantities* involve problems that can be solved in part by using an algebraic or graphical model to represent a relationship between quantities in a general context. The *Algebraic and Graphical Modelling in a General Context 1* course gives adult learners an opportunity to express a connection or a dependency relationship between quantities.

In the situational problems in this course, adult learners derive rules and conditions that determine the number of solutions for the system and make generalizations, they find connections between algebraic and graphical representations in studying a system of equations, and they attempt to extrapolate results using an algebraic rule or a graph.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Environmental Awareness and Consumer Rights and Responsibilities, and Career Planning and Entrepreneurship.

Environmental Awareness and Consumer Rights and Responsibilities

Studying certain functions could help adult learners calculate the real cost of a cell phone and select the best plan, depending on how they intend to use their phone. The course content could help them make better decisions based on their needs and budget. Adult learners could therefore learn to critically assess the choices they make as consumers, which ties in with the educational aim of this broad area of learning.

Career Planning and Entrepreneurship

The content of this course could prove useful in conducting a feasibility study prior to organizing a student activity that involves a certain investment. An analysis of linear functions would make it easier to understand the concepts of break-even point (zeros of the function) and increasing and profitability interval (rate of change). Adult learners could use a greatest-integer function to graph economies of scale associated with purchasing or leasing materials. They could use the concepts learned in this course to master the strategies that apply to their project, which ties in with one of the focuses of development of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Environmental Awareness and Consumer Rights and Responsibilities
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Relationship between quantities
Targeted cross-curricular competency • Is developed at the same time and in the same context as the subject-specific competencies.	• Exercises critical judgment
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the ***Relationship between quantities*** family of learning situations
If an adult learner wants to have a cell phone, this course could enable him or her to compare different plans and determine the one that is most suitable or worthwhile if the phone is used for a certain number of minutes.	**Integrative processes:** *Using an algebraic or graphical model to represent a situation* *Interpolating or extrapolating from an algebraic or graphical model* In carrying out the four phases in the problem-solving process, adult learners could: Representation • Select the most appropriate model (algebraic or graphical representation) and determine the point when a plan, which seemed less attractive at the outset, becomes more worthwhile than another Planning • Use algebraic representation to compare the monthly cost of using a given set of telephone services, through interpolation or extrapolation, as applicable

<table>
<tr><th>Situational problem</th><th colspan="2">Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the Relationship between quantities family of learning situations</th></tr>
<tr><td rowspan="2"></td><td>Activation</td><td>Establish the algebraic rule that relates the various elements of the situation (the number of minutes being the independent variable and the rate of change corresponding to the rate per minute of use) to be able to extrapolate the cost of each plan for a given number of minutes of use
Make a conjecture about the various plans, taking into account the case when the time limits for each plan are exceeded, then verify the conjecture algebraically or graphically
Use a graphing calculator to compare plans and determine the best one, or determine the point at which the plan becomes worthwhile</td></tr>
<tr><td>Reflection</td><td>Conclude that, for extreme values, graphical extrapolation is not very realistic
Validate a graphical extrapolation through algebraic calculation
Decide that it is better not to buy a cell phone and justify their decision mathematically</td></tr>
</table>

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Relationship between quantities*, adult learners represent a situation, interpolate or extrapolate, and use an algebraic or graphical model to generalize a set of situations. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language.*

To represent a situational problem using an algebraic or graphical model, adult learners employ different strategies to identify the problem. They reformulate the situational problem in their own words and determine the key elements and the obstacles to be overcome. They find ways of illustrating the dependency relationship between two quantities, using tables of values, diagrams or Cartesian coordinate graphs. They choose the most accurate representation, aware that it does not necessarily reflect what they have observed, but that it is the best choice given the functions studied in the course. They validate their choice, comparing their solution against existing studies or experiments. For example, they may compare the mathematical model against the physical reality of the situation. In representing the situational problem, they determine the purpose of the message and observe mathematical codes and rules in order to effectively communicate their intention. They choose the register of representation best suited to the situation (e.g. table of values, Cartesian coordinate graph, algebraic equation, real functions covered in the course).

To make decisions, adult learners interpolate or extrapolate results from an algebraic or graphical model. They interpret the model, making connections between the elements of the message and distinguishing between those that are relevant and those that are not. They recognize the purpose of the message and determine its overall meaning. In addition, they use mathematical reasoning to explore the situational problem and to determine questions about the issue. They gather relevant information in order to draw a conclusion. They make one or more conjectures, suggesting probable or plausible ideas and anticipating the implications of these ideas as needed. They use examples to find invariants that will help them formulate their conjecture.

Mathematical reasoning involves using an algebraic model to generalize a set of situations. To do this, adult learners determine questions about the patterns observed. They gather relevant information about the relationships between the quantities (e.g. growth rate of exponential functions, height and length of the steps in a step function). They make conjectures, suggesting equations or formulas, or drawing sketches to identify invariants. They construct and use networks of cognitive resources in order to test their model. They draw conclusions and formulate them correctly, in accordance with the rules and conventions of mathematics. Their mathematical message is clear and accurate, and illustrates as effectively as possible the generalization of the set of situations using quadratic, exponential, step or other types of functions. They identify the weaknesses of the models, as well as the nuances between them and the reality observed.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (relation, function, inverse and systems of linear relations). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

* The solution includes a procedure, strategies and a final answer.
** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*

Course

MTH-4152-1

Data Collection in a General Context

Mathematics

INTRODUCTION

The goal of the *Data Collection in a General Context* course is to enable adult learners to deal with situations that involve collecting or processing data pertaining to a one or a two-variable distribution in a general context.

In this course, adult learners continue to learn about descriptive statistics and begin to make intuitive inferences. In order to make informed decisions in realistic situations, they use their number and operation sense, proportional reasoning and the concept of statistical data to interpret and evaluate conjectures. Their decisions can also be based on an analysis of different sources of bias or the effect of changing certain elements, or on a calculation of other relevant measures. In situational problems, adult learners make decisions based on statistical data. They organize data and study one- or two-variable distributions in which they must determine statistical measures (correlation coefficient, measures of central tendency, dispersion or position). For example, they may be asked to describe a population and draw conclusions about it. Lastly, they may be required to describe a situation based on a graph or table of values. In some cases, they may have to write a series of questions before collecting data or producing a statistical report. In other cases, they can identify and interpret different measures that they themselves or others have taken.

By the end of this course, adult learners will be able to collect data. They will also compare other, similar sets of data when solving a problem that they themselves have defined. They will present the results of their analysis in accordance with the rules and conventions of mathematics. They will use problem-solving strategies to make the best decisions and determine the most accurate solution. In addition, they will be able to use mathematical reasoning to interpret statistical information resulting from the collection of data.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

PROCESS AND STRATEGIES	
REPRESENTATION	
- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. - They use observational and representational strategies that are essential to inductive reasoning. - In attempting to understand the context and the problem, they use deductive reasoning, particularly in situations that involve implicit data.	
Examples of strategies	• Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to find a relationship between variables when looking for a correlation • Organizing data from a sample in order to describe a population and make it easier to process the information • Listing their statistical strategies and knowledge pertaining to the situation • Describing the characteristics of the situation • Gathering relevant information
PLANNING	
- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient. - They develop a plan, taking into account the elements of mathematical language (symbols, terms and notation used, and the different registers of representation). - Through reasoning, they establish organized and functional relationships among different aspects of their knowledge, thus expanding their networks of cognitive resources.	
Examples of strategies	• Systematically determining the correlation model best suited to the situation, bearing in mind that the dispersion of the data must be taken into account in order to make the right choice • Determining the most appropriate measures of central tendency and dispersion in order to establish connections between data values
ACTIVATION	
- When dealing with a situational problem, adult learners can make connections between the graphical representation, the correlation coefficient and the interdependence of the variables involved. - By drawing on their knowledge of statistics, they are able to deduce certain relationships	
Examples of strategies	• Using a table to connect the elements associated with the correlation: ordering the statistical data, finding the median, mean, mean deviation and so on • Drawing the regression line based on the means or medians • Using technology (e.g. spreadsheet program, graphing calculator) to analyze the role of the different parameters of the rule of the correlation line
REFLECTION	
- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution. - Reasoning can be used to reject extrapolations that would yield nonsensical results. - When decoding mathematical elements, adult learners make sure they can distinguish between the everyday meaning of the terms used and their meaning in statistics.	
Examples of strategies	• Checking their solution by substituting the values of the variables in the algebraic expression of the regression line in order to validate a graphical interpolation or extrapolation, etc. • Using a spreadsheet program to validate their work.

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the *Processing data* family of learning situations. Two of these are considered particularly relevant to this course: *Exercises critical judgment* and *Uses information and communications technologies*.

Intellectual Competency

Situations that involve statistics require adult learners to use the competency *Exercises critical judgment* before deciding on the relevance and validity of the information based on given criteria that reflect the goal of a study. For example, checking a correlation between two data values or accurately calculating measures of position and dispersion could help adult learners develop this competency by encouraging them to reject prejudices and preconceived ideas. The study of statistics and probability could help them understand the role that reason and preconceived notions play in the formation of an opinion.

Methodological Competency

The competency *Uses information and communications technologies* is a definite asset for anyone who intends to work with statistical distributions. Among other things, a spreadsheet program is used to calculate standard deviation, draw a graph for a one-variable distribution, calculate a correlation coefficient and draw a scatter plot for a two-variable distribution.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on some of their previously acquired mathematical knowledge. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following three integrative processes:

- **collecting data**
- **comparing collections of data**
- **interpreting data resulting from an experiment**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover all three processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
One-variable distribution	
• Determining and interpreting measures of position and dispersion	The measures of position and dispersion studied are: • percentile • mean deviation *In the analysis and interpretation of a distribution, the adult learners' understanding of mean deviation should be considered more important than the calculations involved.*
• Representing statistical data related to a population or a sample	The register of representation studied is the stem-and-leaf plot.
Two-variable distribution	
• Constructing and interpreting two-variable distributions	In the study of linear correlation, analysis and communication should be considered more important than calculations.
• Drawing a scatter plot	

Mathematical Knowledge	Restrictions and Clarifications
Two-variable distribution (cont.)	
• Representing the regression line by means of a rule or graph	In this course, adult learners need only approximate the equation of the regression line. They could determine the equation of the line using two points in the scatter plot, one of which could be the mean of the *x*- and *y*-intercepts. The regression line can be determined using the median-median line method or the Mayer line method. However, these methods are optional.
• Interpolating or extrapolating using the regression line	
• Approximating and interpreting the correlation coefficient	
• Interpreting a correlation qualitatively and quantitatively	The characteristics of the correlation are: positive, negative, zero, perfect, strong, moderate or weak. Interpretation of the correlation is limited to cases involving linear correlations, which can be estimated using a graphical method (box method or ellipse). The exact value of the correlation coefficient is determined using technology.

Cultural References

Statistics play a major role in our society. Newspapers are full of examples of statistical data: the annual number of deaths related to smoking over the last 20 years, the success rate of an employment program by age or region, or the number of goals and assists scored by National Hockey League players. Data in statistical reports influence the decisions of corporations, municipalities, insurance companies and a variety of other organizations in different fields. In addition, the advent of computers has made it considerably easier to process these data.

Adult learners who like hockey could, for example, use data compiled in sports magazines to check whether there is a correlation between the number of goals scored by a forward and the amount of ice time he gets.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Processing data* involve problems that can be solved in part by collecting or processing data in a general context. The *Data Collection in a General Context* course provides adult learners with an opportunity to learn how to collect and compare data.

In the situational problems in this course, adult learners establish organized and functional relationships among different aspects of their knowledge, thus expanding their networks of cognitive resources, deduce certain relationships in a situation by drawing on their knowledge of correlation, and exclude data from a correlation analysis because they are too far from the scatter plot.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Environmental Awareness and Consumer Rights and Responsibilities, and Career Planning and Entrepreneurship.

Environmental Awareness and Consumer Rights and Responsibilities

The concepts of linear correlation studied in this course could help adult learners make better financial decisions when buying insurance. Adults could establish the correlation between each of the factors that affect the cost of the insurance and the payments to be made in order to be covered. A comparison of various insurance options could then be conducted. Adult learners will be better equipped to make informed consumer decisions, which ties in with one of the focuses of development of this broad area of learning.

Career Planning and Entrepreneurship

Various services such as a cafeteria or a student radio station can be offered in an adult education centre. Before providing these services, it is necessary to make sure that they meet learners' needs. In addition, these services must be adjusted as the student population changes. Studying statistical data—particularly the relationships between different factors and extrapolation based on historical and current data—will help adult learners make informed decisions regarding such a project. Carrying out a project of this scope enables them to fulfill their potential and carve out a place for themselves in society, which ties in with the educational aim of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Environmental Awareness and Consumer Rights and Responsibilities
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Processing data
Targeted cross-curricular competency • Is developed at the same time and in the same context as the subject-specific competencies.	• Exercises critical judgment
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Processing data* family of learning situations
An adult learner wants to buy car insurance. He realizes that there is discrimination involved in determining premiums. For example, the driver’s age and gender affects the amount of the premium. Adult learners are asked to analyze and interpret the different factors that affect the cost of insurance premiums.	**Integrative process:** *Comparing collections of data* In carrying out the four phases in the problem-solving process, adult learners could: Representation • Consult statistical reports on the SAAQ Web site • Obtain information on car insurance rates from one or more insurance companies Planning • Determine the variables that can be correlated, for example: – the amount of the premiums and the driver’s age – the amount of the premiums and the number of claims filed – age and number of accidents – the number of years of driving experience and the cost of the premiums

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Processing data* family of learning situations	
	Activation	• Make conjectures based on the type of correlation involved. For example, adult learners could assume that premiums that are justified by the link between the driver's age and the number of accidents in this age category could also be related to geographic location. This conjecture could also be confirmed through another statistical analysis.
	Reflection	• Draw conclusions from an analysis of the results by distinguishing relevant from irrelevant information

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Processing data*, adult learners collect, compare and interpret data resulting from an experiment. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To collect data, adult learners use problem-solving strategies to identify the issue and define the tasks to be carried out. They determine the important elements and the obstacles to be overcome, with a view to differentiating between one- and two-variable statistical distributions. In addition, in working out their solution, they establish and carry out a plan involving the previously validated steps: data collection and processing (interpretation and analysis). The last two steps require that they use mathematical reasoning to explore the issue in question and identify patterns. They make conjectures based on a correlation line in order to make decisions in the medium or long term. For example, they draw conclusions when they derive laws or rules associated with percentile and mean deviation. Lastly, to produce mathematical messages, they use an appropriate register of representation given the constraints of the situational problem: Mayer line, median-median line, table or stem-and-leaf diagram in the case of a one-variable statistical distribution.

To compare collections of data, adult learners interpret a mathematical message, making connections between the elements of the message, determining its overall meaning, or associating pictures, objects or knowledge with mathematical terms and symbols. In addition, they use mathematical reasoning to compare trends, developing and using networks of mathematical cognitive resources such as percentile, mean deviation and correlation coefficient.

To interpret data resulting from an experiment—one- or two-variable statistical distributions—adult learners decode the elements of mathematical language, distinguishing between the mathematical and everyday meanings of terms. In addition, they interpret mathematical messages, switching from one register of representation to another (e.g. from a stem-and-leaf diagram to a one-variable data table) and making sure they understand what is involved. They use mathematical reasoning to develop networks of mathematical cognitive resources, such as the Mayer line or median-median line, and to determine the correlation coefficient using the box method or ellipse. They make generalizations, derive laws and rules, and deduce propositions in order to make informed decisions.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (one- or two-variable statistical distributions). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

  * The solution includes a procedure, strategies and a final answer.

  ** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*

Course

MTH-4153-2

Geometric Representation in a General Context 1

Mathematics

MTH-4153-2 Geometric Representation in a General Context 1

INTRODUCTION

The goal of the *Geometric Representation in a General Context 1* course is to enable adult learners to use trigonometry to deal with situations that involve the geometric representation of an object or a physical space in a general context.

In this course, adult learners encounter various situational problems that enable them to expand their knowledge of geometry, and trigonometry in particular. They use and improve their knowledge of plane figures, solids, isometries, similarity transformations and projections, and apply and develop their ability to represent and construct figures or determine and deduce measurements. In some situational problems, they must find unknown measurements (length, area, volume) using different metric or trigonometric relations and the properties of right triangles or congruent, similar or decomposable figures. In addition, they validate certain conjectures through simple deductive reasoning based on their knowledge of geometry. They justify their choices and the steps in their process. Lastly, in communicating using mathematical language, they identify and interpret different measurements that they themselves or others have taken, or information contained in drawings and geometric constructions. In geometric contexts, they decode the information in a geometric figure or the elements of the construction of an object based on a two-dimensional representation. They also use their spatial sense, as well as their knowledge of measurement and proportionality, to describe this representation and interpret geometric figures in contexts involving the concepts of similarity or trigonometry.

By the end of this course, adult learners will be able to use the properties of congruent or similar figures and trigonometric relations to represent and describe an object or a physical space. They will also be able to use different strategies and types of reasoning to manage various situations in accordance with the mathematical rules and conventions used in geometry.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

<table>
<tr><th colspan="2">PROCESS AND STRATEGIES</th></tr>
<tr><th colspan="2">REPRESENTATION</th></tr>
<tr><td colspan="2">- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. They use observational and representational strategies that are essential to inductive reasoning.
- In describing the characteristics of the situational problem, they organize the elements that will enable them to plan the main steps in the deductive reasoning they will use to determine whether or not they are dealing with a case of similarity, and master the elements of mathematical language.</td></tr>
<tr><td>Examples of strategies</td><td>• Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to identify a dependency relationship and determine the variables in the situation
• Representing the situational problem mentally or in writing
• Listing their geometry-related strategies and knowledge pertaining to the situation
• Describing the characteristics of the situation
• Gathering relevant information</td></tr>
<tr><th colspan="2">PLANNING</th></tr>
<tr><td colspan="2">- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient.
- They develop a plan, taking into account the elements of mathematical language (key elements, subject of the message, overall meaning of the situation).
- They use different registers of representation to illustrate certain properties of trigonometric ratios.</td></tr>
<tr><td>Examples of strategies</td><td>• Dividing the situational problem into subproblems
• Using lists, tables, diagrams, concrete materials or drawings to plan their solution</td></tr>
<tr><th colspan="2">ACTIVATION</th></tr>
<tr><td colspan="2">- When dealing with a situational problem, adult learners identify patterns by exploring different figures.
- They also observe mathematical codes, symbols and rules.</td></tr>
<tr><td>Examples of strategies</td><td>• Solving certain situational problems by working backwards when the solution consists of several steps or when there is insufficient information
• Analyzing the parameters of a right triangle, for example, to properly understand how they are related to the parameters of any given triangle</td></tr>
<tr><th colspan="2">REFLECTION</th></tr>
<tr><td colspan="2">- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
- They validate their message using new mathematical symbols to describe the organization or representation of their physical environment, consulting different references in the process.
- Validating certain results could involve making conjectures about particular or special cases involving any triangle in order to see the effect on the area or the perimeter of a triangular shape when the angle varies in the cosine law or the sine law formula.</td></tr>
<tr><td>Examples of strategies</td><td>• Checking their solution by means of examples or counterexamples, particularly by using the Pythagorean theorem to validate the lengths of the sides of a triangle to be able to conclude that it is, in fact, a right triangle
• Making sure that their solution makes sense, for example, by using the sine law to determine whether an angle of any given triangle is acute or obtuse
• Using a calculator or geometric modelling software to validate their work</td></tr>
</table>

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Measurement and spatial representation*. Two of these are considered particularly relevant to this course: *Uses creativity* and *Adopts effective work methods*.

Intellectual Competency

Creative individuals find imaginative ways of using the resources and materials available to them. Representing physical spaces and organizing a layout often call for varied and personal approaches, which could lead adult learners to express their ideas and use their intuition. For example, they could determine how they will go about drawing a scale diagram of a room or a piece of land. Since objects are sometimes out of reach and therefore difficult to measure, adult learners must find an innovative and feasible way of measuring them. In this course, the competency *Uses creativity* enables them to explore situational problems in original ways.

Methodological Competency

The competency *Adopts effective work methods* can be developed in dealing with a situational problem that involves the organization of a physical space. For example, a great deal of rigour and precision is required to design a scale model that accurately represents the object in question. In this regard, it is crucial to choose the right measuring instruments. Adult learners take into account all the constraints and analyze their consequences. To see their project through to completion, they carefully plan it and manage their time efficiently. In this way, they use a methodical approach suited to the given context.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of geometry. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following two integrative processes:

- **organizing a physical space**
- **describing an object or a physical space and representing it in two or three dimensions**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover both processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Metric and trigonometric relations in triangles	
• Representing and interpreting situations using triangles	The trigonometric ratios studied are: sine, cosine and tangent. The sine law and Hero’s formula are also studied in this course. The other metric and trigonometric relations are listed in the Principles table that comes after this table.
• Describing the properties of trigonometric ratios	Adult learners intuitively use the properties of trigonometric ratios to justify the steps in their solution, but they are not required to prove these properties.
• Determining the slope, measurements and positions using metric and trigonometric relations in triangles	The measurements and positions studied in this course pertain to: • the angles in a triangle • the altitude to the hypotenuse, the orthogonal projection of the legs on the hypotenuse • the sides of a triangle • the area of a triangle and a quadrilateral • the coordinates of a point (point of division) • the length of a segment • distance (between two points)

Mathematical Knowledge	Restrictions and Clarifications
Similar and congruent triangles	
• Determining the minimum conditions required to conclude that triangles are congruent or similar	These conditions are listed in the Principles table that comes after this table.

Principles
Adult learners must master the following compulsory principles, which may be used in a proof: **P1.** If the corresponding sides of two triangles are congruent, then the triangles are congruent. **P2.** If two sides and the contained angle of one triangle are congruent to the corresponding two sides and contained angle of another triangle, then the triangles are congruent. **P3.** If two angles and the contained side of one triangle are congruent to the corresponding two angles and contained side of another triangle, then the triangles are congruent. **P4.** If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the triangles are similar. **P5.** If the lengths of the corresponding sides of two triangles are in proportion, then the triangles are similar. **P6.** If the lengths of two sides of one triangle are proportional to the lengths of the two corresponding sides of another triangle and the contained angles are congruent, then the triangles are similar. **P7.** In a right triangle, the length of the side opposite an angle of 30° is equal to half the length of the hypotenuse. **P8.** The lengths of the sides of any triangle are proportional to the sines of the angles opposite these sides: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ (sine law). **P9.** The area *A* of a triangle whose sides measure a, b, and c is: $A = \sqrt{p(p-a)(p-b)(p-c)}$ where p is half the perimeter of the triangle (Hero's formula). **P10.** The length of a leg of a right triangle is the geometric mean between the length of its projection on the hypotenuse and the length of the hypotenuse. **P11.** The length of the altitude to the hypotenuse of a right triangle is the geometric mean between the lengths of the segments of the hypotenuse. **P12.** The product of the lengths of the legs of a right triangle is equal to the product of the length of the hypotenuse and the length of the altitude to the hypotenuse.

Cultural References

Geometric figures are found everywhere in our environment, both in human creations (e.g. works of art, various objects, fabrics, wallpaper, architecture, structures) as well as in nature (e.g. crystallography, trajectories).

A number of mathematicians, such as Archimedes, Hero of Alexandria, Galileo and Leonardo da Vinci, designed machines, tools and measuring instruments, some of which are still used today. Adult learners can identify properties of measuring instruments used in drawing, navigation, geodesy or astronomy. They can appreciate how a number of instruments (e.g. balance, odometer, global positioning system, compass, sextant, quadrant) used today or in the past have helped solve real-world problems. Furthermore, surveying equipment, the mirror and shadow technique, the pantograph, the proportional compasses, and Jacob's and Gerbert's staffs can help adult learners develop their understanding of the concept of similarity.

Different branches of geometry have been developed to address various questions and needs. One of the most recent types of geometry to emerge is fractal geometry, which is used to model a variety of things including different natural occurrences such as atmospheric phenomena, floral patterns and geographical features. It is used in the arts and in digital imaging. Examples given during the course will enable adult learners to understand its importance.

FAMILY OF LEARNING SITUATIONS

The situations in the *Measurement and spatial representation* family of learning situations involve problems that can be solved in part through the geometric description or representation of an object or a physical space. The *Geometric Representation in a General Context 1* course provides adult learners with an opportunity to develop their spatial representation skills.

In the situational problems in this course, adult learners determine the corresponding sides and angles of two triangles, identify the corresponding sides of two similar triangles by recognizing common identification codes, and validate their message using new mathematical symbols to describe an organization or representation of their physical environment.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Citizenship and Community Life, and Health and Well-being.

Citizenship and Community Life

During a meeting of the board of their local community centre, adult learners who wish to be active in their community could present a development plan for a playground. The geometry concepts acquired in this course could help them complete this project, which ties in with one of the focuses of development of this broad area of learning.

Health and Well-Being

This course will prove useful for adult learners who spend long hours at their computer, whether it be for study, work or pleasure. Work space arrangement could help them adopt a better, more comfortable posture. Adults learn about certain ergonomic concepts related to computer use (e.g. proper posture in front of the computer, the right distance between the eyes and the screen, the proper angle for the arms and legs). To rearrange their work space and office, they determine how much distance there should be between themselves and their tools and where their tools should be positioned, depending on whether they are right-handed or left-handed. They also calculate the area of their work space and determine the type of lighting that will make it as easy as possible to read the screen. The geometry concepts studied in this course can help them plan the layout of the available space, which will prompt them to be more concerned with safety in accordance with one of the focuses of development of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Citizenship and Community Life
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Measurement and spatial representation
Targeted cross-curricular competencies • Are developed at the same time and in the same context as the subject-specific competencies.	• Uses creativity • Adopts effective work methods
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	**Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Measurement and spatial representation* family of learning situations**	
A local community organization asked the parents of the children who attend the centre to find ways to decrease violent behaviour in the neighbourhood. Since an analysis of the situation showed that there are too few play areas in the neighbourhood, it was recommended that a playground be built.	**Integrative process:** *Describing a physical space and representing it in two dimensions* In carrying out the four phases in the problem-solving process, adult learners could:	
	Representation	• Make note of the relevant information about the space to be used for the playground: dimensions, irregular shapes, whether or not there is a fence, proximity to the street, etc.
	Planning	• Describe the available space, make a sketch of the lot and use appropriate language • Choose the scale to be used to produce a clear and sufficiently detailed plan

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Measurement and spatial representation* family of learning situations	
The community organization has the funds to pay for materials, provided the work is done on a volunteer basis.	Activation	• Calculate the available surface areas for each play structure • Calculate the height of the structures, the distance between them, the angle of elevation of the slides, etc.
Adult learners are asked to draw up the layout plan for the playground.	Reflection	• Validate the resulting measurements by means of trigonometric ratios other than those used to find the measurements

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Measurement and spatial representation*, adult learners describe and represent objects and physical spaces in two or three dimensions, and organize a physical space. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To describe an object or a physical space and represent it in 2-D or 3-D, adult learners interpret and produce sketches, drawings or plans using different relations associated with geometric figures. In situational problems, they find measurements (length, area, volume) using different metric or trigonometric relations involving right triangles or congruent, similar or decomposable figures. They deduce properties based on their knowledge of geometry and validate their conjectures, justifying all the steps in their process. In addition, in producing a mathematical message, they identify the key elements of mathematical language (e.g. scale, dimensions, perimeter, area, volume) and associate images, objects or knowledge with mathematical terms and symbols. They apply their newly acquired knowledge (Hero's formula and the sine law), which enables them to find measurements in unusual situations.

To organize a physical space, adult learners use a variety of strategies such as producing a sketch or drawing, or dividing the task into subtasks. They use a complex process that includes representing the problem and validating their solution based on their knowledge of trigonometry. They use the concept of triangulation to organize a physical space and validate all of their steps using the theorems covered in the course. They find measurements using deductive reasoning, identify results using inductive reasoning, and draw conclusions based on their study of the theorems in question.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (trigonometric and metric relations in triangles, and similar and congruent triangles). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

* The solution includes a procedure, strategies and a final answer.
** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*

6.2.2 Secondary V Courses

Course

MTH-5150-2

Optimization in a General Context

Mathematics

INTRODUCTION

The goal of the *Optimization in a General Context* course is to enable adult learners to deal with optimization situations that involve using graphs or linear programming, or finding measurements in order to design or use three-dimensional objects in a general context.

In this course, adult learners are introduced to linear programming and are required to apply their knowledge of arithmetic and algebra to different situational problems involving specific constraints. They use their ability to translate a situation into equations or inequalities and to work with algebraic expressions. They draw the Cartesian coordinate graph of the corresponding system, interpreting the inequality relations. In optimization situations, they determine the values of the decision variables in the function that optimizes (minimizes or maximizes) a situation involving a number of constraints, which in fact represent limitations related to real-life situations involving optimization.

Adults also learn how to use graphs to model situational problems involving optimization. The situations may involve project planning, communication or distribution networks, circuits, incompatibilities, localizations, strategies and so on. Depending on the situation, adult learners use different types of graphs: trees, directed or undirected, coloured or not coloured, weighted or unweighted. To optimize certain situations, they find the critical path, use graph colouring, or determine trees of minimum value or the shortest path. In addition, they can use graphs to represent or construct labyrinths or winning strategy games. Using a graph representing the outcome of a winning strategy game, adult learners can work backwards to determine which positions could lead to victory.

In this course, adult learners explore situational problems that involve finding certain measurements related to congruent, similar or equivalent figures as well as the properties of figures and metric or trigonometric relations. They compare equivalent figures and determine which one is most appropriate to meet certain objectives (e.g. to maximize or minimize space). They analyze and interpret situations involving measuring instruments, photography, lamps and shadows, etc. For instance, in situations involving packaging, adult learners determine the most economical shape for a container of a given volume, taking into account factors such as ease of storage. They may calculate the ratio between the volume and total area and see a connection between the value of the ratio and the most economical shape.

By the end of this course, adult learners will be able to use half-planes, weighted and directed graphs, or similar, congruent or equivalent figures to represent concrete situations. They will produce clear and accurate work in accordance with the rules and conventions of mathematics. Optimizing situations using systems of first-degree inequalities, inference functions (graphs) or calculations involving geometric data will enable them to make decisions. In addition, they will use different registers of representation to generalize results and extend them to other situations.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

<table>
<tr><th colspan="2">PROCESS AND STRATEGIES</th></tr>
<tr><th colspan="2">REPRESENTATION</th></tr>
<tr><td colspan="2">- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. They use observational and representational strategies that are essential to inductive reasoning.
- In attempting to understand the context and the problem, they use deductive reasoning.
- Certain observations can be made in representing situations involving geometric optimization (e.g. a cube is the right prism with the largest volume).</td></tr>
<tr><td>Examples of strategies</td><td>• Determining the type of optimization most suited to the situation by asking questions
• Using vertices and edges, making an intuitive sketch of a graph that represents the problem
• Listing the mathematical concepts related to graph theory when working with a problem that involves finding the optimal path
• Describing the characteristics of the situation
• Gathering relevant information (e.g. vertices, edges, circuit)
• Determining the nature of the task to be carried out (instructions, expected results, goal, time allotted, etc.) based on specifications, a scale drawing or literal descriptions
• Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to find measurements or spatial representation</td></tr>
<tr><th colspan="2">PLANNING</th></tr>
<tr><td colspan="2">- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient and economical.
- At this stage, they are able to express the constraints of the situation in mathematical language.
- Everything they do is aimed at finding optimal solutions. For example, when attempting to find the optimal path in a graph or a tree, they use their intuition to highlight the edges that could represent this path.
- They may use a graphic representation of the situation to highlight certain metric or trigonometric relations.</td></tr>
<tr><td>Examples of strategies</td><td>• Breaking down the situational problem into subproblems (finding the optimal path involves breaking down the graphs into circuits and paths)
• Finding an algebraic rule that reflects the best relationship between the constraints and possible consequences of the situational problem: determining the relevant parameters of the scanning line or the economic function
• Breaking down the situational problem into subproblems in order to find a measurement using the metric relations in similar figures</td></tr>
<tr><th colspan="2">ACTIVATION</th></tr>
<tr><td colspan="2">- When dealing with a situational problem, adult learners deduce the scale of the axes by analyzing the maximum and minimum values of the variables in order to graph the half-planes resulting from the constraints. They also use simple substitution to deduce certain values of the points of intersection of the boundary lines.
- To avoid confusion, they use the symbols, terms and notation in accordance with their meaning.
- They distinguish between the different types of figures by making sure that the proper codes and rules have been observed.
- They take into account the proportion indicated and use the symbols and conventions related to the concept in question.
- They use a diagram or sketch to illustrate their proof, thereby making it easier for the reader to understand.</td></tr>
<tr><td>Examples of strategies</td><td>• Proceeding by trial and error to mathematize certain constraints or to identify the different paths in the graph
• Enumerating all the possible paths in a graph in order to choose the best solution
• Constructing tables of values in order to find two points to represent the boundary lines of the polygon of constraints
• Trying to identify the figures that optimize the situation in order to fully understand, among other things, the relationship between the characteristics (area and volume) of an object and the constraints that affect the space it occupies</td></tr>
<tr><th colspan="2">REFLECTION</th></tr>
</table>

<table>
<tr><th colspan="2">PROCESS AND STRATEGIES</th></tr>
<tr><td colspan="2">- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
- By reviewing the steps in their work, they hone their ability to use exact mathematical language, particularly when producing a message. They must ensure that their message is clear and that they have observed the relevant codes and conventions.
- They make conjectures about particular or special cases involving any triangle in order to validate certain results using reasoning.</td></tr>
<tr><td>Examples of strategies</td><td>• Checking their solution by, for example, comparing the number of possible solutions for a system of equations with the number of solutions found, or using their intuition to make sure that the coordinates of the points they have found are those of the vertices of the polygon of constraints
• Distinguishing the strategies that are useful in linear programming from those that are used in graph theory
• Determining the strategies for dealing with situational problems in geometry (e.g. applying a rule, referring to a geometry principle, using a formula)</td></tr>
</table>

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Optimizing solutions*. Two of these are considered particularly relevant to this course: *Communicates appropriately* and *Exercises critical judgment*.

Communication-Related Competency

There are many situational problems involving optimization (e.g. planning a market study to predict company earnings, optimizing the expenses involved in planning an advertising campaign, choosing the most economical type of packaging). By developing the competency *Communicates appropriately*, adult learners will be able to approach a problem in a manner that is not purely mathematical. Since communication is an interactive process that demands constant adjustment to a range of possible meanings and mutual expectations, linear programming alone is not sufficient to solve a problem. In addition, knowledge of the methods for producing and distributing media products and the ability to use various techniques, technologies and modes of communication are assets that go beyond the scope of mathematics.

Intellectual Competency

The cross-curricular competency *Exercises critical judgment* can be very useful in a learning situation that involves planning the creation of a media product. In these situations, adult learners become aware of the need to respect intellectual property, defend freedom of expression and respect the privacy and reputation of others. This focus is broader than an approach limited to the mathematization of constraints and the optimization of objective functions. It can also encourage adult learners to overcome prejudice and to go beyond intuitive assumptions. In a learning situation that involves designing packaging, adult learners may be called upon to *exercise their critical judgment* with regard to the sometimes excessive use of packaging in marketing. They therefore become aware that responsible choices made by consumers can save money and energy and, above all, promote better environmental management. Other situations may also enable adult learners to understand the issues surrounding the use of space in different fields, such as advertising or the arts, or in different types of facilities.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of arithmetic and algebra. This knowledge is useful for taking into account constraints in an optimization context. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following three integrative processes:

- **optimizing a situation using linear programming**
- **optimizing a situation using graph theory**
- **optimizing space when designing or using three-dimensional objects**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover all three processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Algebraic expressions	
• Solving first-degree inequalities in two variables	

<table>
<tr><th>Mathematical Knowledge</th><th>Restrictions and Clarifications</th></tr>
<tr><td>Linear programming</td><td></td></tr>
<tr><td>System of first-degree inequalities in two variables</td><td></td></tr>
<tr><td>Representing the constraints and the function to be optimized (objective or economic function)</td><td>Constraints can be represented algebraically or graphically.

In this course, the function to be optimized is expressed solely as an equation of the form Ax + By + C = Z, where A, B and C are rational numbers.</td></tr>
<tr><td>Determining and interpreting the vertices and the feasible region (bounded or unbounded)</td><td></td></tr>
<tr><td>Changing the conditions associated with the situation to provide a more optimal solution</td><td></td></tr>
<tr><td>Graph</td><td></td></tr>
<tr><td>Representing and modelling a situation using a graph</td><td></td></tr>
<tr><td>Comparing different graphs</td><td>The graphs studied in this course, including trees, are the following:
simple graphs (vertices and edges only)
directed graphs
coloured graphs
weighted graphs
connected graphs
complete graphs
The different elements related to the graphs studied in this course are the following: vertex, edge, loop, degree of a vertex, distance, path, circuit, simple path and simple circuit.</td></tr>
<tr><td>Finding Euler and Hamiltonian paths and circuits, a critical path, the shortest path, a tree of minimum or maximum values or the chromatic number</td><td></td></tr>
</table>

Mathematical Knowledge	Restrictions and Clarifications
Finding measurements	
• Equivalent figures	
• Finding measurements: ○ positions ○ angles ○ lengths (segments, chords) ○ areas ○ volumes	These measurements are found by applying the properties of congruent, similar or equivalent figures as well as the properties of figures and trigonometric relations. Metric relations may also be applied.
• Relations in triangles	The trigonometric relations studied involve the cosine law. The trigonometric ratios in right triangles, the cosine law and Hero's formula may also be applied.

Principles

Adult learners must master the following compulsory principles, which may be used in a proof:

P13. A connected graph contains a Euler path if and only if the number of vertices with an odd-numbered degree is 0 or 2.

P14. A connected graph contains a Euler circuit if and only if the degrees of all its vertices are even numbers.

P15. The chromatic number of a graph is less than or equal to $r + 1$, where r is the largest degree of its vertices.

P16. Regular polygons have the smallest perimeter of all equivalent polygons with n sides.

P17. Of two equivalent regular convex polygons, the polygon with the most sides will have the smaller perimeter. (Ultimately, an equivalent circle will have the smaller perimeter.)

P18. Cubes have the largest volume of all rectangular prisms with the same total surface area.

P19. Spheres have the largest volume of all solids with the same total surface area.

P20. Cubes have the smallest total surface area of all rectangular prisms with the same volume.

P21. Spheres have the smallest total surface area of all solids with the same volume.

P22. The square of the length of a side of any triangle is equal to the sum of the squares of the lengths of the other two sides, minus twice the product of the lengths of the other two sides multiplied by the cosine of the contained angle (*cosine law*).

Cultural References

For many years, business and government decision-makers have had to solve problems involving combinatorial analysis, random experiments and corporate competitiveness. This is why many mathematicians have studied the issue.

Swiss mathematician Leonhard Euler (1707-1783), a pioneer in pure and applied mathematics, is regarded as the author of the first theorem stemming from graph theory. Linear programming, which is a branch of optimization commonly used in decision-making, originated with the work of French mathematician Joseph Fourrier (1768-1830) on systems of inequalities, even though these systems have been attributed to American mathematician George Dantzig (1914-2005). While in the United States Air Force during the Second World War, Dantzig developed a technique for solving the army's logistical problems at a minimum cost. This technique, which combines power and flexibility, was soon adopted in business and industry. Businesses used it to solve major economic problems, while industry applied it to production management.

Since the 1970s, linear programming has been applied in a variety of fields such as health care, the environment, agriculture, communications, the oil industry, chemistry, computer science, energy, transportation, industrial production and finance. This breakthrough is the result of advancements in computer technology, which made it possible to deal with situations involving an astronomical number of calculations. Examples given during the course will enable adult learners to understand the importance of linear programming.

Graph theory is another tool that has been used to make transportation and freight companies more profitable. Adult learners who are interested in learning more about the applications of this theory could contact one or more transportation companies to understand how they use graphs to save time and money when determining their routes.

Since prehistoric times, people have used packaging techniques to preserve and transport food. First, they used animal hides, leaves and shells and then began to make packaging: amphora, jars, baskets, and glass and metal containers. With the industrial revolution (end of the 19th century and beginning of the 20th century), throwaway containers made their appearance. Packaging also acquired some additional functions—namely, to make storage easier and inform the consumer. Packaging designers must meet industry demands at the least cost by finding measurements to solve the optimization problems submitted to them.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Optimizing solutions* involve problems that can be solved in part through optimization using linear programming or graph theory or by finding measurements. The *Optimization in a General Context* course provides adult learners with an opportunity to learn how to maximize a profit, a process or a number of objects or people, and to minimize costs or losses.

In the situational problems in this course, adult learners list the mathematical concepts related to graph theory when working with a problem that involves finding the optimal path, use their intuition to highlight the edges that could represent the optimal path in a graph or a tree, and go back to the problem to check whether the solution is closely related to the vertices or the boundary of the polygon of constraints. In a geometrical context, they must distinguish between the different types of figures by making sure that the proper mathematical rules and codes have been observed.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Media Literacy, and Environmental Awareness and Consumer Rights and Responsibilities.

Media Literacy

Some learning situations can involve examining the role the media plays in marketing a product in order to help adult learners become more aware and exercise their critical, ethical and aesthetic judgment with respect to the media. For example, adult learners could try to optimize an investment in an advertising campaign, while taking into account the constraints of the problem. Such factors as the gender, age and income of the target group must be taken into account in mathematizing these constraints. This type of situation ties in with the educational aim of this broad area of learning.

Environmental Awareness and Consumer Rights and Responsibilities

Some of the learning situations encountered in this course could lead adult learners to examine the use of plastics in packaging. For example, they could compare the amount of packaging used by volume depending on whether the product is sold individually or in a family-sized package. They might also consider studying a larger population. In addition, they could try to determine whether there is a relationship between the size and shape of the packaging, the amount of space containers occupy in refrigerated display cases and the cost of transportation. This exercise is aimed at making them aware of the impact of daily consumer choices so that they can develop a more active relationship with their environment. It also enables them to maintain a critical attitude toward consumption and the exploitation of the environment, which ties in with the educational aim of this broad area of learning. In other situations, adult learners may have to organize a company’s delivery schedule. In doing so, they may assess the cost of fuel based on different routes, using graph theory, and thereby determine the possible cost savings and benefits for the environment.

Example of a learning situation

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Media Literacy
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Optimizing solutions
Targeted cross-curricular competencies • Are developed at the same time and in the same context as the subject-specific competencies.	• Communicates appropriately • Exercises critical judgment
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Optimizing solutions* family of learning situations
A company wants to launch a new product. The marketing director must draw up a budget to promote the product in the media. The first step is to develop an advertising plan. Naturally, the director wants the most media coverage at the least cost. Once the most suitable medium has been chosen, adult learners are asked to choose the least expensive advertising plan from among four possibilities.	**Integrative process:** *Optimizing a situation using linear programming* In carrying out the four phases in the problem-solving process, adult learners could: Representation • Determine the key elements to be considered: the number of days required to complete each step (design and implementation) and the cost of each one • Indicate the obstacles to be overcome in implementing the plan Planning • Refer to the solution of a similar situational problem in drawing up the plan • Determine the mathematical knowledge needed to deal with the situation: identify the variables, determine the constraints, establish a system of first-degree inequalities in two variables

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Optimizing solutions* family of learning situations	
To optimize the advertising plan, adult learners must first analyze the four possibilities submitted and graph the constraints associated with each plan (e.g. the minimum number of employees required and their hourly wage, the maximum cost of the materials or the cost of designing the plan).	Activation	• Mathematize the constraints (e.g. the price charged by each supplier) using inequalities • Draw a polygon of constraints to represent the situation and then optimize the expenses • Determine the vertex of the polygon of constraints that represents the lowest cost • Calculate the cost associated with this vertex
	Reflection	• Compare their solution and results with those of others in order to identify the strengths and weaknesses of the model(s), etc. • Determine the number of suppliers required to ensure that the model is reliable and realistic • Make sure the solution is realistic • Examine how a change in one of the constraints will determine which supplier provides the best plan (e.g. Will an increase in the minimum wage result in a different optimal solution?) • Indicate what change a company must make to its plan in order to obtain the contract

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Optimizing solutions*, adult learners optimize a situation using linear programming or graph theory, or for the purpose of designing or using three-dimensional objects. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To use linear programming to solve a situational problem involving optimization, adult learners decode the elements of mathematical language, use symbols and mathematical rules to represent the constraints of the situation in the form of a system of inequalities, produce a graph to illustrate the polygon of constraints, and determine the coordinates of the vertices. Then they examine every possible solution using the scanning line and distinguish between discrete and continuous solutions. Lastly, they take the time to validate their solution according to the context, and distinguish the vertices that belong to or limit inequalities. Faced with a conjecture, they compare, evaluate and critically examine choices or processes and establish proofs, if applicable. After taking a position, they choose the best solution. They justify all the steps in their process (solution and result) and determine either an optimal solution or reasons for rejecting the conjecture. They also explain the possible effects of changing certain constraints and generalize the situations, as needed.

By using graph theory to solve situational problems involving optimization, adult learners can clearly represent the situation using a graph, identify the vertices and edges that correspond to the context, and judge whether or not to assign a value or direction to the edges. They count the possible number of paths and select the critical path by analyzing their solution and comparing it with the context of the situational problem. In addition, when proving principles related to graphs, they use the three theorems covered in the course to find results through deductive or inductive reasoning.

In exploring situational problems involving the optimization of space when designing or using three-dimensional objects, adult learners compare equivalent figures and determine which ones best meet certain objectives (maximize or minimize the space). They establish relationships between the total surface areas and volume of solids. They employ various strategies to find an algebraic solution to the situational problem, applying their mathematical knowledge related to functions. They use their knowledge of geometry to design and produce plans and objects. They determine different measurements based on established definitions, properties, formulas or principles in the case of triangles, plane figures, or congruent, similar or equivalent solids. In addition, when producing and validating their solution, they rigorously justify the different steps in their work, providing formal proofs.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (linear programming, graphs and finding measurements). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

  * The solution includes a procedure, strategies and a final answer.
  ** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*

Course

MTH-5151-1

Algebraic and Graphical Modelling in a General Context 2

Mathematics

INTRODUCTION

The goal of the *Algebraic and Graphical Modelling in a General Context 2* course is to enable adult learners to deal with situations that involve using an algebraic or graphical model to represent a dependency relationship between quantities in a general context.

In Secondary V, adult learners continue to study real numbers and to expand their knowledge of powers and logarithms. Some of the situations they encounter may require them to determine the approximate value of an exponent (logarithm) by using a graph, a table of values or a calculator. They may have to change exponential notation to logarithmic notation and vice versa.

In some situations, adult learners apply their knowledge of real functions such as the second-degree polynomial function ($f(x) = ax^2$, where $a \neq 0$) and the exponential function ($f(x) = ab^x$, where $a \neq 0$ and $b > 0$). The purpose of these functions is to help adult learners to compare, analyze and recognize the characteristics of the curve in order to be able to select the function that is best suited to the situation. Thus, adult learners may use such strategies as the second difference of the *y*-values (second-degree polynomial function) or strategies that involve multiplication of *y*-values when the data are presented in a table of values (exponential function). With respect to the real functions previously seen in the course *Algebraic and Graphical Modelling in a General Context 1* (MTH-4151-1), adult learners may be required to calculate values, make graphs and analyze the properties of a particular function, without having to provide an algebraic solution to the situational problem.

Furthermore, the purpose of this course is to introduce adult learners to financial mathematics and to familiarize them with the related vocabulary. Adult learners will be required to calculate and analyze the future value of a sum of money (capital) invested for one period at a fixed annual interest rate as well as determine the future value of capital invested at an annual compound interest rate for several periods. Adult learners may also be required to compare interest rates with a view to determining the most advantageous one so that they can make informed decisions.

At the end of this course, adult learners will be able to represent concrete situations using exponents or logarithms and to analyze economic (e.g. personal finances), social, technical and everyday situations. They will produce clear and accurate work in accordance with the rules and conventions of mathematics. By algebraically or graphically representing a situation using real functions and operations on these functions, they will be able to induce results through interpolation or extrapolation. They can interpolate or extrapolate using a table of values, a graph or algebra when the algebraic rule is given. Lastly, adult learners will use different registers of representation (tables of values, graphs or algebraic rules) to generalize a model so that it can be applied to a range of situations.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly while observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phase in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

<table>
<tr><th colspan="2">PROCESS AND STRATEGIES</th></tr>
<tr><th colspan="2">REPRESENTATION</th></tr>
<tr><td colspan="2">- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. They use observational and representational strategies that are essential to inductive reasoning.
- They increase their knowledge of mathematical notation and symbols related to functions and inverse functions expressed in the general form.</td></tr>
<tr><td>Examples of strategies</td><td>• Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to identify a dependency relationship and determine the variables in the situation
• Using examples involving numbers, determining the types of relationship that exist between the variables in the situation
• Using a logarithmic scale</td></tr>
<tr><th colspan="2">PLANNING</th></tr>
<tr><td colspan="2">- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient.
- They attempt to extrapolate results using an algebraic rule or a graph, thus expanding their networks of cognitive resources.
- To correctly plan their solution, they decode elements of mathematical language, such as the meaning of the symbols, terms and notation used, as well as the different registers of representation.</td></tr>
<tr><td>Examples of strategies</td><td>• Drawing a concept map showing the different steps in the solution
• Referring to a list of elements to be considered in consolidating their work plan (e.g. the scale of the axes, the increasing and decreasing intervals, the maximum or the minimum, if any)
• Exploring registers of representation that highlight the linear nature of the data</td></tr>
<tr><th colspan="2">ACTIVATION</th></tr>
<tr><td colspan="2">- When dealing with a situational problem, adult learners use reasoning to establish structured and functional relationships among different aspects of their knowledge, thus expanding their networks of mathematical cognitive resources.
- They use different strategies by associating pictures, objects or concepts with mathematical terms and symbols and by switching from one register of representation to another.</td></tr>
<tr><td>Examples of strategies</td><td>• Changing perspective
• Systematically determining the general form of the algebraic rule of a function
• Finding combinations in order to determine the rule of a quadratic function
• Creating a linear model from a non-linear one by replacing the values of the independent (X) or dependent (Y) variable, or both, with their logarithm (ideally, it is preferable to first recognize the model that seems to best fit the scatter plot, and then verify whether the model is correct)</td></tr>
<tr><th colspan="2">REFLECTION</th></tr>
<tr><td colspan="2">- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
- Through reasoning, they could make conjectures about particular or special cases to validate certain results.
- They use different strategies to make sure that the dependent and independent variables are properly defined, that the axes are correctly scaled, that no unit of measure has been omitted and that the data have been correctly transcribed.</td></tr>
<tr><td>Examples of strategies</td><td>• Checking their solution by, for example, making sure that the resulting values satisfy the range of the function, or substituting the values of the variables in the algebraic expression in order to validate a graphical interpolation or extrapolation</td></tr>
</table>

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Relationship between quantities*. Two cross-curricular competencies are considered particularly relevant to this course: *Uses information and communications technologies* and *Uses information*.

Methodological Competencies

Adult learners who wish to compile and analyze data related to a situation may use computer tools such as a spreadsheet program or graphing software. These tools make it easier to produce graphs and to change or work with parameters in order to carry out simulations and extrapolations. Through the competency *Uses information and communications technologies*, adult learners will realize that the ability to master these technologies will make their work considerably more interesting.

Intellectual Competency

The information in studies on financial matters is not always clearly presented in running text or tables according to the rules and conventions of mathematics. Data may be collected through polls or surveys and, in this case, must be organized so that it can be interpreted as accurately as possible in order to provide the information required. Adult learners could therefore learn to use information consisting of raw data. The competency *Uses information* will help them distinguish between data and information, and understand that the proper organization of data makes it possible to correctly interpret a situation.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of arithmetic and algebra. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following three integrative processes:

- **using an algebraic or graphical model to represent a situation**
- **interpolating or extrapolating from a graphical model**
- **using an algebraic or graphical model to generalize a set of situations**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover all three processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Numerical and algebraic expressions	
• Real numbers ○ Powers ○ Logarithms	An arithmetic approach to exponents and logarithms is promoted. Adult learners manipulate expressions and convert them to the same base (base 10, for use with a calculator) so that the exponents can be compared. If necessary, they use such equivalencies as: • $a^b = c \Leftrightarrow \log_a c = b$ • $\log_a c = \frac{\log_b c}{\log_b a}$
Relation, function and inverse	
• Solving exponential and logarithmic equations using a change of base, if necessary	Adult learners can represent and write numbers in logarithmic notation by using the following equivalency, if necessary: $\log_a x = n \Leftrightarrow a^n = x$

Mathematical Knowledge	Restrictions and Clarifications
Financial mathematics	
• Calculating, interpreting and analyzing financial situations	Financial calculations are limited to the following concepts: • simple and compound interest (i) • interest period (n) • discounting (present value – C_0) • compounding (future value – C_n) Compounding (future value) is determined using the following formula: $C_n = C_0(1 + i)^n$ Discounting (present value) is determined using the following formula: $C_0 = C_n(1 + i)^{-n}$ The interest rate (i) is determined using the following formula: $i = \left(\frac{C_n}{C_0}\right)^{1/n} - 1$ The interest period is determined using the following formula: $n = \frac{\log\left(C_n/C_0\right)}{\log(1 + i)}$ Compound interest is presented using graphs or compiled data tables. The following aspects may figure in situations involving personal finances: • types of income (e.g. remuneration, salary, commission, contracts, tips and gratuities) • different types of taxes (e.g. income tax, property tax and withholding tax) • types of financing (e.g. in-store financing, personal loans and mortgages), taking into account the charges associated with each one • cost of services (e.g. telephone, electricity)

Cultural References

Many mathematicians in the late 16th century lamented the fact that scientific progress was significantly slowed down by the need to do long and difficult calculations. It was around this time that the mathematician John Napier invented logarithms. Logarithmic tables and the rules of calculation were later developed to facilitate financial calculations, for example. The invention of logarithms had a significant impact on the structure of mathematics. A logarithmic scale is a practical way to represent numbers on a graph, especially when their orders of magnitude differ greatly. Interestingly, in music theory, logarithms are used to describe music intervals.

In science, logarithms are often used in formulas. An example of this is the natural logarithm, in which the mathematical constant e is the base. The natural logarithm is used in physics to interpret many natural phenomena. The base 10 logarithm is used to model earthquakes, and the binary logarithm (base 2) is commonly used in computer science and information theory. Lastly, it should be remembered that the work of physicist Ludwig Boltzmann on entropy and heat transfer led him to deduce his famous formula, which relates entropy S to the number of possible microstates W, i.e. $S = k \log W$, where k is the Boltzmann constant. This formula is carved on his tombstone.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Relationship between quantities* involve problems that can be solved in part by using an algebraic or graphical model of a function to represent a relationship between quantities. The *Algebraic and Graphical Modelling in a General Context 2* course provides adult learners with an opportunity to learn how to express a connection or a dependency relationship between quantities.

In the situational problems in this course, adult learners become more familiar with the mathematical symbols and notation related to functions and their inverse expressed in their general form, extrapolate results using an algebraic rule or a graph, and use a scale appropriate to the context so that the graph they draw in solving the situational problem makes sense in the context.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Environmental Awareness and Consumer Rights and Responsibilities, and Career Planning and Entrepreneurship.

Environmental Awareness and Consumer Rights and Responsibilities

Adult learners interested in natural disasters such as earthquakes could, through a learning situation on this topic, establish a relationship between a logarithmic function and the calculation of the magnitude of an earthquake. They would discover that this data relates to a continuous logarithmic function rather than an ordinary proportional scale. Because of the logarithmic nature of this phenomenon, when the energy generated by an earthquake varies by a factor of 10, this corresponds to a one-unit change in magnitude. For example, an earthquake with a magnitude of seven on the Richter scale is ten times stronger than an earthquake with a magnitude of six. Adult learners could use this situation to become more knowledgeable about their environment and improve their understanding of certain phenomena, which ties in directly with one of the focuses of development of this broad area of learning.

Career Planning and Entrepreneurship

In a learning situation involving financial mathematics, adult learners could be asked to determine an annual rate of interest and, if the initial amount invested is given, the value of a term deposit for varying years of investment and its value ten years later. This situation enables adult learners to use their knowledge of exponential functions to develop a more practical understanding of this type of function, while learning about the principles of saving. In this way, they could develop strategies that will be useful in carrying out a personal plan, which ties in directly with one of the focuses of development of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Environmental Awareness and Consumer Rights and Responsibilities • Career Planning and Entrepreneurship
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Relationship between quantities
Targeted cross-curricular competencies • Are developed at the same time and in the same context as the subject-specific competencies.	• Uses information and communications technologies • Uses information
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

<table>
<tr><th>Situational problem</th><th>Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the Relationship between quantities family of learning situations</th></tr>
<tr><td>An adult learner wants to find out more about the work of a traffic accident reconstruction expert. She wants to become familiar with the concepts related to this type of reconstruction.

In addition to gathering information about a particular event, interpreting physical evidence found at the accident site, taking photographs of the scene and making sketches, such an expert draws on certain mathematical concepts.</td><td>Integrative process: Using an algebraic or graphical model to generalize a set of situations
In carrying out the four phases in the problem-solving process, adult learners could:

Representation
• Select the relevant information (mass and acceleration in this case) and disregard superfluous information (e.g. tire traction, reaction time, type of surface, weather conditions)
• Reflect on the need to refer to several similar experiments to be able to come to a generalization

Planning
• Choose several similar experiments involving acceleration and deceleration
• List the elements needed to draw the graph (mass and acceleration in this case)

Activation
• Make a table of the data related to the situation, taking into account the limitations and precision of the measuring instruments used
• For a given initial speed, find the algebraic rule showing the relationship between acceleration and mass</td></tr>
</table>

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the ***Relationship between quantities*** family of learning situations	
For example, using data resulting from experiments, the adult learner determines the relationship (rule) between the acceleration (or deceleration) of a vehicle and its mass and whether it is possible to generalize this rule, especially when the initial speed is changed.		• Repeat the operation with different initial speeds • Compare the resulting relationships in order to derive a general rule of correspondence between acceleration and mass (the rule should be valid regardless of the initial speed)
	Reflection	• Suggest probable or plausible reasons that the equation is not perfectly consistent with the data analyzed (e.g. human error, measurement errors, limitations of the instruments used to take the measurements)

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Relationship between quantities*, adult learners represent a situation, carry out interpolations and extrapolations and use an algebraic or graphical model to generalize a set of situations. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

When representing a situation using an algebraic or graphical model, adult learners describe, symbolize, code, decode, explain or illustrate information contained in a table of values or a graph of an exponential (or logarithmic) equation. They combine different registers of representation as needed to produce a message in accordance with the notation, rules and conventions of mathematical language. They use problem-solving strategies to make comparisons, propose corrections, present favourable or optimal solutions, or issue recommendations. They formulate constructive criticism and make informed decisions in accordance with the conclusions they have drawn from the mathematical treatment of the situational problem.

In interpolating or extrapolating results from an algebraic or graphical model, adult learners use their knowledge of different types of functions and strategies, combining reasoning and creativity to overcome obstacles and make decisions. They use structured deductive reasoning in order to extract the linear quality of the data in situations that involve an exponential or logarithmic component.

To generalize a set of situations using an algebraic or graphical model, adult learners specify the purpose of their communication and switch from one register to another as needed. They demonstrate their understanding of the mathematical concepts in question using a wide range of communication strategies, which enables them to regulate the transmission of a message based on the specific reactions of the audience or to take new requirements into account. They learn and correctly use language that appropriately combines common mathematical terms. Lastly, by analyzing a process of generalization, they are able to induce laws and formulas specific to the world of finance, for example.

Throughout the problem-solving process, adult learners make an effort to apply their mathematical knowledge (basic second-degree polynomial functions and exponential functions). They construct new knowledge by induction and generalize a set of situations that they validate using different sources so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

  * The solution includes a procedure, strategies and a final answer.
  ** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*

Course

MTH-5152-1

Vote Distribution Models and Random Experiments

Mathematics

INTRODUCTION

The goal of the *Vote Distribution Models and Random Experiments* course is to enable adult learners to deal with situations that involve processing data from a random experiment in a general context.

In this course, adult learners compare different voting procedures with a view to determining which is the fairest from a democratic point of view. By studying different electoral systems, they refine their critical judgment with respect to politics and broaden their knowledge of world democracies. Vote distribution models are often used in situations that involve making social, political or economic choices. Analyzing different electoral systems requires that adult learners study different voting procedures, as well as their advantages and limitations.

In the different contexts, they enumerate possibilities or calculate probabilities in discrete or continuous cases, or use the concept of mathematical expectation to calculate the possibility of a gain or a loss. Adult learners also calculate the probability of compound events, which means that they calculate the probability of event A, given that event B has occurred. This probability concept, better known as conditional probability, enables them to gain a better understanding of random events. The different situations enable them to learn and use the language of sets. They use Venn diagrams, tree diagrams or schematic drawings to understand and convey messages. They make connections with logical connectors, including “and” and “or.” In order to develop their critical judgment, they learn to predict results, comment on behaviours or beliefs, and make decisions that they can explain or justify using different probability concepts.

By the end of this course, adult learners will be able to conduct a comparative analysis of social choice models so that they can make the fairest possible decisions in a given context. They will present the results of their analysis in accordance with the rules and conventions of mathematics, and determine the most efficient solution using problem-solving strategies. In addition, they will be able to interpret probability data from a random experiment and make decisions that reflect their mathematical reasoning.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

PROCESS AND STRATEGIES

REPRESENTATION

- Adult learners examine the situational problem to identify the context, the problem and the task to be performed.
- They use observational and representational strategies that are essential to inductive reasoning.
- In attempting to understand the context and the problem, they use deductive reasoning, which will enable them to make a conjecture.

Examples of strategies	• Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to make decisions concerning probability data • Using examples involving numbers, determining the type of relationship that exists between the odds of winning and the probability of winning or determining the difference between conditional probability and theoretical or experimental probability

PLANNING

- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient.
- Their number sense is influenced by their mathematical knowledge or understanding of conditional probability.
- In this course, they are required to make a decision in the context of a social choice.

Examples of strategies	• Systematically determining the voting procedure that is the most appropriate given the situational problem • Finding an appropriate counting method in the context of a study involving the use of subjective probabilities or in a random experiment involving events that are nonmutually exclusive

ACTIVATION

- When dealing with a situational problem, adult learners can use reasoning to apply mathematical knowledge related to mutually exclusive events.
- When calculating mathematical expectation, they can also use the algebraic form of the equation and the idea of whether or not contestants in a game of chance get to keep the money they bet.
- By drawing on their knowledge of the properties of experimental probability when dealing with a situation, they are able to deduce certain relationships.
- To avoid confusion, they use the symbols, terms and notation in accordance with their meaning.

Examples of strategies	• Compiling the results of an opinion poll in a table, taking into account age groups, years of education and so on • In light of the data gathered, comparing different voting procedures such as the runoff method, approval voting or proportional representation • Using technology (e.g. spreadsheet program, graphing calculator) to analyze the role of conditional probability in a random experiment that involves calculating probabilities

REFLECTION

- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
- They may have to reconsider the voting procedure and the method of analyzing the results if they realize that the conclusions drawn from a study are in complete contradiction with public opinion.

Examples of strategies	• Checking their solution by, for example, making sure that the sum of the probabilities of an event and its complementary event is always equal to 1 and that the possible values of an event are always physically possible, or identifying possible combinations of mathematical operations in an effort to understand the results of random experiments

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the *Processing data* family of learning situations. Two of these are considered particularly relevant to this course: *Uses information and communications technologies* and *Uses information*.

Methodological Competency

Probability calculations often involve very large numbers, especially when it is a question of calculating possible lottery number combinations consisting of more than 40 numbers. This provides an ideal opportunity to use computers, which are very effective for performing tedious and repetitive calculations. Adult learners could be required to use a spreadsheet program to help them carry out the steps in their work. In this way, they will discover the effectiveness of the technologies available to them and develop the competency *Uses information and communications technologies.*

Intellectual Competency

It is sometimes difficult to find clear information on the political systems of foreign countries, and to compile, synthesize and interrelate it. In addition, the vast amount of information and research results available on the Internet may seem daunting to even the bravest Web users. Adult learners can develop the ability to refine their questions, choose the appropriate keywords and ask the right questions. Developing the competency *Uses information* will enable them to make the most of the information at hand, which could help them avoid hours of fruitless research.

SUBJECT-SPECIFIC CONTENT

In order to deal effectively with situational problems, adult learners master the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following two integrative processes:

- **interpreting data resulting from a random experiment**
- **making decisions concerning social choices**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover both processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Probability	
• Distinguishing among theoretical, experimental and subjective probability	Subjective probability is used when it is impossible to calculate the theoretical or experimental probability. Such cases call for judgment, perceptiveness or experience. For example, weather reports involve the subjective evaluation of probabilities.
• Distinguishing between probability and odds	
• Approximating and predicting results	
• Calculating and interpreting mathematical expectation	
• Calculating and interpreting conditional probability	
• Distinguishing between mutually exclusive and nonmutually exclusive events	
• Distinguishing between dependent and independent events	

Mathematical Knowledge	Restrictions and Clarifications
Probability (cont.)	
• Representing random events	Events are represented using: • tables • trees • Venn diagrams Factorial notation is optional.
• Counting and enumerating possibilities	*Finding and using counting formulas is not part of the curriculum.* Factorial notation is optional in the *Cultural, Social and Technical* option.
Fair distribution model	
• Weighted mean	
• Comparing and interpreting different voting procedures	Adult learners compare and analyze: • majority rule • plurality voting • Borda count • Condorcet method • approval voting • elimination or runoff method • proportional representation *In cases that involve aggregating individual preferences (social choice theory), situations will be limited to no more than 4 "candidates."*

Cultural References

Probability calculations originated in the 17th century. In 1654, Blaise Pascal and Pierre Fermat calculated the number of favourable outcomes from among all the possible outcomes in a game of dice. Probability calculations were later used to determine human life expectancy (Christian and Louis Huygens, 1669) and the cost of buying an annuity (Jan De Witt, 1671). In 1696, English astronomer Edmond Halley drew up a mortality table and began work that would lead to the development of modern actuarial science.

In 1714, however, Swiss mathematician Jacob Bernoulli made the connection between statistics and probability with the publication of *Ars conjectandi*, in which he outlined the *law of large numbers*. According to this law, the probability of a result in an experiment is “practically equal” to the frequency with which this result occurs when the same experiment is repeated a large number of times.

The development of probability theory and its more rigorous application led to it being used in a wide range of fields. By the end of the 18th century, Condorcet had shown that probability calculations could be applied to the study of economic and social phenomena. Toward the end of the 19th century, probability theory was associated with progress in medicine and biology, and more specifically, the study of heredity. In the 20th century, it was widely used in the field of quantum mechanics.

Today, probability theory is commonly used and recognized as instrumental in carrying out activities in a variety of areas (e.g. assessing symptoms according to their importance when diagnosing a disease; developing new vaccines and gauging their effectiveness; managing investment risk; using encryption to guard against copyright violations by making it impossible to copy HD-DVD disks; using passwords in computer security; managing peak periods and waiting lists to develop customer loyalty; managing quality control to reduce losses and make factories profitable; developing a hiring policy according to predictions regarding the number of employees expected to retire). Examples given during the course will enable adult learners to understand the role probability plays in everyday life.

As a project, adult learners could conduct a study on the role of probability theory in an area of particular interest to them (e.g. quality control methods on an assembly line). Their analysis could help them better understand the usefulness of probability theory in the workplace.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Processing data* involve problems that can be solved in part by collecting or processing data. The *Vote Distribution Models and Random Experiments* course provides adult learners with an opportunity to learn how to collect and compare data.

When dealing with situational problems, adult learners use reasoning to apply mathematical knowledge related to mutually exclusive events, identify possible combinations of mathematical operations in an effort to understand the results of random experiments, and reconsider their counting method in order to correct their solution if, in determining conditional probability, they realize, for example, that their result is greater than 1.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Citizenship and Community Life, and Career Planning and Entrepreneurship.

Citizenship and Community Life

Adult learners taking this course could organize an election or a referendum in their centre. An awareness of the different voting procedures, the extent to which they are democratic and their limitations ties in with the educational aim of this broad area of learning, while enabling adult learners to apply probability concepts. For example, adult learners could compare three countries, each with a different voting system, in order to choose the one they feel is most suitable. They could also analyze the consequences of changing a given system. By better understanding how an electoral system works, adult learners develop a greater appreciation for democratic institutions and are more likely to become actively involved in them.

Career Planning and Entrepreneurship

Adult learners who wish to learn more about democracy and voting systems on a smaller scale could conduct an opinion poll, taking into account the limitations of a small sample. In order to become familiar with sociology, ethnology, psychology, anthropology and other fields, they could test different types of uninominal and plurinominal systems. Such an activity involves expanding their interests and may lead them to consider a career choice in this regard. They also learn to complete projects that help them develop their potential and integrate into society, which ties in with the educational aim of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Citizenship and Community Life
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Processing data
Targeted cross-curricular competency • Is developed at the same time and in the same context as the subject-specific competencies.	• Uses information
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Processing data* family of learning situations
To determine the best place to build a new hospital, the Ministère de la Santé et des Services sociaux has asked for the opinion of adjoining municipalities. Each municipality has a different population, and each one wants the hospital to be located as close to it as possible.	**Integrative process:** *Making decisions concerning social choices* In carrying out the four phases in the problem-solving process, adult learners could: Representation • Compile the available information • Determine the constraints, if any • Identify the relevant information (the municipalities involved, the percentage of the population eligible to vote in each city, etc.) and the irrelevant information (e.g. the gender of the voters or their occupation) Planning • Look for possible solutions (e.g. existing voting procedures) • Compare these procedures to find the most appropriate solution, and draw up a plan of action in this regard

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Processing data* family of learning situations	
Adult learners are asked to make a fair decision by using the given data to select the city where the hospital will be located. They must provide a clear justification for their choice.	Activation	• Refer to a similar situation studied previously • Analyze and compare the different vote distribution rules: majority rule, plurality voting, etc. • Determine the city (result) where the hospital will be located, using the chosen voting procedure
	Reflection	• Determine in which case the results will be the same regardless of the procedure used

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Processing data*, adult learners interpret data resulting from a random experiment and make decisions concerning social choices. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems, Uses mathematical reasoning* and *Communicates by using mathematical language*.

To interpret data resulting from a random experiment, adult learners organize information with a view to conducting a thorough analysis. They use the appropriate representations to clearly illustrate the constraints related to the context of the situational problem. They use the language of sets to simplify their solutions. They use different strategies to illustrate their line of reasoning. They plan and choose the most appropriate process, taking the purpose of the message into account. They use certain paradoxes (e.g. the boy or girl paradox, the Bertrand paradox, the prisoner’s dilemma) to explain the limitations of conditional probability. They validate their reasoning against reliable sources in order to evaluate and adjust their process and to plan smoother and more elegant solutions.

To choose an electoral system with a view to making a decision concerning social choices, adult learners distinguish between uninominal (one name) and plurinominal (a list of names) systems, and between majority rule and proportional representation. They use the Borda count and the Condorcet method as needed in situations involving weighted votes. They justify their reasoning based on predetermined definitions in the case of approval votes or when they use proportional representation to ensure fairness and equality.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (probability and fair distribution model). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

  * The solution includes a procedure, strategies and a final answer.

  ** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*

6.3 Technical and Scientific Option

The *Technical and Scientific* option allows adult learners to explore mathematics through the use of manual skills and intellectual abilities associated, among other things, with the operation of technical instruments. Adult learners make connections between mathematics and different occupations. It is important to create learning situations that help learners discover the different roles played by mathematics. These roles contribute to the development of the skills required for higher education in technical fields related to biology, physics, chemistry, business administration, the agri-food industry and the graphic arts. This option also opens the doors to vocational and technical training. The context associated with the different courses in this option involves applied mathematics.

Adult learners hone their competencies in a number of ways: they compare their solutions with those of their peers or their teacher; consider various points of view; exercise their critical judgment when validating a solution or a conjecture; look for the causes of a problem, for mistakes or for anomalies in solutions, algorithms or assembly drawings (e.g. architecture, landscaping); and issue recommendations with a view to taking corrective measures or making their actions more efficient. Case studies are of particular interest in this option. Adult learners examine a problem by analyzing certain factors. They may compare cases pertaining to the same theme, enabling them to consider different crucial aspects of that theme and to make informed decisions. Case studies give adult learners an opportunity to observe, work with and formulate conjectures and to verify them.

Among other things, these case studies raise issues relating to business management in the field of science and technology. They pertain to operational research, the production of bids or a process of generalization based on the observation of a variety of specific cases.

Furthermore, one of the goals of this option is to make students aware of various financial considerations. Faced with these types of situations, they develop an understanding of financial management, as it pertains both to business and to their personal lives, and become familiar with basic concepts in business administration.

Actions related to modelling, adjustment, validation and decision-making processes are an important part of the learning content for this option. Adult learners develop their critical thinking skills by validating a model and determining its limitations. They use different types of proofs and alternate between experimental reports and formal proofs. They become aware of the rigour associated with the rules and conventions involved in producing these reports or proofs. They learn to identify the principal steps in a line of reasoning, to consider different aspects or points of view and to emphasize them when communicating.

Several actions or projects can help establish the adult learner’s profile and characterize his or her learning. Activities could also involve special guest speakers, visits to various establishments, films or the production of scale models.

The study of instruments stemming from the application of mathematical concepts provides many opportunities to help students develop intellectually and become aware of the usefulness of mathematics, its widespread use in everyday life and its impact on humankind (e.g. pan scale, clock, pendulum, bow, compass, headlights, tensiometer). It is also an interesting way of introducing students to various trades and occupations.

In addition, the suggested cultural references help adult learners situate mathematical concepts in a historical and social context and identify the needs they helped meet, as well as the issues that gave rise to the acquisition of certain types of knowledge. The study of cultural references allows adult learners to appreciate the role of mathematics in daily life and in work situations, as well as the contribution of numerous people to the development of this subject.

Adult learners who choose this option are given regular opportunities to reflect on the steps in their work, to explore different points of view, to act in accordance with the constraints of a situation or to adjust these constraints in order to achieve a particular result. They are encouraged to adopt attitudes and develop abilities that are in high demand in the labour market, particularly in technical fields (whether or not they involve the use of instruments). They learn to cope with change, deal with complex situations, show creativity and engage in constructive cooperation, which enables them to grow into responsible and informed citizens.

6.3.1 Secondary IV Courses

Course

MTH-4261-2

Algebraic and Graphical Modelling in an Applied Context 1

Mathematics

INTRODUCTION

The goal of the *Algebraic and Graphical Modelling in an Applied Context 1* course is to enable adult learners to deal with situations that involve using an algebraic or graphical model to represent a dependency relationship between quantities in an applied context.

In this course, adult learners solve situational problems in which they must ask questions when it comes to choosing the most appropriate function. For example, does one use a step function, a first-degree function or a piecewise function to represent a per-minute long-distance telephone rate? Adults learn to analyze concrete situations using periodic functions, piecewise functions or step functions. While square root and logarithmic functions are represented graphically, the associated concepts are usually taught as inverse operations involved in solving second-degree or exponential equations and inequalities related to a given situation. Operations on functions can be approached intuitively, as needed. In some situations, adult learners produce, analyze or compare parts of a bid that require mathematical processing. They can use their critical judgment to analyze drawings, algorithms or possible solutions in order to evaluate efficiency and, if applicable, to identify errors and anomalies, take corrective measures, suggest improvements or issue recommendations. Other situations, when necessary, involve using the appropriate instrument to work out a solution, taking into account the precision with which the solution can be validated. In some situational problems, adult learners perform a variety of mental operations involving comparison, exploration, experimentation and simulation. These operations enable them to make conjectures, come up with an interpretation or conclusion, or establish proofs. Many contexts require a thorough knowledge of the concepts and processes needed to use reasoning to compare and comment on solutions, identify errors and anomalies, and suggest changes in line with the objectives of the situation. The situations proposed require that adult learners produce structured explanations or justifications in order to illustrate how they reached their conclusions. With respect to communicating using mathematical language, some situational problems require that adult learners identify and analyze how someone else's reasoning is organized. In the situational problems in this course, adult learners share information, a description, an explanation or an argument verbally or in writing, by developing either an activity, a communication plan or a report on an experimental procedure (e.g. laboratory report, logbook).

By the end of this course, adult learners will be able to use algebra to represent concrete situations and produce clear and accurate work in accordance with the rules and conventions of mathematics. By algebraically or graphically representing a situation using real functions or their inverse, they will be able to deduce results through interpolation or extrapolation. In addition, they will use different registers of representation to generalize the similar characteristics of a range of situations.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

PROCESS AND STRATEGIES	
REPRESENTATION	
- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. - They use observational and representational strategies that are essential to inductive reasoning. - Using representational strategies, they see whether there are any trends or patterns and determine if they persist for each iteration. Different lines of deductive reasoning could lead them to generalizations.	
Examples of strategies	• Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to find a dependency relationship and determine the variables in the situation • Using examples involving numbers, determining the type of relationship that exists between the variables in a situation • Gathering relevant information
PLANNING	
- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient. - They use different types of reasoning to work out the steps involved. They can refer to similar situations they have resolved in the past. - In order to better understand the relationship between the quantities, they could use a table or a graph to represent a relation.	
Examples of strategies	• Systematically determining the functional model best suited to the situation, bearing in mind the limitations regarding the model's precision • Finding an algebraic rule that reflects the best relationship between the constraints and possible consequences of the situational problem
ACTIVATION	
- When dealing with a situational problem, adult learners use reasoning to establish organized and functional relationships among different aspects of their knowledge, thus expanding their networks of mathematical cognitive resources. - They use the appropriate scale so that the graph they draw in solving the situational problem makes sense in light of the context.	
Examples of strategies	• Proceeding by trial and error to determine certain properties of functions • Dividing the situational problem into subproblems to work out a solution • Deducing the positive interval of the function by progression
REFLECTION	
- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution. - They make sure that the dependent and independent variables are correctly defined, that the axes are properly scaled, that no unit of measure has been omitted and that the data have been accurately transcribed.	
Examples of strategies	• Checking their solution by, for example, making sure that the resulting values satisfy the range of the function, comparing the number of possible solutions for a system of equations with the number of solutions found, or substituting the values of the variables in the algebraic expression in order to validate a graphical interpolation or extrapolation.

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Relationship between quantities*. Two of these are considered particularly relevant to this course: *Uses information and communications technologies* and *Adopts effective work methods*.

Methodological Competencies

Calculation software such as a spreadsheet program can be used to develop a graphical model to represent a situation. This tool can make it considerably easier to design and carry out tasks, providing more time for the analysis and interpretation of changes in certain parameters of functions. The development of the competency *Uses information and communications technologies* could help adult learners use this tool to work with parameters.

When studying functions and generalization by means of a functional model, especially when this model is constructed using experimental data, adult learners must work systematically in planning tasks and taking measurements. The development of the competency *Adopts effective work methods* is particularly important for adult learners who wish to study in scientific or technical fields.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of arithmetic and algebra. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following three integrative processes:

- **using an algebraic or graphical model to represent a situation**
- **interpolating or extrapolating from an algebraic or graphical model**
- **using an algebraic or graphical model to generalize a set of situations**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover all three processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Manipulating numerical and algebraic expressions	
• Operations on numerical and algebraic expressions	Operations on algebraic expressions are limited to: • multiplication • division of polynomials by a binomial (with or without a remainder) • the simplification of rational expressions (rational fractions) *When finding the common denominator in order to add two rational expressions, adult learners in this option will deal only with the case in which the denominator of one fraction is the multiple of the denominator of the other fraction.* The numbers can be expressed using: • rational exponents • radicals (n^{th} root) • powers of bases 2 and 10 (base change)

Mathematical Knowledge	Restrictions and Clarifications
Manipulating numerical and algebraic expressions (cont.)	
• Constructing and interpreting tables of values consisting of positive rational numbers written in base 2 and base 10	In finding the approximate value of an exponent (logarithm), adult learners use a graph, a table of values (base 2 or 10) or technology. They manipulate expressions and convert them to the same base (base 10, for the calculator) so that the exponents can be compared. If necessary, they use equivalences such as: • $a^b = c \Leftrightarrow \log_a c = b$ • $\log_a c = \frac{\log c}{\log a}$
• Expanding and factoring	The types of factoring covered in this course are factoring by grouping and the use of second-degree algebraic identities (perfect square trinomial and difference of two squares).
• Solving equations and inequalities in one variable: second-degree, square root, exponential, logarithmic (including the properties of radicals, exponents and logarithms)	

Mathematical Knowledge	Restrictions and Clarifications
Relation, function and inverse	
• Experimenting with real functions as well as observing, interpreting, describing and representing them	The real functions covered are: • second-degree polynomial functions $f(x) = ax^2$ or $f(x) = (bx)^2$ or $f(x) = a(bx)^2$ • exponential functions $f(x) = ac^{bx}$ where $a \neq 0$ and $c > 0$ • square-root functions $f(x) = a\sqrt{bx}$ *This function is introduced in connection with second-degree functions (as an inverse).* • periodic functions • logarithmic functions $f(x) = a \log_c bx$ where $c > 0$ *This function is introduced in connection with exponential functions (as an inverse).* • greatest-integer functions $f(x) = a[bx]$ • piecewise functions • step functions The functions can be represented using: • a table of values • an algebraic rule • a graph (produced with or without technology)
• Solving and graphing first-degree inequalities in two variables	
• Describing and interpreting the properties of real functions	The properties of real functions studied in this course are: • the domain and codomain (range) • increasing and decreasing intervals • extrema • sign • x- and y-intercepts
• Interpreting the multiplicative parameter	

Mathematical Knowledge	Restrictions and Clarifications
System	
• Representing a situation using straight lines or half-planes	The course covers the properties of the following lines: • parallel lines • intersecting lines • coincident lines • perpendicular lines The equation of a line can be expressed in: • general form Ax + By + C = 0 • standard form f(x) = ax + b *The symmetric form* $\left(\frac{x}{a} + \frac{y}{b} = 1 \right)$ *of the equation of a line is optional in the* Technical and Scientific *option.*
• Solving systems of first-degree equations in two variables	System of equations may be solved by means of: • a table of values • an algebraic method chosen by the adult learner • a graphical method, with or without the use of technology

Cultural References

Modelling is a way of understanding reality and establishing dependency relationships between quantities. Dependency relationships led to the development of the concepts of relation and function. The history of mathematics shows that Diophantus of Alexandria defined the concept of an unknown number more than 18 centuries ago. Diophantus even went so far as to work with up to ten unknowns. Mathematical concepts have sometimes given rise to a great deal of controversy before gaining acceptance, and their development has resulted from intellectual jousting between philosophers and scientists throughout history. When a concept is finally attributed to a given mathematician, it is often the case that many other people worked on it beforehand. For example, many mathematicians contributed to the development of algebra, including Oresme, who established the equation of a line three centuries before Descartes invented analytic geometry.

Mathematical reasoning and geometric modelling are used to design many commonly used instruments, and their use requires graphical representations. The evolution of this branch of mathematics has played a vital role in the development of several of today's essential instruments (e.g. sphygmomanometer used to measure blood pressure, radar, multimeter). Algebraic and graphical modelling is of prime importance when using these instruments in professional or technical occupations in the sciences. Examples given during the course will enable adult learners to understand their importance.

Analytic geometry has made it possible to represent many physical phenomena. Depending on their interests, adult learners could use this mathematical approach in a project to study the behaviour of a golf ball, the orbit of the planets in our solar system or the operation of a measuring instrument. They could then use equations to represent these physical phenomena and analyze them to derive useful information.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Relationship between quantities* involve problems that can be solved in part by using an algebraic or graphical model to represent a relationship between quantities in an applied context. The *Algebraic and Graphical Modelling in an Applied Context 1* course provides adult learners with an opportunity to express a connection or a dependency relationship between quantities.

In the situational problems in this course, adult learners see whether there are any trends or patterns and determine if they persist for each iteration, derive and generalize the rules and conditions that determine the number of solutions for the system, and make sure that the dependent and independent variables are correctly defined, that the axes are properly scaled, that no unit of measure has been omitted, and that the data have been accurately transcribed.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Health and Well-Being, and Environmental Awareness and Consumer Rights and Responsibilities.

Health and Well-Being

This course could help adult learners understand the way bacteria spread on door handles, telephones or computer mice and keyboards. After collecting data during an experiment, they could graph the increase in the number of bacteria as a function of time or the number of users. They could also compile their results in a table of values and graph them. Their conclusions could provide an explanation for the transmission of bacteria in different environments and encourage them to extrapolate in order to simulate an epidemic and become aware of the importance of a healthy lifestyle, which ties in with the educational aim of this broad area of learning.

Environmental Awareness and Consumer Rights and Responsibilities

The study of functions could also help adult learners understand the consequences of their driving habits. For example, by analyzing their gas consumption as a function of their driving speed, they would be able to assess the cost and energy expenditure that result from driving fast. Moreover, they could calculate braking distance as a function of speed. In short, adult learners could be encouraged to take a critical attitude toward consumption and the exploitation of the environment, which ties in with the educational aim of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Health and Well-Being
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Relationship between quantities
Targeted cross-curricular competencies • Are developed at the same time and in the same context as the subject-specific competencies.	• Uses information and communications technologies • Adopts effective work methods
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

<table>
<tr><th>Situational problem</th><th colspan="2">Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the Relationship between quantities family of learning situations</th></tr>
<tr><td rowspan="3">In 2006, a study conducted in Québec showed that the C. difficile bacterium, the main cause of infectious diarrhea among hospital patients in industrialized countries, was indirectly responsible for 108 deaths in six months. This infection, which is the most common in hospitals and long-term care facilities, also affects hospital staff. As is the case for all infectious diseases, one of the best means of preventing infection is to frequently wash one’s hands with warm soapy water for at least 20 seconds.</td><td colspan="2">Integrative process: Using an algebraic or graphical model to represent a situation

In carrying out the four phases in the problem-solving process, adult learners could:</td></tr>
<tr><td>Representation</td><td>Describe the situation in their own words
Put forward a hypothesis on the type of relationship that exists between the number of bacteria and elapsed time (if the number of people who touch the door handle doubles, the number of bacteria will also double) and then confirm or refute their hypothesis</td></tr>
<tr><td>Planning</td><td>Refer to a similar situational problem previously studied in class (pyramid sales schemes, uncontrolled chain reaction resulting from nuclear fission) to begin their analysis
Decide, according to the information provided, to begin by representing the relationship using a graphical model rather than an algebraic one</td></tr>
</table>

<table>
<tr><th>Situational problem</th><th colspan="2">Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the Relationship between quantities family of learning situations</th></tr>
<tr><td rowspan="3">Adult learners are asked to become aware of the importance of being careful about occupational health and safety and to raise awareness of this issue. They will have to show the importance of hand-washing by illustrating the phenomenal growth of harmful bacteria on door handles.

For their demonstration, adult learners will use data provided by their teacher or found on the Internet.</td><td></td><td>Determine which is the independent (number of bacteria) and the dependent (time elapsed) variable</td></tr>
<tr><td>Activation</td><td>Draw a Cartesian coordinate graph of the situation, using an appropriate scale
Draw a curve through the data values
Associate this curve with an exponential relationship</td></tr>
<tr><td>Reflection</td><td>Compare their solution and results with those of their teacher or classmates in order to identify the strengths and weaknesses of their model
Determine whether there is a critical number beyond which the situation no longer reflects the algebraic model
Check whether the bacterial growth model is still valid under different conditions (e.g. if the temperature increases or decreases markedly)</td></tr>
</table>

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Relationship between quantities*, adult learners represent a situation, interpolate or extrapolate, and generalize a set of situations using an algebraic or graphical model. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To represent a situational problem using an algebraic or graphical model, adult learners employ real functions or their inverse, and select relevant information in order to find a pattern or a law that takes into account the best relationship between the constraints and consequences involved. They choose the algebraic model that is best suited to the situation, giving examples with numerical values as needed in order to determine the type of relationship that exists between the variables. In addition, they recognize and choose the mathematical symbols, terms and notation with a view to representing the situation accurately. They produce precise mathematical messages using the rules and conventions specific to the functions covered in this course.

Adult learners determine questions before interpolating or extrapolating from an algebraic or graphical model. These questions help them establish organized and functional relationships among certain aspects of mathematical knowledge, in particular the relationships between the parameters of a given function or the effect of a change in a parameter on a family of functions. They then suggest probable or plausible ideas in order to deduce propositions related to the situation and validate their conjectures through interpolation or extrapolation, substituting numerical values in the algebraic rule they have modelled. They also associate the appropriate graph with the sequences of numbers in question (unlike a geometric progression, which must be graphed using an exponential relation, an arithmetic progression can be graphed using a linear relation).

Adult learners model several situations using real functions, and consider whether or not the properties of these situations can be generalized. To do this, they must determine the important elements and the obstacles to be overcome and refer to the solution of one or more similar situational problems. They find invariants through trial and error, which enables them to make generalizations and derive laws, rules or properties. They validate their solution using examples or counterexamples to test their deductive reasoning. In addition, solving systems of first-degree equations in two variables provides them with the tools they need to generalize results that lead to the properties of different types of lines, be they parallel, perpendicular, coincident or intersecting.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (manipulating numerical and algebraic expressions, function, inverse and system). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

* The solution includes a procedure, strategies and a final answer.
** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*

Course

MTH-4262-2

Data Collection in an Applied Context

Mathematics

INTRODUCTION

The goal of the *Data Collection in an Applied Context* course is to enable adult learners to deal with situations that involve collecting or processing data pertaining to a one- or two-variable distribution in an applied context.

In this course, adult learners continue to develop probabilistic thinking skills by studying the concept of conditional probability and one- or two-variable statistical distributions. By exploring the concept of fairness, adults learn to distinguish between the concepts of chance, odds and probability. In analyzing the rules of certain games, they can determine the *odds for* or *odds against* a player and change these rules, if necessary, to make the situation fair or more favourable for that player. The concept of weighted mean leads to the concept of mathematical expectation, which adult learners use to make decisions. When analyzing situations, including games of chance, they change the parameters of the equation to make the game fair or to optimize a gain or a loss in order to meet certain objectives. In situations where chance is a factor, their decisions are based on conditional probability or mathematical expectation. They might have to make changes to the parameters of a situation (e.g. rules of the game, size of a gain, event) in order to make the game fair or optimize a gain or a loss in order to meet certain objectives. By exploring the concepts of conditional probability and mathematical expectancy to validate conjectures involving the concept of fairness or the optimization of a gain or a loss, adult learners identify the dependency relationship between events. In some situations, they may have to justify their choices or conclusions in a statistical study or judge how representative or reliable the study is. Lastly, adult learners may have to use various representations (e.g. tree diagrams, graphs, tables of values) to interpret, produce and convey mathematical messages. They can also use Venn diagrams, which are associated with conditional probability, and scatter plots in statistics.

By the end of this course, adult learners will be able to collect data and compare other one- or two-variable distributions when solving a problem that they themselves have defined. They will present the results of their analysis in accordance with the rules and conventions of mathematics. They will use problem-solving strategies in order to determine the most efficient solution. In addition, studying situations consisting of several interrelated variables and constraints encourages them to draw on the concept of conditional probability to simulate a simple model for predicting outcomes.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

<table>
<tr><th colspan="2">PROCESS AND STRATEGIES</th></tr>
<tr><th colspan="2">REPRESENTATION</th></tr>
<tr><td colspan="2">- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. They use observational and representational strategies that are essential to inductive reasoning.
- In attempting to understand the context and the problem, they use deductive reasoning, particularly when they work with implicit data.</td></tr>
<tr><td>Examples of strategies</td><td>• Using examples involving numbers, determining the type of relationship that exists between the odds and the probability of winning (e.g. when analyzing whether a game is fair)
• Listing their statistical and probability-related strategies and knowledge pertaining to the situation
• Describing the characteristics of the situation
• Gathering relevant information</td></tr>
<tr><th colspan="2">PLANNING</th></tr>
<tr><td colspan="2">- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient.
- Reasoning enables them to establish organized and functional relationships among different aspects of their knowledge as, for example, when they attempt to extrapolate results using an algebraic rule or a correlation graph.
- To correctly plan their solution, they decode the elements of mathematical language such as the meaning of the symbols, terms and notation used, as well as the different registers of representation.</td></tr>
<tr><td>Examples of strategies</td><td>• Systematically determining the correlation model best suited to the situation, bearing in mind the limitations regarding the model’s precision
• Finding an appropriate counting method in the context of a study on designing a fair game</td></tr>
<tr><th colspan="2">ACTIVATION</th></tr>
<tr><td colspan="2">- When calculating mathematical expectation in a situational problem, adult learners make certain connections between the algebraic form of the equation and the idea of whether or not contestants in a game of chance get to keep the money they bet.
- By drawing on their knowledge of the properties of experimental probability, they make certain deductions (e.g. the greater the value of a lottery prize, the lower the probability of winning).
- They use various association strategies when interpreting codes and rules, for example, to distinguish between the probability and the odds of winning an amount of money.</td></tr>
<tr><td>Examples of strategies</td><td>• Using a table to connect the elements associated with the correlation: ordering the statistical data, finding the vertex, axis of symmetry, rate of change, and so on
• Using relevant data, drawing the functional model best suited to the situation
• Using technology to analyze the role of the different parameters of the rule of the correlation line or another model</td></tr>
<tr><th colspan="2">REFLECTION</th></tr>
<tr><td colspan="2">- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
- It is important for them to review their work, especially when they make conjectures about particular or special cases. They take the time to validate certain results such as the effect on the graph of changing the slope of the regression line.</td></tr>
<tr><td>Examples of strategies</td><td>• Checking their solution by, for example, making sure that the resulting values satisfy the range of the function in the case of a correlation
• Determining the strategies used to deal with the situation</td></tr>
</table>

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Processing data*. Two of these are considered particularly relevant to this course: *Exercises critical judgment* and *Adopts effective work methods*.

Intellectual Competency

Dealing with a situation that involves statistics or probability requires that adult learners use the competency *Exercises critical judgment* before deciding on the relevance and validity of the information or the credibility of the author. They must identify the data needed to deal effectively with the situation. They make connections between two sets of data and draw conclusions based on persuasive mathematical arguments.

Methodological Competency

Carrying out a study involving statistical or probability data fosters the development of the competency *Adopts effective work methods*, which can be used in other areas of adult learners' lives. To deal with such situations, learners must consider all aspects of a task, use the available resources (people, materials, technology, personal knowledge), adapt their actions to the context and the work method (research, analysis, representation or communication) and carry them out carefully. Organized planning leaves little room for the unexpected, thereby making it possible to see the task through to completion.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on some of their previously acquired knowledge of statistics. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following three integrative processes:

- **collecting data**
- **comparing collections of data**
- **interpreting data resulting from an experiment**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover all three processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
One-variable distribution	
• Determining and interpreting measures of position and dispersion	The measures of dispersion studied in this course are: • mean deviation • standard deviation
Two-variable distribution	
• Constructing and interpreting two-variable distributions	
• Drawing a scatter plot	
• Representing and determining the equation of the regression line or curves related to the functional models under study	The functional models studied in this course are those covered in *Algebraic and Graphical Modelling in an Applied Context 1*.
• Interpolating or extrapolating using the regression line	
• Approximating and interpreting the correlation coefficient	Interpretation of the correlation is limited to cases involving linear correlations, which can be estimated using a graphical method (box method or ellipse). The exact value of the correlation coefficient is determined using technology.

Mathematical Knowledge	Restrictions and Clarifications
Two-variable distribution (cont.)	
• Interpreting a correlation qualitatively and quantitatively	The characteristics of the correlation are: positive, negative, zero, perfect, strong, moderate or weak.
• Interpolating and extrapolating using the functional model best suited to the situational problem	
Probability	By using factorial notation, adult learners will find it easier to write out certain operations and can make efficient use of their calculator.
• Calculating probabilities using statistical reports	
• Representing and determining conditional probability	The situations explored should not involve the use of formulas, but enable students to use their reasoning and to represent the situation by using a: • Venn diagram • tree diagram
• Determining the *odds for* or the *odds against*	
• Calculating and interpreting mathematical expectation	
• Changing the value of parameters or conditions	The value of the conditions or the parameters is changed in order to optimize a gain or a loss, or to make the situation fair.
• Distinguishing among mutually exclusive, nonmutually exclusive, independent and dependent events	

Cultural References

Throughout history, humans have collected data, made inventories and taken censuses. The best-known example in this regard is the Roman census, which coincided with the birth of Jesus of Nazareth and which, according to historians, was conducted in the years roughly corresponding to the beginning of the Common Era. However, it was not until the 17th century that statistical tools used for extrapolation were developed on the basis of demographic data relating to public health, among other things. Compared with algebra and geometry, statistics and probability are relatively new branches of mathematics whose emergence and development resulted from the need to understand phenomena, validate observations or intuition, and predict an outcome in the more or less immediate future.

Adult learners can appreciate the importance that statistical information has in society today. Opinion polls during elections are an obvious example of this, not to mention the results of regularly published studies dealing with different issues of public interest. In addition, each year Statistics Canada conducts nationwide studies based on censuses, surveys and administrative data. Their results are accessible to the general public, including secondary school students.

Computers have made it possible to process larger amounts of data and to crosscheck different sets of data. Today, no science could progress without using statistics and probability theory.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Processing data* involve problems that can be solved in part by collecting or processing data in an applied context. The *Data Collection in an Applied Context* course provides adult learners with an opportunity to learn how to collect and compare data.

In the situational problems in this course, adult learners determine the sum of the other cases and subtract it from 1 if probability calculations pertain to a complementary event, make certain connections between the algebraic form of the equation and the idea of whether or not contestants in a game of chance get to keep the money they bet, and use a scatter plot instead of a contingency table to represent the data in order to identify a trend in a two-variable statistical distribution.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Career Planning and Entrepreneurship, and Environmental Awareness and Consumer Rights and Responsibilities.

Career Planning and Entrepreneurship

The statistical concepts introduced in this course could help adult learners become more familiar with the trades and occupations that interest them. For example, if they are interested in different construction trades, they could compare and assess the training requirements for each one. Any important correlation they notice could help them make a career choice. Adult learners could therefore become more familiar with the world of work and the requirements related to different trades and occupations, which is one of the focuses of development of this broad area of learning.

Environmental Awareness and Consumer Rights and Responsibilities

Statistical or probability calculations inform adult learners about how social factors influence the cost of insurance. For example, the cost of car insurance decreases with the driver’s age, while the cost of life insurance increases with the age of the insured. A simulation of the premium calculation that takes mathematical expectation into account shows how probability can be related to statistical data. This could help adult learners understand how certain behaviours can affect statistical data, and therefore the cost of insurance premiums. In this way, they could be made aware of the social and economic aspects of consumption, which ties in with one of the focuses of development of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Environmental Awareness and Consumer Rights and Responsibilities
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Processing data
Targeted cross-curricular competencies • Are developed at the same time and in the same context as the subject-specific competencies.	• Exercises critical judgment • Adopts effective work methods
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Processing data* family of learning situations
Global warming has been a major issue for many years now. Despite all the scientific data on the subject, experts still do not agree on what causes global warming besides the increase in greenhouse gases.	**Integrative process:** *Collecting data* In carrying out the four phases in the problem-solving process, adult learners could: Representation • Put forward a hypothesis to the effect that rising average temperatures worldwide are related to the increase in the number of motor vehicles in the world • Identify the focus of their analysis: the geographic location or time period covered Planning • Prioritize the steps in their work • Establish an experimental procedure • Describe the characteristics of the problem in order to determine a data-collection method (one or two variables)

<table>
<tr><th>Situational problem</th><th colspan="2">Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the Processing data family of learning situations</th></tr>
<tr><td rowspan="2">In order to become familiar with statistical analysis, adult learners are required to conduct a case study on the subject. They are asked to put forward a hypothesis regarding climate change, collect data that supports the hypothesis and rigorously validate their analysis.

Lastly, they must write a report presenting their research results.</td><td>Activation</td><td>• Gather information related to the situational problem: temperatures, time intervals considered, etc. For example, adult learners could consult the Environment Canada Web site
• Establish organized and functional relationships among concepts and processes in order to construct contingency tables
• Draw a regression line or a curve to represent the model in question
• Use technology to draw a scatter plot representing the distribution
• Calculate the standard deviation to determine if there really is a correlation between the variables (e.g. the number of vehicles and the rise in temperature)</td></tr>
<tr><td>Reflection</td><td>• Decide to rule out data that come from sources whose reliability is questionable
• Critically assess the reliability of the information
• Analyze the role of the different parameters of their function. (For example, if the number of vehicles doubles, will the temperature also double?)</td></tr>
</table>

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Processing data*, adult learners collect, compare and interpret data resulting from an experiment. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To collect data, adult learners use problem-solving strategies to define the problem and identify the tasks involved. They determine the important elements of information and the obstacles to be overcome in order to differentiate between one- and two-variable statistical distributions. In working out their solution, they establish and implement a plan, validating and following each of the steps, namely collecting and processing (interpreting and analyzing) data. This last step requires that they use mathematical reasoning, explore the problem in question and identify patterns. Adult learners make conjectures using a correlation line or curves in order to make decisions in the medium and long term. They draw conclusions based on laws and rules related to the measure of dispersion used (mean or standard deviation). Lastly, they produce mathematical messages, using the appropriate register of representation given the constraints of the situational problem. They choose the functional model best adapted to the situation, a Venn diagram or tree diagram in the case of a one-variable distribution.

To compare collections of data, adult learners interpret mathematical messages by making connections between the elements of the message, determining its overall meaning or associating pictures, objects or knowledge with mathematical terms and symbols. In addition, they use mathematical reasoning by developing and using networks of cognitive resources in order to compare trends, for example the rate of change, the rate of growth, or any other characteristic of the functions covered in this course, such as the mean deviation, standard deviation or correlation coefficient.

To interpret data resulting from an experiment—predicting an event using conditional probability, a one- or two-variable distribution—adult learners decode the elements of mathematical language, distinguishing between the mathematical and everyday meanings of terms. In addition, they interpret mathematical messages by switching from one register of representation to another, for example, from a Venn diagram to a tree diagram, and vice versa, bearing in mind that the data they are transposing are not of the same nature (sample space vs. conditional probability). They use mathematical reasoning, developing networks of mathematical cognitive resources, such as the functional model best suited to the situation, a Venn diagram or a tree diagram in the case of a one-variable distribution. They generalize, derive laws and rules and deduce propositions that help them make informed decisions.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (one- or two-variable statistical distributions and conditional probability). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

* The solution includes a procedure, strategies and a final answer.
** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*

Course

MTH-4263-2

Geometric Representation in an Applied Context 1

Mathematics

INTRODUCTION

The goal of the *Geometric Representation in an Applied Context 1* course is to enable adult learners to use trigonometry to deal with situations that involve the geometric representation of an object or a physical space in an applied context.

In this course adult learners encounter various situational problems that enable them to expand their knowledge of geometry, and trigonometry in particular. Both an empirical and a formal approach are required to derive the properties of certain figures and to justify or validate the truth of various statements. Adult learners deduce metric relations in right triangles or in triangles that they split into right triangles. Using the concept of similarity, they derive the minimum conditions required to conclude that figures are congruent or similar. They solve situational problems in geometry, making drawings or constructing objects in accordance with instructions. This requires that they use their spatial and measurement sense. In addition, as needed, they develop models and find optimal solutions using the concepts of line, distance and point of division. They use geometrical concepts (Euclidean or Cartesian plane) to deduce measurements or propose optimal solutions. In some situations, they convey messages using various symbols, types of notation, units, logical connectors, quantifiers or literal expressions, in accordance with the rules and conventions of mathematics.

By the end of this course, adult learners will be able to use different metric and trigonometric relations to represent and describe an object or a physical space in accordance with the mathematical rules and conventions used in geometry. They will also be able to use different strategies and types of reasoning to organize a physical space, taking into account different constraints.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

<table>
<tr><th colspan="2">PROCESS AND STRATEGIES</th></tr>
<tr><th colspan="2">REPRESENTATION</th></tr>
<tr><td colspan="2">- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. They use strategies that are essential to inductive reasoning.
- They organize the elements that will enable them to make a conjecture.
- They distinguish between the mathematical and everyday meaning of the terms used so that they can understand concepts.</td></tr>
<tr><td>Examples of strategies</td><td>• Illustrating their understanding of the situational problem by trying to make connections between their mathematical knowledge and the task to be performed
• Representing the situational problem mentally or in writing
• Listing their geometry-related strategies and the metric relations pertaining to the situation
• Describing the characteristics of the situation
• Determining questions about the situation</td></tr>
<tr><th colspan="2">PLANNING</th></tr>
<tr><td colspan="2">- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient.
- Through mathematical reasoning, they choose different registers of representation to illustrate certain properties of trigonometric ratios.
- By making connections between the elements of the message and by giving a literal description of the ratios of the corresponding sides of two plane figures, adult learners are able to construct a figure based on their description.</td></tr>
<tr><td>Examples of strategies</td><td>• Dividing the situational problem into subproblems
• Using lists, tables, diagrams, concrete materials or drawings to plan their solution</td></tr>
<tr><th colspan="2">ACTIVATION</th></tr>
<tr><td colspan="2">- When dealing with a situational problem, adult learners show discernment by making rigorous use of the elements of mathematical language.
- When producing a scale plan of an architectural structure, they take into account the proportions indicated by the scale and use the related symbols and conventions.</td></tr>
<tr><td>Examples of strategies</td><td>• Using the parameters of a function, making a sketch to predict results
• Solving certain situational problems by working backwards when the solution consists of several steps or when there is insufficient information
• Analyzing the parameters of a right triangle to properly understand how they are related to the parameters of any given triangle</td></tr>
<tr><th colspan="2">REFLECTION</th></tr>
<tr><td colspan="2">- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
- Reasoning can help them review their work and make conjectures about particular or special cases involving any triangle in order to validate certain results. Reasoning also enables them to reject extrapolations that would yield nonsensical results.
- They validate their mathematical message by consulting different sources of information.</td></tr>
<tr><td>Examples of strategies</td><td>• Checking their solutions by means of examples or counterexamples, particularly by using the Pythagorean theorem to validate the lengths of the sides of a triangle to be able to conclude that it is, in fact, a right triangle
• Recognizing the strategies for dealing with situational problems in geometry (e.g. applying a rule, referring to a geometry principle)
• Using a calculator or geometric modelling software to validate their work</td></tr>
</table>

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Measurement and spatial representation*. Two of these are considered particularly relevant to this course: *Communicates appropriately* and *Adopts effective work methods*.

Communication-Related Competency

Adult learners who wish to organize a space in their adult education centre (e.g. a student café) must first convince the governing board of the feasibility of their project and the needs it will meet. They must then provide an estimate of the cost involved. The competency *Communicates appropriately* will be very useful to them in this situation. Adult learners could design a sufficiently precise plan that enables them to visualize the project. By having an expert check their plan beforehand, they will ensure that it is consistent with the rules of geometry. They should indicate the constraints involved and their consequences, give appropriate answers to any questions asked and take into account their audience's reactions. The way that adult learners make their presentation will for the most part determine how the members of the governing board respond to the plan.

Intellectual Competency

In this course, adult learners might need to know a length that they cannot physically measure. They could use triangulation, which involves dividing an area into triangles, directly measuring one of the sides, and then using trigonometric relations to determine the other measurements. In another situation, they could be asked to determine the distance between a celestial body and the Earth. They could do this by creating an angle between the celestial body and two lines drawn from two different observation points and find information such as the diameter of the Earth. Given the diversity of learning situations they encounter in this course, adult learners will discover a variety of problem-solving approaches. Some approaches will be more appropriate given the context and the resources available. The competency *Adopts effective work methods* is therefore essential.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of geometry. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following two integrative processes:

- **organizing a physical space**
- **describing an object or a physical space and representing it in two or three dimensions**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover both processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Metric and trigonometric relations in triangles	
• Representing and interpreting situations using triangles	The trigonometric ratios studied in this course are sine, cosine and tangent. *In the* Technical and Scientific *option, the sine law and the cosine law are covered in Secondary V.* The other metric and trigonometric relations are listed in the Principles table that comes after this table.
• Describing the properties of trigonometric ratios	Adult learners use the properties of trigonometric ratios in a formal manner to justify the steps in their solution.

Mathematical Knowledge	Restrictions and Clarifications
Metric and trigonometric relations in triangles (cont.)	
• Determining the slope, measurements and positions using metric and trigonometric relations in triangles	The measurements and positions studied pertain to: • the angles in a triangle • the altitude to the hypotenuse, the orthogonal projection of the legs on the hypotenuse • the sides of a triangle • the area of a triangle • the coordinates of a point (point of division) in the Euclidean and Cartesian planes • the length of a segment • the perpendicular bisector of a segment • the distance (between two points) • areas of triangles, given the measure of an angle and the lengths of two sides or given the measures of two angles and the length of one side
Similar and congruent triangles	
• Determining the minimum conditions required to conclude that triangles are congruent or similar	These conditions are listed in the Principles table that comes after this table.

Principles

Adult learners must master the following compulsory principles, which may be used in a proof:

P1. If the corresponding sides of two triangles are congruent, then the triangles are congruent.

P2. If two sides and the contained angle of one triangle are congruent to the corresponding two sides and contained angle of another triangle, then the triangles are congruent.

P3. If two angles and the contained side of one triangle are congruent to the corresponding two angles and contained side of another triangle, then the triangles are congruent.

P4. If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the triangles are similar.

P5. If the lengths of the corresponding sides of two triangles are in proportion, then the triangles are similar.

P6. If the lengths of two sides of one triangle are proportional to the lengths of the two corresponding sides of another triangle and the contained angles are congruent, then the triangles are similar.

P7. Transversals intersected by parallel lines are divided into segments of proportional lengths.

P8. In a right triangle, the length of the side opposite an angle of 30° is equal to half the length of the hypotenuse.

P9. The segment joining the midpoints of two sides of a triangle is parallel to the third side and its length is one-half the length of the third side.

P10. The length of a leg of a right triangle is the geometric mean between the length of its projection on the hypotenuse and the length of the hypotenuse.

P11. The length of the altitude to the hypotenuse of a right triangle is the geometric mean between the lengths of the segments of the hypotenuse.

P12. The product of the lengths of the legs of a right triangle is equal to the product of the length of the hypotenuse and the length of the altitude to the hypotenuse.

Cultural References

We owe Euclidean geometry to Euclid, a Greek mathematician who developed an organized body of geometry principles. In using deductive reasoning, adult learners will learn how to construct proofs. In the process, they may discover that, for Aristotle, deductive reasoning was the best way to acquire knowledge and that, for Galileo and Descartes, it made it possible to produce mathematical explanations of physical phenomena.

In developing their measurement sense, adult learners could learn to appreciate how a number of instruments (e.g. odometer, global positioning system, compass, sextant, quadrant) used today or in the past have helped solve many problems. Furthermore, surveying equipment, navigation and astronomical instruments, the mirror and shadow technique, the pantograph, the proportional compass, and Jacob's and Gerbert's staffs could help adult learners develop their understanding of the concept of similarity or make connections with the field of science. With respect to computers, adult learners may discover that visual on-screen representation involves trigonometry, and that animation in the development of video games requires geometric transformations.

In this course, adult learners are introduced to analytic geometry. The combination of loci (geometry) and equations (algebra) makes it easier to compare mathematical objects. Given that astronomy is a science that combines algebra and trigonometry, and that robotics, mechanics, automotive production and 3-D description involve combining sets of loci with algebra, all these fields could be incorporated into the learning process because they are likely to pique the interest and curiosity of adult learners.

FAMILY OF SITUATIONS

The situations in the family *Measurement and spatial representation* involve problems that can be solved in part through the geometric description or representation of an object or a physical space. The *Geometric Representation in an Applied Context 1* course provides adult learners with the opportunity to develop their spatial representation skills.

In the situational problems in this course, adult learners distinguish between the mathematical and everyday meanings of the terms used in order to understand certain concepts, give several examples before drawing conclusions when attempting to prove geometry principles pertaining to right triangles, and make conjectures about particular or special cases involving any triangle in order to validate certain results.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Career Planning and Entrepreneurship, and Environmental Awareness and Consumer Rights and Responsibilities.

Career Planning and Entrepreneurship

The *Geometric Representation in an Applied Context 1* course could be useful to adult learners who are interested in becoming machinists. A learning situation could involve visiting a vocational training centre and taking part in an exploration activity related to machining techniques. Adult learners would then have an opportunity to represent the different views of a metal part in order to design a plan. The learning situation allows adult learners to explore future prospects based on their interests and to become familiar with related job tasks and working conditions. This type of situation ties in with one of the focuses of development of this broad area of learning.

Environmental Awareness and Consumer Rights and Responsibilities

Many adult learners might be considering home renovations. Reading or designing plans, deciding what materials they will need and organizing the work involved could require knowledge of basic trigonometric concepts. For example, adult learners may need to be familiar with relations in right triangles and know how to calculate areas in order to plan the purchase of ceramic or parquet tiles. Furthermore, designing a staircase or an access ramp requires some knowledge of trigonometry in order to be able to assess the amount of space and materials needed. This course could help adult learners perform such calculations so that they can make informed choices, which ties in with one of the focuses of development of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Career Planning and Entrepreneurship
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Measurement and spatial representation
Targeted cross-curricular competencies • Are developed at the same time and in the same context as the subject-specific competencies.	• Communicates appropriately • Adopts effective work methods
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Measurement and spatial representation* family of learning situations
In a machine shop, the complete drawing of a part made up of several triangles was added to the specifications for a major project. The shop supervisor said that it was impossible to manufacture the part based on the documents and plan provided. After checking the documentation, the supervisor observed that while the measurements were accurate, the scale varied with the different views of the part. In addition, he noted that some of the angular measures were not specified and that some of the angles were not accurately drawn.	**Integrative process:** *Describing an object and representing it in two dimensions* In carrying out the four phases in the problem-solving process, adult learners could: Representation • Describe, in their own words, the errors found in the initial plan: the scale is different for each view of the part, certain measurements have been omitted • Determine the task to be carried out: produce a new plan in a cavalier perspective rather than manufacture the part • Make a sketch that takes into account the required modifications

<table>
<tr><th>Situational problem</th><th colspan="2">Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the Measurement and spatial representation family of learning situations</th></tr>
<tr><td rowspan="3">The supervisor has requested that a new plan be drawn, with the same scale used for the front, top and side views of the part and with the measure of each angle and each side indicated.

Adult learners must identify the errors in the initial plan and then produce a correct plan that meets the specified criteria.</td><td>Planning</td><td>• List the instruments needed to modify the plan: a set square, a protractor, a compass, a ruler or a computer and the appropriate software
• Determine the appropriate method and the order of operations: calculate the unknown angular measures using trigonometric ratios before drawing these angles, and indicate their measures on the plan, etc.</td></tr>
<tr><td>Activation</td><td>• Calculate the value of the unknown angular measures by using the lengths of the sides and the trigonometric ratios
• Carefully draw the different views or use technology to draw a complete plan for the part to be machined</td></tr>
<tr><td>Reflection</td><td>• Make sure that the changes made to a view are correct even when considering a different view
• Find the sum of the angles in the various triangles to make sure that they add up to 180 degrees
• Determine whether another method would have been faster or more effective</td></tr>
</table>

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Measurement and spatial representation*, adult learners describe an object or a physical space and represent it in two or three dimensions, and organize a physical space. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To describe and represent an object or a physical space, adult learners interpret and produce sketches or drawings using complex figures that can be broken down into right triangles or other types of triangles. They identify the key elements of mathematical language (e.g. scale, dimensions, perimeter, area) and associate pictures, objects or knowledge with mathematical terms and symbols. In addition, they apply newly acquired mathematical knowledge such as metric and trigonometric relations in triangles, which enables them to determine unknown measurements in unusual situations.

To organize a physical space, adult learners use a variety of strategies: making a sketch or drawing, dividing the task into subtasks, etc. They use a complex process—representing the problem, validating the solution and everything in between—applying their knowledge of trigonometry. They use the concept of triangulation to organize a physical space and validate every step against the theorems covered in the course. They deduce unknown measurements, determine results through inductive reasoning and draw conclusions based on their study of the theorems. When these conclusions involve the properties of certain figures, they prove their accuracy by developing a formal proof.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (trigonometric and metric relations in triangles, similar and congruent triangles, and equivalent figures). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

* The solution includes a procedure, strategies and a final answer.
** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*

6.3.2 Secondary V Courses

Course

MTH-5160-2

Optimization in an Applied Context

Mathematics

INTRODUCTION

The goal of the *Optimization in an Applied Context* course is to enable adult learners to use linear programming to deal with situations that involve optimization in an applied context.

In this course, adult learners solve situational problems that enable them to enhance their repertoire of strategies. They learn how to conduct case studies. They make comparisons, suggest corrections, propose favourable or optimal solutions, or issue recommendations. They formulate constructive criticism and make informed decisions concerning problems in various areas, including technical fields (e.g. graphics, biology, physics, administration). They apply their knowledge of arithmetic and algebra to different situational problems involving specific constraints which, in fact, represent limitations related to real-life situations involving optimization. In addition, they apply their knowledge of systems of first-degree inequalities in order to solve situational problems using linear programming. They use the simplex method to develop networks of cognitive resources.

By the end of this course, adult learners will be able to use linear programming to solve situational problems involving optimization. They will also be able to distinguish between explicit and implicit information, plan their solution based on the steps in the simplex method, apply their solution (process and outcome) taking constraints into account, and validate it according to the context of the situation.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

PROCESS AND STRATEGIES

REPRESENTATION

- Adult learners examine the situational problem to identify the context, the problem and the task to be performed.
- They use observational and representational strategies that are essential to inductive reasoning.
- In attempting to understand the context and the problem, they also use deductive reasoning.

Examples of strategies	• Determining the nature of the task involved and presenting the information in a table • Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to determine the economic or technical constraints involved in mathematizing the problem • Describing the situation in their own words and comparing their understanding of the problem with that of their classmates and teacher • Describing the characteristics of the situation • Determining questions about the situation

PLANNING

- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient.
- Planning the solution correctly involves decoding the elements of mathematical knowledge such as the meaning of symbols, terms and notation.

Examples of strategies	• Finding an algebraic rule that takes into account the best relationship between the constraints and consequences associated with the situational problem: determining the relevant parameters of the scanning line or the economic function • Sketching the Cartesian coordinate graph

ACTIVATION

- When dealing with a situational problem, adult learners use mathematical reasoning to graph the half-planes resulting from the constraints.
- They deduce the scale of the axes by analyzing the maximum and minimum values of the variables.
- They make rigorous use of mathematical language and, to avoid confusion, they use the symbols, terms and notation in accordance with their meaning.

Examples of strategies	• Proceeding by trial and error to mathematize certain constraints • Referring to previously solved situational problems in order to graph the half-planes resulting from the constraints • Constructing tables of values in order to find two points to represent the boundary lines of the polygon of constraints

REFLECTION

- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
- They go back and forth between the graph and the economic function when the solutions are integers.
- They express their ideas in accordance with mathematical codes and conventions, taking the constraints of the situation into account in formulating their messages.

Examples of strategies	• Comparing their results with the expected results and those of others • Checking their solution by, for example, comparing the number of possible solutions for a system of equations with the number of solutions found, or using their intuition to make sure that the coordinates of the points they have found are those of the vertices of the polygon of constraints • Identifying the strategies used to solve the situational problem

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Optimizing solutions*. Two of these are considered particularly relevant to this course: *Achieves his/her potential* and *Adopts effective work methods*.

Social Competency

Achieving career goals is a daily concern for adults who go back to school. This course offers a range of learning situations that help adult learners fulfill their potential. Exploring trades and occupations gives adult learners the opportunity to reflect on and implement their career plan. More and more occupations related to engineering are open to them. Meaningful and contextualized learning situations allow them to identify abilities that can open up new avenues. Operational research and linear programming are valuable assets in the process of discovering certain technical trades and occupations: adults learn to recognize what is possible for them. Having the opportunity to utilize their personal resources enables them to develop the competency *Achieves his/her potential* because they are then able to develop self-knowledge and the motivation to use their abilities to the fullest.

Methodological Competency

Through the study of linear programming, this course gives adult learners the opportunity to use the competency *Adopts effective work methods*. This type of programming provides a very sequential method of optimizing linear functions of the form $z = ax + by$ subject to several constraints. This course helps adult learners consider all aspects of a task, examine numerous possibilities and find ways of doing things or methods that are best suited to a particular situation or context. They can then adjust their actions as required.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of arithmetic and algebra. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following integrative process:

- **optimizing a situation using linear programming**

This process, which is applied in the learning situations in this course, fosters the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve this integrative process. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Linear programming	
• System of first-degree inequalities in two variables	
• Representing constraints and the function to be optimized (objective or economic function)	Constraints can be represented algebraically or graphically. In this course, the function to be optimized is expressed solely as an equation of the form $Ax + By + C = Z$, where A, B and C are rational numbers.
• Determining and interpreting the vertices and the feasible region (bounded or unbounded)	
• Changing the conditions associated with the situation to provide a more optimal solution	

Cultural References

Swiss mathematician Leonhard Euler (1707-1783), a pioneer in pure and applied mathematics, left his mark on numerous fields, including the theory of numbers, geometry, optics and astronomy. The origin of mathematical optimization lies in the principle of least action (i.e. to explain the world in terms of optimization), which is one of the boldest ideas in science.

Linear programming, which is a branch of optimization, originated with the work of French mathematician Joseph Fourrier (1768-1830) on systems of inequalities, even though these systems have been attributed to American mathematician George Dantzig (1914-2005). While in the United States Air Force during the Second World War, Dantzig developed a technique for solving the army's logistical problems at a minimum cost, but he published his work only in 1947.

Adult learners could examine a fictitious logistical problem on a smaller scale. The optimal solution should take into account the constraints of the situation. In this way, adult learners will become aware of the difficulties involved in coming to the rescue of the victims of a natural disaster. They will better understand the importance of coordinated action to save as many lives as possible.

Linear programming, which combines power and flexibility, was soon adopted in business and industry. Businesses used it to solve major economic problems, while industry applied it to production management.

Since the 1970s, linear programming has been applied in a variety of fields such as health care, the environment, agriculture, communications, the oil industry, chemistry, computer science, energy, transportation, industrial production and finance. This breakthrough is the result of advancements in computer technology, which made it possible to deal with situations involving an astronomical number of calculations. Examples given during the course enable adult learners to understand the importance of linear programming in everyday life.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Optimizing solutions* involve problems that can be solved in part through optimization using linear programming. The *Optimization in an Applied Context* course provides adult learners with an opportunity to learn how to maximize a profit, a process or a number of objects or people, and to minimize costs and losses.

In the situational problems in this course, adult learners make connections between literal expressions and inequality symbols by using examples involving numbers, determine the half-planes that represent the constraints and their impact on the economic function, and deduce certain values of the points of intersection of the boundary lines using simple substitution.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Environmental Awareness and Consumer Rights and Responsibilities, and Career Planning and Entrepreneurship.

Environmental Awareness and Consumer Rights and Responsibilities

Adult learners could be required to optimize the use of a piece of farmland by taking into account the surface area allotted to certain crops and the cost of fertilizers and fungicides. Such a learning situation will help make them aware of the interdependence between the environment and human activity, which ties in directly with one of the focuses of development of this broad area of learning.

Career Planning and Entrepreneurship

New and increasingly challenging trades and occupations regularly emerge in the job market. This course could enable adult learners to investigate these emerging trades and occupations. For example, linear programming could help familiarize them with the organization and analysis involved in agricultural and food engineering. Learning situations that involve applying the rules of linear programming to agricultural and food engineering will give adult learners the opportunity to explore one aspect of this field: using exact mathematics to find solutions. This type of learning situation encourages adult learners to undertake and carry out plans designed to develop their potential and help them integrate into society, which is the educational aim of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Environmental Awareness and Consumer Rights and Responsibilities
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Optimizing solutions
Targeted cross-curricular competency • Is developed at the same time and in the same context as the subject-specific competencies.	• Adopts effective work methods
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

<table>
<tr><th>Situational problem</th><th colspan="2">Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the Optimizing solutions family of situations</th></tr>
<tr><td rowspan="4">A farmer wants to plant corn and wheat on his land. He knows that in Québec the season is often too short to grow corn, unless nitrogen is added to the young plants to make them grow faster. Wheat must be treated to prevent such fungal diseases as leaf blotch, since up to 40% of the crop can be lost as a result of these diseases.</td><td colspan="2">Integrative process: Optimizing a situation using linear programming</td></tr>
<tr><td colspan="2">In carrying out the four phases in the problem-solving process, adult learners could:</td></tr>
<tr><td>Representation</td><td>Determine the key elements to be considered: the surface area of the farmer’s land, the cost of such products as seeds, fungicide and nitrogen fertilizer</td></tr>
<tr><td>Planning</td><td>Break down the problem into subproblems to identify the relationships between the constraints of the situation and the problem: corn production, wheat production, corn crop maintenance, wheat crop maintenance and fixed costs
State the constraints to be observed and what must be optimized</td></tr>
</table>

<table>
<tr><th>Situational problem</th><th colspan="2">Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the Optimizing solutions family of situations</th></tr>
<tr><td rowspan="2">Adult learners are required to find the surface area that should be allotted to corn and to wheat in order to maximize the farmer’s output. They will have to take into account the cost of nitrogen fertilizer and fungicide, as well as the number of hectares of land the farmer has.</td><td>Activation</td><td>• Mathematize the constraints related to farming and to corn and wheat crop maintenance
• Draw a Cartesian coordinate graph of these equations to find the vertices
• Find the optimal solution by determining what the surface area of the corn and wheat fields should be to provide the maximum output</td></tr>
<tr><td>Reflection</td><td>• Make sure the maximum solution corresponds to the highest vertex of the polygon of constraints</td></tr>
</table>

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Optimizing solutions*, adult learners optimize a situation using linear programming. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To use linear programming to solve a situational problem, adult learners apply different mathematical models and different types of strategies, combining reasoning and creativity to overcome obstacles. They decode relevant information in order to find an optimal solution. They translate the different constraints using a system of inequalities in two variables, and give an algebraic definition of the function to be optimized. They graph the polygon of constraints and the feasible region. They use algebra to determine the coordinates of the vertices using matrices, or approximate an answer using a graph. To prove a conjecture, they use structured deductive reasoning and correctly use the codified form required for their proof. They illustrate, explain or justify their arguments. To develop a proof, they use different types of reasoning, including proof by exhaustion. They observe real-life situations and make generalizations. Lastly, in some situations, they analyze data in order to identify the necessary and sufficient conditions for drawing a conclusion, make decisions and determine how best to approach, optimize or adjust the situation.

When conducting case studies, synthesizing information, constructing proofs or making presentations in order to deal with situational problems related to linear programming, adult learners must accurately identify the purpose of the mathematical messages to be conveyed or interpreted. They select the medium, the type of discourse and the register of representation best suited to the audience and the purpose of the message. They switch easily from one register to another. They use a wide range of communication strategies that enable them to regulate the transmission of a message based on the specific reactions of the audience or to take new requirements into account. They adopt language that appropriately combines everyday, mathematical, technical and scientific terms.

Throughout the problem-solving process, adult learners apply their knowledge of linear programming. Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

  * The solution includes a procedure, strategies and a final answer.
  ** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*

Course

MTH-5161-2

Algebraic and Graphical Modelling in an Applied Context 2

Mathematics

INTRODUCTION

The goal of the *Algebraic and Graphical Modelling in an Applied Context 2* course is to enable adult learners to deal with situations that involve using an algebraic or graphical model to represent a dependency relationship between quantities in an applied context.

In this course, adult learners interpret parameters in different registers of representation. They learn to model certain situations by means of a periodic function. While their exploration of the standard unit circle introduces the concept of the sinusoidal function, it also helps adult learners make a connection between radians and degrees and calculate arc lengths in different units. Only the sinusoidal model, however, is analyzed in all registers. Operations on functions are examined in the context of concrete situations. In addition to learning about the general form and the factored form of second-degree functions, adult learners discover that the factored form $h(x)$ can be obtained by finding the product or the sum of two functions $f(x)$ and $g(x)$. They also learn that a rational function can be obtained by finding the quotient of two polynomial functions. Adult learners continue to analyze situations in which the rate of change varies according to the interval in question. This type of analysis was introduced in Secondary III and IV. Adult learners can use several functional models to describe how two variables behave in a given interval.

By the end of this course, adult learners will be able to use different functions, including the sinusoidal function, to represent concrete situations. They will produce clear and accurate work in accordance with the rules and conventions of mathematics. By algebraically or graphically representing a situation using real functions and operations on real functions, they will be able to employ inductive reasoning to obtain results through interpolation or extrapolation. In addition, they will use different registers of representation to generalize results and extend them to other situations.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

<table>
<tr><th colspan="2">PROCESS AND STRATEGIES</th></tr>
<tr><th colspan="2">REPRESENTATION</th></tr>
<tr><td colspan="2">- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. They use observational and representational strategies that are essential to inductive reasoning.
- They become more familiar with the mathematical symbols and notation related to functions and their inverse expressed in their general form.</td></tr>
<tr><td>Examples of strategies</td><td>• Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to identify a dependency relationship and determine the variables in the situation
• Using examples involving numbers, determining the type of relationship that exists between the variables in a situation
• Sketching a Cartesian coordinate graph to represent the dependency relationship between the variables
• Making false assumptions to identify an inconsistency or an absurdity to corroborate their perceptions or call them into question</td></tr>
<tr><th colspan="2">PLANNING</th></tr>
<tr><td colspan="2">- Adult learners look for ways of approaching the problem and choose those that seem the most efficient.
- They attempt to extrapolate results using an algebraic rule or a graph, thus expanding their networks of cognitive resources.
- To correctly plan their solution, they decode the meaning of the symbols, terms and notation used, as well as the different registers of representation.</td></tr>
<tr><td>Examples of strategies</td><td>• Drawing a concept map showing the different steps in the solution
• Referring to a list of elements to be considered in consolidating their work plan (e.g. the scale of the axes, the increasing and decreasing intervals, the maximum or the minimum, if any)</td></tr>
<tr><th colspan="2">ACTIVATION</th></tr>
<tr><td colspan="2">- When dealing with a situational problem, adult learners use reasoning to establish organized and functional relationships among different aspects of their knowledge, thus expanding their networks of mathematical cognitive resources.
- They use different strategies by associating pictures, objects or concepts with mathematical terms and symbols and by switching from one register of representation to another.</td></tr>
<tr><td>Examples of strategies</td><td>• Changing perspective
• Systematically determining the general form of the algebraic rule of a function
• Finding combinations in order to determine the rule of a quadratic function</td></tr>
<tr><th colspan="2">REFLECTION</th></tr>
<tr><td colspan="2">- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
- Through reasoning, they could make conjectures about particular or special cases to validate certain results.
- They use different strategies to make sure that the dependent and independent variables are properly defined, that the axes are correctly scaled, that no unit of measure has been omitted and that the data have been correctly transcribed.</td></tr>
<tr><td>Examples of strategies</td><td>• Checking their solution by, for example, making sure that the resulting values satisfy the range of the function, or substituting the values of the variables in the algebraic expression in order to validate a graphical interpolation or extrapolation</td></tr>
</table>

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Relationship between quantities*. Two of these are considered particularly relevant to this course: *Uses information and communications technologies* and *Uses information*.

Methodological Competency

Adult learners who wish to compile and analyze data related to a situation could use computer tools such as a spreadsheet program or graphing software. These tools make it easier to produce graphs and to change or work with parameters in order to carry out simulations and extrapolations. Through the competency *Uses information and communications technologies*, adult learners will realize that the ability to master these technologies will make their work considerably more interesting.

Intellectual Competency

The information in studies on physical and natural phenomena is not necessarily presented in text or table form. Data can be collected through probes, and in this case, it must be organized so that it can be interpreted as accurately as possible in order to generate the required information. Adult learners could therefore learn to use information consisting of raw data. The competency *Uses information* will help them distinguish between data and information, and understand that proper organization makes it possible to correctly interpret a situation.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of arithmetic and algebra. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following three integrative processes:

- **using an algebraic or graphical model of a function to represent a situation**
- **interpolating or extrapolating from an algebraic or graphical model**
- **using an algebraic or graphical model of a function to generalize a set of situations**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover all three processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Numerical and algebraic expressions	
• Completing the square	Completing the square is used for factoring second-degree polynomial functions and switching from one way of writing them to another.
• Dividing second-degree polynomials in one or two variables by a first-degree binomial	The polynomials have a maximum of four terms.
Relation, function and inverse	
• Experimenting with real functions and their inverse as well as observing, interpreting, describing and representing them	Functions can be represented: • verbally • using a table of values • algebraically • graphically

<table>
<tr><th>Mathematical Knowledge</th><th>Restrictions and Clarifications</th></tr>
<tr><td>Relation, function and inverse (cont.)</td><td></td></tr>
<tr><td>
Experimenting with real functions and their inverse as well as observing, interpreting, describing and representing them (cont.)
</td><td>
The real functions studied are:

second-degree polynomial functions (general, standard and factored form)
$f(x) = ax^2 + bx + c$
$f(x) = a(x - h)^2 + k$
$f(x) = a(x - x_1)(x - x_2)$
exponential functions
$f(x) = a\, c^{b(x - h)} + k$
logarithmic functions
$f(x) = a\log_c b(x - h) + k$
rational functions (standard form)
$f(x) = a\left(\frac{1}{b(x - h)}\right) + k$
and also of the following form:
$f(x) = \frac{ax + b}{cx + d}$ where a, b, c and d $\in \mathbb{R}$ and $cx + d \neq 0$
square root functions
$f(x) = a\sqrt{b(x - h)} + k$
sinusoidal functions
$f(x) = a \sin b(x - h) + k$ and
$f(x) = a \cos b(x - h) + k$
tangent functions
$f(x) = a \tan b(x - h) + k$
greatest integer functions
$f(x) = a\,[b(x - h)] + k$

Experimental data are modelled by using curves related to the functional models under study and associating them with scatter plots.

The study of exponential and logarithmic functions should focus on bases 2, 10 and e.
</td></tr>
</table>

Mathematical Knowledge	Restrictions and Clarifications
Relation, function and inverse (cont.)	
• Experimenting with real functions and their inverse as well as observing, interpreting, describing and representing them (cont.)	The concept of inverse is further studied in Secondary V; it is mainly associated with logarithmic, rational, exponential and square root functions. The second-degree polynomial function was introduced in previous courses and is now studied in standard form. Converting an expression to the factored form involves using the factoring methods studied in Secondary IV. Converting an expression to the general form involves expanding the standard form of the expression and makes it possible to establish a correspondence between the parameters. In order to switch from the general form to the standard form, adult learners refer to the established correspondences or complete the square.
• Operations on functions	The four operations are studied in addition to the composition of functions.
• Describing and interpreting the properties of a function	The properties of real functions covered in this course are: • domain and codomain (range) • increasing and decreasing intervals • extrema • sign • x- and y-intercepts
• Interpreting additive parameters in the different registers of representation	The registers of representation studied are: • tables of values • rules • graphs

Mathematical Knowledge	Restrictions and Clarifications
Relation, function and inverse (cont.)	
• Solving equations and inequalities in one variable	The following equations and inequalities are studied: • trigonometric equations and inequalities of the first degree containing a sine, a cosine or a tangent • second-degree equations and inequalities • square root equations and inequalities • rational equations and inequalities • exponential and logarithmic equations and inequalities that involve applying the properties of exponents and logarithms *The concepts of arcsine, arccosine and arctangent are studied mainly as inverse operations involved in solving equations or inequalities. The same is true for the concepts of square root and the logarithm introduced in previous courses.*
System	
• Finding the graphical solution for situations consisting of systems of equations or inequalities involving different functional models	

Cultural References

Human beings have always invented instruments to make their lives easier. In developing their mathematical competencies, adult learners may discover, among other things, that modelling is used to design a number of instruments and machines, that mathematical reasoning plays a role in their manufacture and that different registers of graphical representation must be used to operate them.

By studying the evolution of certain modern instruments (e.g. sphygmomanometer used to measure blood pressure, multimeter), adult learners can make connections between algebraic modelling and the use of these instruments in professional or technical occupations in the sciences. For example, they could analyze a digital camera. Using experimentation and graphs, they could study the relationships between the camera’s resolution, format, pixels, size and storage capacity to determine whether these are functional relationships. They attempt to determine the type of function involved, if any.

In addition, adult learners may discover that the search for all sorts of precise measurements has been a constant concern throughout history.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Relationship between quantities* involve problems that can be solved in part by using an algebraic or graphical model of a function to represent a relationship between quantities. The *Algebraic and Graphical Modelling in an Applied Context 2* course provides adult learners with an opportunity to express a connection or a dependency relationship between quantities.

In the situational problems in this course, adult learners become more familiar with the mathematical symbols and notation related to the functions and their inverse expressed in their general form, extrapolate results using an algebraic rule or a graph, and use the appropriate scale so that the graph they draw in solving the situational problem makes sense in light of the context.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Environmental Awareness and Consumer Rights and Responsibilities, and Career Planning and Entrepreneurship.

Environmental Awareness and Consumer Rights and Responsibilities

Adult learners interested in natural disasters such as earthquakes could, through a learning situation on this topic, establish a relationship between a logarithmic function and the calculation of the magnitude of an earthquake. They would discover that this data value is related to a continuous logarithmic function rather than an ordinary scale. Because of the logarithmic nature of this phenomenon, when the energy generated by an earthquake varies by a factor of 10, this corresponds to a one-unit change in magnitude. For example, an earthquake with a magnitude of seven on the Richter scale is ten times stronger than an earthquake with a magnitude of six. Adult learners could use this situation to become more knowledgeable about their environment and improve their understanding of certain phenomena, which ties in directly with one of the focuses of development of this broad area of learning.

Career Planning and Entrepreneurship

In a learning situation involving financial mathematics, adult learners could be asked to determine an annual rate of interest and the value of a term deposit for different investment years, given the initial amount invested and its value ten years later. This situation enables adult learners to use their knowledge of exponential functions to develop a more practical understanding of this function, while learning about the principles of saving. In this way, they could develop strategies that will be useful in carrying out a personal plan, which ties in directly with one of the focuses of development of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Career Planning and Entrepreneurship
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Relationship between quantities
Targeted cross-curricular competencies • Are developed at the same time and in the same context as the subject-specific competencies.	• Uses information and communications technologies • Uses information
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Relationship between quantities* family of learning situations
An adult learner wants to find out more about the work of a traffic collision reconstruction expert. She wants to become familiar with the concepts related to this type of reconstruction. In addition to gathering information about a particular event, interpreting physical evidence found at the collision site, taking photographs of the scene and making sketches, such an expert draws on certain mathematical concepts.	**Integrative process:** *Using an algebraic or graphical model of a function to generalize a set of situations* In carrying out the four phases in the problem-solving process, adult learners could: Representation • Select the relevant information (mass and acceleration in this case) and disregard superfluous information (e.g. tire traction, reaction time, type of surface, weather conditions) • Reflect on the need to refer to several similar experiences to be able to come to a generalization Planning • Choose several similar experiments involving acceleration and deceleration • List the elements needed to draw the graph (mass and acceleration in this case)

<table>
<tr><th>Situational problem</th><th colspan="2">Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the Relationship between quantities family of learning situations</th></tr>
<tr><td rowspan="2">For example, using data resulting from experiments, the adult learner determines the relationship between the acceleration (or deceleration) of a vehicle and its mass and whether it is possible to generalize this rule, especially when the initial speed is changed.</td><td>Activation</td><td>Make a table of the data related to the situation, taking into account the limitations and precision of the measuring instruments used
For a given initial speed, find the algebraic rule showing the relationship between acceleration and mass
Repeat the operation with different initial speeds
Compare the resulting relationships in order to derive a general rule of correspondence between acceleration and mass (the rule should be valid regardless of the initial speed)</td></tr>
<tr><td>Reflection</td><td>Suggest probable or plausible reasons that the equation is not perfectly consistent with the data analyzed (human error, measurement errors, limitations of the instruments used to take the measurements, etc.)</td></tr>
</table>

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Relationships between quantities*, adult learners represent a situation, interpolate or extrapolate, and generalize a set of situations using an algebraic or graphical model. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To represent a situational problem using an algebraic or graphical model, adult learners describe, symbolize, code, decode, explain or illustrate the information contained in tables of values or algebraic rules. They combine different registers of representation as needed to produce a message in accordance with the notation, rules and conventions of mathematical language. They use problem-solving strategies to make comparisons, propose corrections, present favourable or optimal solutions, or issue recommendations. They formulate constructive criticism and make informed decisions about issues in a variety of fields, including technical fields (e.g. graphics, biology, physics, administration).

To interpolate or extrapolate results from an algebraic or graphical model in order to make decisions, adult learners use different types of functional and strategic models, combining reasoning and creativity to overcome obstacles. They use structured deductive reasoning and become familiar with the codified form required for their proof. They illustrate, explain or justify their arguments. They use different types of proofs and different lines of reasoning, including proof by exhaustion. The latter is used in particular in analyzing or conducting case studies, or in applying a generalization process leading to the validation of a conjecture. Adult learners also observe specific real-life cases and generalize their observations.

To generalize a set of situations using an algebraic or graphical model, adult learners specify the purpose of their communication and switch from one register to another as needed. They demonstrate their understanding of the problems in question using a wide range of communication strategies, which enables them to regulate their transmission of a message based on the specific reactions of the audience or to take new requirements into account. They learn and correctly use language that appropriately combines everyday, mathematical, technical and scientific terms. They deduce new algebraic rules by combining the different operations on functions they have mastered, and prove them, justifying all the steps in their procedure. In addition, they make effective use of the parameters of the functions to illustrate generalities about a set of functions.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (relations, functions, inverse and system of equations). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

* The solution includes a procedure, strategies and a final answer.
** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*

Course

MTH-5163-2

Geometric Representation in an Applied Context 2

Mathematics

MTH-5163-2 Geometric Representation in an Applied Context 2

INTRODUCTION

The goal of the *Geometric Representation in an Applied Context 2* course is to enable adult learners to deal with situations that involve describing and representing an object or a geometric locus in an applied context.

In this course, adult learners broaden their network of concepts to include equivalent figures, metric relations in circles and trigonometry in triangles. They are introduced to new mathematical knowledge related to vectors and geometric representation, which help them make connections with the sciences. They find the resultant and make connections with a composition of translations, triangles and parallelograms. Trigonometric relations are used in situations involving the orthogonal projection of a vector. As with vectors, studying the relative position of two circles and constructing the segment representing the distance from a point to a circle or an ellipse allows adult learners to apply the concept of distance to other situations. An intuitive approach is used to introduce mathematical concepts related to loci and relative positions. Adult learners continue to develop this mathematical knowledge through exploration and observation activities that involve finding the figure that corresponds to the description of a locus. Conversely, they describe the locus corresponding to a given figure. The emphasis is on describing a geometric locus so as to identify the necessary and sufficient conditions that make it possible to understand and use it. When adult learners define a locus, they first describe it in terms of the concept of distance. They then use their understanding of algebraic expressions as well as familiar operations to modify the expression without changing its meaning. Adult learners formulate and validate conjectures relating to a locus, i.e. the possible position of a set of points that meets specific conditions. They construct loci using properties and devising mechanisms or procedures; they draw or modify loci using geometric transformations. Constructing knowledge related to the concept of geometric locus involves exploring several different loci and recognizing that a given locus can be generated in different ways. Connections with the sciences and with vocational and technical training can readily be made through the study of this concept.

By the end of this course, adult learners will be able to use trigonometric relations, the properties of equivalent figures and metric relations in circles to describe and represent geometric transformations. Used in conjunction with matrices, analytic geometry makes it possible to algebraically model certain geometric transformations of objects. Adult learners will also be able to use vectors to describe, represent and generalize certain characteristics of geometric loci in the Cartesian plane in accordance with the mathematical notation and conventions employed in geometry.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the notation and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

<table>
<tr><th colspan="2">PROCESS AND STRATEGIES</th></tr>
<tr><th colspan="2">REPRESENTATION</th></tr>
<tr><td colspan="2">- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. They use strategies essential to inductive reasoning.
- They conceive of probable or plausible relationships that can then be expressed as a formal conjecture. To do this, they use strategies to clarify the different patterns and invariants.
- They distinguish between the mathematical and everyday meanings of the terms used so that they can understand what is meant by focus, angle, vertex, arc and so on. They use different sources of information to provide a correct representation of the problem.</td></tr>
<tr><td>Examples of strategies</td><td>• Using an estimate, a scale plan or a literal description to determine the nature of the task involved
• Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to find measurements or provide a spatial representation
• Representing the situational problem mentally or in writing
• Describing the situational problem in their own words to show that they have understood it</td></tr>
<tr><th colspan="2">PLANNING</th></tr>
<tr><td colspan="2">- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient.
- They use reasoning when they employ different registers of representation to highlight the properties of geometric transformations.
- They use a literal expression to describe the image of an object resulting from a geometric transformation, taking into account the elements of mathematical language.</td></tr>
<tr><td>Examples of strategies</td><td>• Dividing the situational problem into subproblems, for example, in order to find a measurement using the metric relations in a circle
• Using lists, tables, diagrams, concrete materials or drawings to work out their solution</td></tr>
<tr><th colspan="2">ACTIVATION</th></tr>
<tr><td colspan="2">- When dealing with a situational problem, adult learners make rigorous use of the elements of mathematical language and, to avoid confusion, they use the symbols, terms and notation in accordance with their meaning.
- When using vectors to prove a geometric proposition, they identify patterns by exploring different cases related to the properties of vectors.</td></tr>
<tr><td>Examples of strategies</td><td>• Using the characteristics of a conic to make a sketch in order to predict results
• Solving certain situational problems by working backwards when the solution consists of several steps or when there is insufficient information
• Analyzing the effects of a geometric transformation on a plane figure to properly understand its relationship with the parameters of the algebraic or matrix rule, for example</td></tr>
<tr><th colspan="2">REFLECTION</th></tr>
<tr><td colspan="2">- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
- Reasoning helps them make conjectures about particular or special cases involving any triangle in order to validate certain results.
- They make sure that their message is clear and that they have observed the relevant notation and conventions.</td></tr>
<tr><td>Examples of strategies</td><td>• Checking their solution by means of examples or counterexamples
• Determining the strategies for dealing with situational problems in geometry (e.g. applying a rule, referring to a geometry principle, using a formula)</td></tr>
</table>

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Measurement and spatial representation*. Two of these are considered particularly relevant to this course: *Uses information* and *Uses information and communications technologies*.

Intellectual Competency

In a situation related to architecture or urban planning, adult learners could be asked to find information on conics related to these contexts. They would have to find and assess this information and organize it in accordance with the constraints of the problem. In this way, adult learners would be developing the competency *Uses information*.

Methodological Competency

By studying the movements associated with a computer animation, adult learners might decide to try their hand at this type of work using specialized software. They would first have to become familiar with a new computer environment so that they could perform geometric transformations involving simple objects. They would then use their knowledge of this medium to produce a more complex animation. In this way, adult learners would be able to develop the competency *Uses information and communications technologies*.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of geometry and algebra. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following three integrative processes:

- **describing an object or a physical space and representing it in two or three dimensions**
- **describing geometric loci and representing them graphically and algebraically**
- **using vectors to generalize geometry principles**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover all three processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Geometric transformations	
• Describing and representing geometric transformations	Geometric transformations are represented using: • algebraic rules • matrices *The matrix form is introduced in order to enable adult learners to synthesize what they have learned.*
Trigonometric relations in triangles	
• Representing and interpreting situations using triangles	The sine law and cosine law will be studied in this course. Trigonometric ratios in right triangles may also be applied. *The other metric and trigonometric relations are listed in the Principles table that comes after this table.*

Mathematical Knowledge	Restrictions and Clarifications
Equivalent figures **(plane figures or solids)**	
• Finding measurements: ○ lengths of segments ○ areas ○ volumes ○ capacities	These unknown measurements are found by applying the properties of congruent, similar or equivalent figures. *The other relations associated with equivalent figures are listed in the Principles table that comes after this table.*
Metric relations in circles	
• Finding measurements: ○ arcs or angles (degrees or radians) ○ lengths (segments, chords)	
Standard unit circle	
• Finding measurements: ○ arcs or angles (radians)	
• Finding the coordinates of points associated with important angles	Important angles are multiples of $\frac{\pi}{6}$, $\frac{\pi}{4}$ and $\frac{\pi}{2}$. Finding these coordinates might require the use of trigonometric relations in right triangles and only requires the use of the identity $\sin^2 a + \cos^2 a = 1$. The properties of periodicity and symmetry are studied in this course.
Trigonometric identities	
• Manipulating simple trigonometric expressions using definitions	*Only the Pythagorean identities and the properties of periodicity and symmetry are studied in this course.*

Mathematical Knowledge	Restrictions and Clarifications
Geometric loci and relative positions	
• Describing, representing and constructing geometric loci ○ plane loci	The plane loci studied involve lines or circles only.
○ conics studied: ▪ parabola (centred at the origin and translated) ▪ circle (centred at the origin and translated) ▪ ellipse (centred at the origin and translated) ▪ hyperbole (centred at the origin and translated)	The elements described are: • the radius • the axes • the directrix • the vertices • the foci • the asymptotes • the regions *Conics are described using only the standard form of the algebraic rule.*
Vectors	
• Resultant and projection	
• Operations on vectors	Geometric and free vectors are studied in this course. Operations on vectors are limited to the following: • adding and subtracting vectors • multiplying a vector by a scalar • the scalar product of two vectors

Principles

Adult learners must master the following compulsory principles, which may be used in a proof:

P13. The medians of a triangle determine six equivalent triangles.

P14. The midpoint of the hypotenuse of a right triangle is equidistant from the three vertices.

P15. The lengths of the sides of any triangle are proportional to the sines of the angles opposite these sides (*sine law*)

P16. The square of the length of a side of any triangle is equal to the sum of the squares of the lengths of the other two sides, minus twice the product of the lengths of the other two sides multiplied by the cosine of the contained angle (*cosine law*).

P17. Regular polygons have the smallest perimeter of all equivalent polygons with *n* sides.

P18. Of two equivalent regular convex polygons, the polygon with the most sides will have the smaller perimeter. (Ultimately, an equivalent circle will have the smaller perimeter.)

P19. Cubes have the largest volume of all rectangular prisms with the same total surface area.

P20. Spheres have the largest volume of all solids with the same total surface area.

P21. Cubes have the smallest total surface area of all rectangular prisms with the same volume.

P22. Any diameter perpendicular to a chord divides that chord and each of the arcs that it subtends into two congruent parts.

P23. The measure of an inscribed angle is one-half the measure of its intercepted arc.

P24. If a line is perpendicular to a radius of a circle at the endpoint of the radius in the circle, the line is tangent to the circle. The converse is also true.

P25. In a circle or in congruent circles, two congruent chords are equidistant from the centre and vice versa.

P26. Two parallel lines, be they secants or tangents, intercept two congruent arcs of a circle.

P27. If point P is located outside circle O, and if segments PA and PB are tangents to that circle at points A and B respectively, then OP bisects angle APB and the length of segment PA is equal to the length of segment PB.

P28. The measure of an angle located between the circumference and the centre of a circle is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

P29. The measure of an angle located outside a circle is one-half the difference of the measures of the intercepted arcs.

P30. If two chords of a circle intersect in its interior, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

P31 If secants PAB and PCD of a circle have the same external endpoint P, then $m\overline{PA} \times m\overline{PB} = m\overline{PC} \times m\overline{PD}$.

Cultural References

Although urban planning has probably existed since the advent of the first cities, it was raised to the status of a science in 1867 with the publication of Ildefons Cerdà's work entitled *General Theory of Urbanization*. However, this discipline was developed only in the 20th century through the creation of specialized organizations and schools. Moreover, it was in the 19th century that Baron Haussmann redeveloped a part of Paris not only to beautify it, but also to make it easier for pedestrians to get around and to improve the air quality. Today, urban planning deals with such matters as the organization of living space and the design of traffic circles that are similar to conics.

We can easily imagine the difficulty involved in managing a city the size of Montréal. Fortunately, specialized software helps engineers in their planning and representational tasks. An introduction to urban planning could encourage adult learners to carry out different projects suggested in the course.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Measurement and spatial representation* involve problems that can be solved in part through the mathematical description or representation of geometric transformations of objects or geometric loci. The *Geometric Representation in an Applied Context 2* course provides adult learners with an opportunity to develop their spatial representation skills.

In the situational problems in this course, adult learners prove geometry principles related to metric relations in a circle, giving several examples before drawing conclusions; apply the properties of geometric transformations; and validate their message by consulting different sources of information or by comparing their understanding of the message with that of their classmates.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: *Citizenship and Community Life*, and *Media Literacy*.

Citizenship and Community Life

Some learning situations could involve using a tracking system to locate an object or an individual, thereby helping adult learners become more aware of their surroundings and develop an attitude of openness to the world and respect for diversity. For example, imagine that a missing person with a cell phone must be found. By means of sensors, the person can be located using triangulation. In addition to dealing with the situation mathematically, adult learners must weigh the advantages and disadvantages of global positioning systems used for recreational, efficiency or security reasons. They will therefore be better able to critically assess the unintended consequences of such systems and to strike a balance between public security and individual freedom. This issue ties in with one of the focuses of development of this broad area of learning.

Media Literacy

Some learning situations could provide adult learners with the opportunity to learn how to produce media documents, including those that involve computer animation techniques. For example, a learning situation could require that adult learners create an animation whose movements would be programmed using the appropriate software. Producing this type of animated sequence would enable adult learners to integrate their knowledge of geometric transformations and matrices in linear programming. This approach ties in with one of the focuses of development of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Citizenship and Community Life
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Measurement and spatial representation
Targeted cross-curricular competencies • Are developed at the same time and in the same context as the subject-specific competencies.	• Uses information • Uses information and communications technologies
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Measurement and spatial representation* family of learning situations
In some European countries, the movement of tectonic plates can produce a marked change in the local landscape. In Switzerland, for example, maps showing the boundaries of land parcels must be reviewed and corrected periodically because of these changes in the landscape.	**Integrative process:** *Describing an object or a physical space and representing it in two or three dimensions* In carrying out the four phases in the problem-solving process, adult learners could: Representation • Decode the elements that can be processed mathematically • Illustrate the shift in the boundary points of a piece of land by sketching a Cartesian coordinate graph Planning • Review their knowledge of vectors and geometric transformations • Outline the different steps involved in working out their solution, taking into account the constraints of the situation Activation • Using vectors, calculate the position of the boundary points of the piece of land and determine the components of the matrix that models the disturbance described in the

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Measurement and spatial representation* family of learning situations	
To understand this phenomenon, adult learners must use matrices and vectors to describe and represent the changes in values of the coordinates (*x*, *y*) of the points that define the boundaries of a piece of land, which shift as a result of natural disturbances. In addition, they must test their model by predicting what will happen to the boundary points as a result of these disturbances if they are considered constant over time (e.g. a 2-m southward shift and a 3-m eastward shift every 125 years). A diagram with a legend explains the natural disturbances involved.	Reflection	problem; these components can be rotations, dilatations or translations • Apply matrix rules and perform vector operations in solving the problem • Submit their solution for critical assessment by sharing it with their teacher and classmates so that it can be modified • Validate their predictions by having their classmates review their work • Consult reference materials in order to obtain background information that will help them better understand the problem

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Measurement and spatial representation*, adult learners describe geometric transformations of objects and geometric loci, represent them graphically and algebraically, and use vectors to generalize geometry principles. To do this, they use the three subject-specific competencies: *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To describe and represent an object or a physical space, adult learners interpret and produce drawings, plans or graphical representations by using solids or complex figures that can be broken down into triangles or circles. They also distinguish between the key elements of mathematical language (perimeter, area, equivalent solids, chords, etc.) and associate images, objects or knowledge with mathematical terms and symbols. They also apply their newly acquired knowledge (sine or cosine law and equivalent figures), which enables them to find unknown measurements in various situations.

To describe geometric loci and represent them graphically or algebraically, adult learners use different mathematical models and types of strategies, combining reasoning and creativity to overcome obstacles. Adult learners observe specific real-life cases and generalize their observations. They analyze data in order to identify the necessary and sufficient conditions for drawing a conclusion, making a decision, and determining how best to approach, optimize or adjust a situation. In addition, they use a Euclidean or Cartesian plane in order to determine measurements, optimize distances, construct geometric loci, represent the relative positions of figures or justify recommendations.

To prove a geometry theorem using vectors, adult learners translate the hypotheses and thesis into vector expressions and construct an equality. They develop the equality and use the Chasles relation to reduce the initial equality to its simplest form. As needed, they apply the properties of the scalar product of two vectors. They make connections between vector notation and the properties of geometric figures. In addition, they justify all the steps in their solution so as to communicate clearly and concisely.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (geometric transformations, trigonometric relations in triangles, equivalent figures, metric relations in circles, the unit circle, trigonometric identities, geometric loci and relative position, and vectors). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

* The solution includes a procedure, strategies and a final answer.
** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*

6.4 Science Option

In the *Science* option, adult learners continue to develop their competencies, use and expand their knowledge, and become familiar with new networks of mathematical cognitive resources. Their capacity for abstract thinking enables them to make a variety of connections among the different branches of mathematics, in particular between algebra and geometry. They make formal and more proficient use of symbols, rules and conventions, and they construct proofs.

This option emphasizes the modelling process. By learning to devise a mathematical model to represent a situation, adult learners develop the ability to work with various types of dependency relationships, geometric figures and statistical and probabilistic processes. They analyze a situation, a phenomenon or a behaviour and notice related patterns or trends. They interpolate, extrapolate and generalize elements. These activities may involve simulations or making connections between statistical and algebraic concepts. In this way, adult learners discover how useful mathematics can be to society. The *Science* option prepares adult learners to continue their studies in science, research and vocational and technical training. The context associated with the different courses in this option involves fundamental mathematics.

Adult learners encounter situations that require them to use their knowledge of mathematics and other subject areas. They sometimes work with purely mathematical contexts, while continuing to deal with concrete situations, particularly of a scientific nature. The learning situations should preferably be related to the sciences, since they enable students to develop methods used in scientific research and investigation. The variety of situations that may be studied can focus on such things as:

- contexts involving biology, such as cell multiplication or epidemics
- contexts involving economics, such as the study of different financing rates and terms
- cyclical occurrences (e.g. tides, the seasons, physiological data, mechanisms that generate movement, the changes in a person's position or a pendulum in motion)
- demographic or biological contexts
- contexts associated with physics, such as the analysis of situations involving successive displacements, forces or velocities

The *Science* option also involves situations related to surveying, topography, geodesy, biology, biometrics, optics, mechanics, electricity, chemistry, meteorology and computer science. In addition, adult learners can carry out science-related learning activities in order to emphasize the scientific aspect of this option as well as the contribution of mathematics to society. Among other things, they could organize exhibits, interview a physicist or a mathematician, or visit pharmaceutical, meteorological or robotic centres. A mathematics tutoring committee could also be created to help adult learners experiencing difficulties with the subject. Such activities will enable adult learners to develop a positive attitude toward mathematics, to better understand different concepts that are based on mathematics and to find new ways of being actively involved in their work.

The learning process can also be enhanced by technology. By piquing their curiosity, the use of technology encourages adult learners to reason and formulate various conjectures. The visual nature of these tools helps them form a mental image of the situations they encounter. Some types of interactive software allow them to observe what happens when the values of certain parameters are changed. These different tools give them an opportunity to generate various conjectures that in turn give rise to other questions and gradually lead to a process of exploration that will help them construct a proof.

In the *Science* option, adult learners have many opportunities to refine their work methods, explore research procedures, discuss science, and develop mathematical and cross-curricular competencies in meaningful situations. As the main architects of their own education, adult learners carry out these activities autonomously. This option offers them an intellectual education that prepares them to function effectively in a changing world.

6.4.1 Secondary IV Courses

Course

MTH-4271-2

Algebraic and Graphical Modelling in a Fundamental Context 1

Mathematics

INTRODUCTION

The goal of the *Algebraic and Graphical Modelling in a Fundamental Context 1* course is to enable adult learners to deal with situations that involve using an algebraic or graphical model to represent a dependency relationship between quantities in a fundamental context.

In this course, adult learners expand their knowledge of algebra in order to analyze all the nuances of the functions in question. Step functions give them the opportunity to further develop their understanding of real numbers and their reasoning abilities, especially when they represent and compare *greatest integer, truncation* and *round* functions, as well as the *fractional part* function. In order to determine the parameters of the functions to be studied, adult learners will normally be required to write the standard form of the equation. To make the parameters more meaningful, they analyze their role in the function, their effect on graphs (transformation of the initial function) and their relationship with the given information. Observations and manipulations can be carried out with or without technological tools, depending on the educational goals involved. Technology makes it possible to find the best model more quickly and to focus on its analysis and justification rather than on algebraic manipulations. In situational problems, adult learners identify data (in verbal, algebraic or graphical form or in a table of values), create models, recognize patterns, and interpolate or extrapolate in order to conduct an in-depth analysis of the situation. In many of the situational problems, they must demonstrate their ability to work with algebraic expressions. To arrive at one or more solutions, they use mathematical rigour and deductive reasoning. In addition, the situational problems include tasks in which adult learners must validate and correct the solutions as needed. Other situations involve formal proofs associated with different items of knowledge, in particular the properties and manipulation of algebraic expressions. Still others allow adult learners to analyze a model by determining and interpreting the value of the parameters. Situations involving the concept of correlation help adult learners develop a line of reasoning which, supported by an understanding of dependency relationships and a capacity for abstraction, lead them to recognize cause-and-effect relationships. Situations involving systems of equations or inequalities require that adult learners describe and interpret information. Lastly, some situations require that data be processed in a given register of representation, in particular when writing the rules of second-degree functions in standard, general or factored form, while others require translation from one register to another.

By the end of this course, adult learners will be able to use algebra to represent concrete situations. They will produce clear and accurate work in accordance with the rules and conventions of mathematics. By algebraically or graphically representing a situation using real functions and their inverse, they will be able to employ inductive or deductive reasoning to obtain results through interpolation or extrapolation. In addition, adult learners will use different registers of representation to generalize results and extend them to other situations.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

<table>
<tr><th colspan="2">PROCESS AND STRATEGIES</th></tr>
<tr><th colspan="2">REPRESENTATION</th></tr>
<tr><td colspan="2">- Adult learners examine the situational problem to identify the context, the problem and the task to be performed.
- They use observational and representational strategies that are essential to inductive reasoning.
- Through different lines of deductive reasoning, they could test their hypothesis by assigning increasingly large values to one variable to determine their effect on the value of the other variable.</td></tr>
<tr><td>Examples of strategies</td><td>• Using a table to display the variables involved
• Sketching a Cartesian coordinate graph
• Using words or symbols to reformulate the dependency relationship between the variables
• Exploring the situational problem by substituting numerical values in algebraic fractions to observe the changes in the quotient</td></tr>
<tr><th colspan="2">PLANNING</th></tr>
<tr><td colspan="2">- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient.
- Reasoning enables them to establish organized and functional relationships among different aspects of their knowledge, thus expanding their networks of mathematical cognitive resources.
- For example, they attempt to extrapolate results using an algebraic rule or a graph.</td></tr>
<tr><td>Examples of strategies</td><td>• Listing the main steps of the solution in point form
• Identifying the elements needed for the graphical and algebraic representation of the situation, depending on the register of representation chosen: characteristics of the function, graduation of the axes of the graph, dependent variable and independent variable</td></tr>
<tr><th colspan="2">ACTIVATION</th></tr>
<tr><td colspan="2">- When dealing with a situational problem, adult learners use reasoning to determine the relationships between the change in the parameters of the rule of a function and the transformation of the corresponding Cartesian coordinate graph.
- They can also identify the necessary elements such as the scale, properties and constraints related to the domain of the function to draw the graph of the function, bearing in mind the meaning of mathematical symbols, terms and notation.</td></tr>
<tr><td>Examples of strategies</td><td>• Proceeding by trial and error in assigning values to the variables so as to determine the mathematical constraints and properties of algebraic fractions
• Solving a problem step by step</td></tr>
<tr><th colspan="2">REFLECTION</th></tr>
<tr><td colspan="2">- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
- Using reasoning, they make conjectures about particular or special cases in order to validate certain results.
- They make sure that the dependent and independent variables are properly defined, that the axes are correctly scaled, that no unit of measure has been omitted and that the data have been correctly transcribed.</td></tr>
<tr><td>Examples of strategies</td><td>• Proceeding by trial and error to determine whether a function is increasing or decreasing by substituting different values into the rule of that function over a given interval
• Checking the solution of a system of relations by making sure that an ordered pair in the feasible region is the solution for both the first and second relations
• Calling into question the method they have used if negative radicals are obtained in determining the zeros when a real-number solution is expected</td></tr>
</table>

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the *Relationship between quantities* family of learning situations. Two of these are considered particularly relevant to this course: *Uses information and communications technologies* and *Uses information*.

Methodological Competency

When adult learners are faced with a problem involving any kind of complexity, they need a method of visualizing it to be able to find the appropriate model more quickly. For example, to understand the relationship between the level of radioactivity of uranium and the distance from the mine to their homes, adult learners can use the competency *Uses information and communications technologies* to create and manipulate graphs by modifying some of their parameters. The use of technology allows them to focus on analyzing the situation.

Intellectual Competency

Adult learners are often placed in a learning situation involving an experiment whose results have already been published. When searching for information related to the situation they wish to model, they discover a large amount of data that are contradictory or of unequal value. This is an ideal opportunity to apply the competency *Uses information*. In looking for appropriate information, they assess the reliability of their sources. In addition, they must organize this information, a process that calls for a great deal of intellectual rigour.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of arithmetic and algebra. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following three integrative processes:

- **using an algebraic or graphical model to represent a situation**
- **interpolating or extrapolating from an algebraic or graphical model**
- **using an algebraic or graphical model to generalize a set of situations**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover all three processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Manipulating algebraic expressions	
• Operations on algebraic expressions	Operations on algebraic expressions are limited to: • multiplication • division of polynomials by a binomial (with or without a remainder) • the simplification of rational expressions (rational fractions)
• Expanding, simplifying or substituting expressions using significant algebraic identities	The significant algebraic identities of the second degree are: • perfect square trinomial • difference of two squares
• Completing the square	Completing the square is used for factoring second-degree polynomial functions and switching from one way of writing them to another.
• Factoring trinomials using roots	Trinomials are factored using the roots of the polynomial, if any: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Mathematical Knowledge	Restrictions and Clarifications
Manipulating algebraic expressions (cont.)	
• Solving first-degree equations and inequalities in one or two variables and second-degree equations and inequalities in one variable	Equations and inequalities are solved: • algebraically • graphically
Relation and function	
• Experimenting with real functions as well as observing, interpreting, describing and representing them	The real functions studied in this course are: • second-degree polynomials – general form $f(x) = ax^2 + bx + c$ – factored form $f(x) = a(x - x_1)(x - x_2)$ – standard form $f(x) = a(x - h)^2 + k$ • step functions (greatest integer not greater than *x*) $f(x) = a[b(x - h)] + k$ Functions can be represented: • verbally • using a table of values • algebraically • graphically, with or without the use of technology
• Describing and interpreting the properties of real functions	The properties of real functions covered in this course are: • domain and codomain (range) • increasing and decreasing intervals • extrema • sign • x- and y-intercepts
• Interpreting the multiplicative and additive parameters	
• Switching from one form to another in writing second-degree polynomial functions	

Mathematical Knowledge	Restrictions and Clarifications
System	
• Representing a situation using straight lines or half-planes	The properties of the following lines are studied: • parallel lines • intersecting lines • coincident lines • perpendicular lines The equation of the line can be expressed in: • general form $Ax + By + C = 0$ • standard form $y = ax + b$ • symmetric form $\left(\frac{x}{a} + \frac{y}{b}\right) = 1$
• Solving systems of first-degree equations in two variables	Systems may be solved: ▪ using a table of values ▪ algebraically ▪ graphically
• Solving systems composed of a first-degree equation and a second-degree equation in two variables	Systems may be solved: ▪ using a table of values ▪ algebraically ▪ graphically

Cultural References

Mathematics is responsible for the existence and effectiveness of a growing number of objects, tools and techniques used on a daily basis. For example, recent developments in weather forecasting, digital image processing, data fusion related to aerial and space-based surveillance, the control of rail transport, the optimization of cellular telephone networks and hydroelectric management of a power station or region all involve mathematical modelling.

Mathematical modelling also makes it possible to understand why a hydroelectric dam shaped like a parabola can better withstand the enormous pressure exerted by the water in the upstream reservoir than a dam of another shape. Graphical representation can enable adult learners to gain insight into the importance of mathematics and modelling in building structures. They could also compare present-day structures with those built a thousand years ago and discover that these principles were already known back then.

Adult learners can explore numerous examples of phenomena modelled using algebraic equations and realize that algebraic concepts are used in many fields such as air traffic control, operational research, computer science, cryptography and economics, to name just a few.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Relationship between quantities* involve problems that can be solved in part by using an algebraic or graphical model to represent a relationship between quantities in a fundamental context. The *Algebraic and Graphical Modelling in a Fundamental Context 1* course provides adult learners with an opportunity to express a connection or a dependency relationship between quantities.

In the situational problems in this course, adult learners test their hypothesis by assigning increasingly large values to one variable to determine their effect on the value of the other variable, determine the relationships between the change in the parameters of the rule of a function and the transformation of the corresponding Cartesian coordinate graph, and distinguish between the mathematical and everyday meanings of the terms used.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Media Literacy, and Environmental Awareness and Consumer Rights and Responsibilities.

Media Literacy

Adult learners interested in car chases could conduct a study on the variation in parameters such as maximum speed and acceleration in order to derive algebraic equations from graphical models. They could then prepare a multimedia presentation to explain the conclusions they have drawn from their observations. By acting ethically, adult learners can help their audience distinguish between facts and misconceptions, thereby fostering an understanding of media representations of reality. This is one of the focuses of development of this broad area of learning.

Environmental Awareness and Consumer Rights and Responsibilities

Adult learners who are concerned about environmental problems and interested in systems for generating renewable energy could use certain functions to analyze the efficiency of a wind turbine or a solar panel in relation to their cost. The performance of photovoltaic cells varies with the surface area of the panel, the amount of incident sunlight and various other factors. Adult learners could estimate their electricity requirements and, using graphs or tables of values, explore the potential merit of investing in the technology or select the size of the solar panel or wind turbine that will meet their needs. This could make them aware of their electricity consumption and prompt them to consider alternatives for using resources wisely. In this way, they could make informed choices, which ties in with one of the focuses of development of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Media Literacy
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Relationship between quantities
Targeted cross-curricular competencies • Are developed at the same time and in the same context as the subject-specific competencies.	• Uses information and communications technologies • Uses information
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Relationship between quantities* family of learning situations
Car chases are very popular in movies. One car is travelling along a highway and, far behind it, another car sets off in pursuit. When the driver of the first car realizes he is being pursued, he accelerates in an effort to get away. How will the chase unfold?	**Integrative process:** *Using an algebraic or graphical model to generalize a set of situations* In carrying out the four phases in the problem-solving process, adult learners could: Representation • Describe the characteristics of the situation in their own words to have a clear idea of the problem • Put forward a hypothesis about the relationship between the position of the cars and time • Make an intuitive assumption that if the acceleration increases, the minimum distance between the two cars will decrease Planning • Refer to a similar situational problem already analyzed in class when modelling the relations on the basis of the data provided • Determine the order in which the different steps are carried out: for example, plot the given data on a graph before looking for the relationship between the data and then generalize the system of equations

<table>
<tr><th>Situational problem</th><th colspan="2">Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the Relationship between quantities family of learning situations</th></tr>
<tr><td rowspan="2">It is assumed that the two cars have the same maximum speed and that the car being pursued has a constant acceleration until it reaches its maximum speed. Adult learners are provided with enough data on the two cars to determine the type of relationship that exists between the position of the cars and their travel time.

Adult learners are asked to generalize this type of situation by establishing a system of equations and to determine what the initial minimum distance between the two cars would have to be to enable the first car to accelerate and get away. The speed of the car in pursuit is assumed to be constant.

Adult learners will have to give a multimedia presentation to show their findings.</td><td>Activation</td><td>• Sketch a Cartesian coordinate graph showing the position of the two cars as a function of time
• Determine the algebraic rule for each car
• Determine through extrapolation when the two vehicles will meet
• Determine what the initial distance would have to be in order for the first car to get away (using technology if possible)
• Vary the acceleration and modify the graph accordingly, and then determine the minimum initial distance between the two cars that would enable the first car to get away (as a function of the speed of the pursuer and the acceleration of the car being pursued)
• Repeat this final step with a view to generalizing the system of equations
• Use formal mathematical language to generalize the situation</td></tr>
<tr><td>Reflection</td><td>• Compare their solution and results with those of their classmates in order to identify the strengths and weaknesses of their model
• Examine whether or not they have made appropriate choices: could the second-degree function have been replaced by an exponential function and, if so, why? Is the initial assumption (the minimum distance decreases if acceleration increases) correct?
• Answer the following question: would the modification of a parameter other than the initial distance between the cars make it possible for the car being pursued to get away?</td></tr>
</table>

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Relationships between quantities*, adult learners represent a situation, interpolate or extrapolate, and generalize a set of situations using an algebraic or graphical model. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To represent a situational problem using an algebraic or graphical model, adult learners use real functions and their inverse, and select relevant information in order to derive a pattern or law that will take into account the best relationship between the constraints and consequences of the situation. They choose the algebraic model best suited to the situation, giving examples with numerical values as needed in order to make a decision concerning the type of relation that exists between the variables of the situation. In addition, they recognize and choose mathematical symbols, terms and notation with a view to representing a situation accurately. They produce accurate mathematical messages in accordance with the mathematical rules and conventions associated with the functions covered in the course. If they are required to solve systems of first- and second-degree equations, they validate their solutions or intuitions algebraically, sometimes using a graph. They are also able to justify all of the steps in their process using mathematical language.

Adult learners formulate questions related to the situation by interpolating or extrapolating from an algebraic or graphical model. These questions help them establish organized and functional relationships among different aspects of mathematical knowledge (e.g. the relationships between the parameters of a given function or how a change in a parameter affects a family of functions). Adult learners suggest probable or plausible ideas in order to deduce propositions related to the situation. They then validate their conjectures through interpolation or extrapolation, substituting numerical values in the algebraic rule they have modelled. They find the rule using the zeros of the function or the characteristics of the step function.

When modelling several situations using a real function, adult learners consider whether or not their properties can be generalized. To do this, they determine the important elements and the obstacles to be overcome and refer to the solution of one or more similar situational problems. They find invariants through trial and error, which enables them to make generalizations that lead to laws, rules or properties. They validate their solution using examples or counterexamples to test their deductive reasoning. In addition, solving systems of first-degree equations in two variables provides the tools they need to generalize results that lead to the properties of different types of lines, be they parallel, perpendicular, coincident or intersecting. Lastly, to perform operations on algebraic expressions, adult learners use factoring involving significant identities: perfect square trinomial or difference of two squares. They easily identify the specific characteristics of algebraic fractions, and can illustrate their conclusions using a graph.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (manipulation of numerical and algebraic expressions, function, inverse and system). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

  * The solution includes a procedure, strategies and a final answer.
  ** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*

Course

MTH-4272-2

Data Collection in a Fundamental Context

Mathematics

INTRODUCTION

The goal of the *Data Collection in a Fundamental Context* course is to enable adult learners to deal with situations that involve collecting or processing data pertaining to a one- or two-variable distribution in a fundamental context.

In this course, adult learners continue to develop their statistical thinking skills by applying prior learning as well as the concept of dispersion to the study of two-variable statistical distributions. In some situations, they learn about measurement and measurement errors by collecting data related to a two-variable distribution. Elsewhere, they compare the data collected in different situations. To determine the rule that best corresponds to the analysis of their results, they represent the data in a contingency table or a scatter plot, and use the concept of linear correlation to verify the strength of the relationship between two quantities. Analyzing the strength of the relationship makes it possible to describe and characterize the correlation in a qualitative fashion (perfect, strong, weak, zero, positive, negative). In situational problems, adult learners must validate and correct the solutions as needed, and organize and interpret statistical data in order to be able to represent it using linear correlation or second-degree polynomial functions. They become aware that handling and measurement errors affect the results of experiments, and that the resulting graphs are not always “perfect” curves. By analyzing various situations or conducting experiments, they learn that a mathematical model, such as a function, can be associated with a scatter plot. Situations involving the concept of correlation help adult learners develop a line of reasoning which, supported by an understanding of dependency relationships and a capacity for abstraction, lead them to recognize cause-and-effect relationships.

By the end of this course, adult learners will be able to collect and compare data pertaining to one- or two-variable distributions in order to answer a question related to a problem that they themselves have defined. They will present the results of their analysis in accordance with the rules and conventions of mathematics. They will use problem-solving strategies to determine the most efficient solution. In addition, by conducting an experiment using technological tools, they will be able to test the way that they use statistical analysis to deal with a situation.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

<table>
<tr><th colspan="2">PROCESS AND STRATEGIES</th></tr>
<tr><th colspan="2">REPRESENTATION</th></tr>
<tr><td colspan="2">- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. They use observational and representational strategies that are essential to inductive reasoning.
- In attempting to understand the context and the problem, they use deductive reasoning, particularly in situations that involve implicit data.
- When they study a case of correlation using a particular model, they determine the parameters needed to find the algebraic rule or to draw the graph.</td></tr>
<tr><td>Examples of strategies</td><td>• Describing the situation in their own words and comparing their understanding of the problem with that of their classmates or teacher
• Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to find a relationship between the variables when looking for a correlation using the model best suited to the problem
• Listing their statistical and probability-related strategies and knowledge pertaining to the situation
• Describing the characteristics of the situation
• Gathering relevant information</td></tr>
<tr><th colspan="2">PLANNING</th></tr>
<tr><td colspan="2">- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient.
- Reasoning enables them to establish organized and functional relationships among different aspects of their knowledge when they attempt to extrapolate results using an algebraic rule or a correlation graph.
- They find the elements needed to transpose data from one register of representation to another, for instance when switching from a statistical distribution to a stem-and-leaf plot, and vice versa.</td></tr>
<tr><td>Examples of strategies</td><td>• Systematically determining the correlation model best suited to the situation, bearing in mind the limitations regarding the model’s precision
• Finding an effective proof method to compare two correlation models</td></tr>
<tr><th colspan="2">ACTIVATION</th></tr>
<tr><td colspan="2">- When dealing with a situational problem, adult learners establish organized and functional relationships among different aspects of their knowledge, for example, when checking and describing the correlation between two variables.
- They use different strategies by associating pictures, objects or concepts with mathematical terms and symbols and by switching from one register of representation to another.</td></tr>
<tr><td>Examples of strategies</td><td>• Using a table to connect the properties of the correlation
• Using relevant data, drawing the functional model best suited to the situation
• Using technology to analyze the role of the different parameters of the rule of the correlation line or another model</td></tr>
<tr><th colspan="2">REFLECTION</th></tr>
<tr><td colspan="2">- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
- Reasoning can be used to reject extrapolations that would yield nonsensical results.
- Adult learners hone their ability to use exact mathematical language, especially when producing a message.</td></tr>
<tr><td>Examples of strategies</td><td>• Comparing their results with the expected results and those of others
• Checking their solution by making sure, for example, that the resulting values satisfy the range of the function in the case of a correlation
• Determining the strategies used to deal with the situation</td></tr>
</table>

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Processing data*. Two of these are considered particularly relevant to this course: *Adopts effective work methods* and *Exercises critical judgment*.

Methodological Competency

Presenting a statistical profile requires organized planning to avoid possible sources of bias. Adult learners must therefore conduct their study by observing the criteria for ensuring the validity of the data they collect. They must examine the data rigorously to ensure that their interpretation and analysis reflect the reality of the situation and are not influenced by prejudice or misconceptions. They must properly plan the tasks to be performed and carry them out in a logical order. They allot the time needed to complete these tasks. The competency *Adopts effective work methods* is very useful for this course, particularly for adult learners who wish to go on to study science or eventually go into research.

Intellectual Competency

The media discuss statistical studies and report on situations that must be analyzed with a certain degree of objectivity to be able to separate fact from opinion. Bearing in mind that a study is reliable only if it is objective, adult learners must determine the motivation behind the decision to promote one type of information over another. The competency *Exercises critical judgment* is very useful, for example, when interpreting the correlation between two variables and determining how they are actually related.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of statistics. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following three integrative processes:

- **collecting data**
- **comparing collections of data**
- **interpreting data resulting from an experiment**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover all three processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Two-variable distribution	
• Constructing and interpreting two-variable distributions	
• Drawing a scatter plot	
• Representing and determining the equation of the regression line	
• Interpolating or extrapolating using the regression line	
• Interpreting a correlation qualitatively and quantitatively	The characteristics of a correlation are: positive, negative or zero; perfect, strong, moderate or weak. Interpretation of the correlation is limited to cases involving linear correlations, which can be estimated using a graphical method (box method or ellipse). The exact value of the correlation coefficient is determined using technology.

Mathematical Knowledge	Restrictions and Clarifications
Two-variable distribution (cont.)	
• Interpolating and extrapolating using the functional model best suited to the situational problem	The functional models studied in this course are those covered in the *Algebraic and Graphical Modelling in a Fundamental Context 1* course.

Cultural References

From the first rudimentary livestock inventories recorded in the 23rd century B.C.E. in China and the 18th century B.C.E. in Egypt, statistics remained a simple data-collecting system until the 17th century C.E. It finally came into its own in the 19th century with the formulation of precise rules for collecting and interpreting data.

In the 20th century, industrial applications were developed first in the United States and then in Europe after the First World War. Computerization made it possible to cross-reference different types of data sequences. Methods became more sophisticated, and more and more studies were conducted. Today, statistics is a science in and of itself. Canada has become world-renowned for advances in this discipline and its applications in fields such as science, technology, business and public administration. Canada has played a leading role in this area thanks to the quality and commitment of its researchers. The decision to locate the headquarters of the UNESCO Institute for Statistics in Montréal was motivated by our country’s excellent reputation in this field. This institute is responsible for collecting and publishing statistics related to science, technology and education for the United Nations.

Today, the world of statistics is no longer the preserve of a small group of experts: it is accessible to everyone thanks in large part to the Internet. Adult learners should now be able to understand statistics that are of public interest such as all the data that Statistics Canada produces annually for Canadian citizens. Depending on their interests and on the needs of the course, adult learners can check whether or not there are causal links between two variables in a given area.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Processing data* involve problems that can be solved in part by collecting or processing data in a fundamental context. The *Data Collection in a Fundamental Context* course provides adult learners with an opportunity to learn how to collect and compare data.

In the situational problems in this course, adult learners decode the meaning of mathematical symbols, terms and notation, interpret codes and rules to distinguish between the probability and odds of winning an amount of money in a game of chance, and correctly interpret the intensity and sign of the correlation coefficient.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Health and Well-Being, and Career Planning and Entrepreneurship.

Health and Well-Being

Many situations that involve statistics can be explored while bearing in mind the educational aim of this broad area of learning. For example, adult learners can monitor their own progress in a structured fitness program aimed at achieving optimal health. This exercise could motivate them to make sound decisions with respect to their health and to adopt healthy lifestyle habits.

Career Planning and Entrepreneurship

Adult learners interested in scientific fields could conduct an experiment that would require them to interpret their results using the statistical methods and concepts learned in this course. The experimental and scientific approach involves compiling, analyzing and interpreting results. Drawing a scatter plot to represent data makes it possible to identify extreme or aberrant results and to find possible errors made in processing information. Exploring a situation related to their interests and aptitudes in order to master the strategies involved in carrying out a scientific experiment ties in with one of the focuses of development of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Health and Well-Being
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Processing data
Targeted cross-curricular competency • Is developed at the same time and in the same context as the subject-specific competencies.	• Exercises critical judgment
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the ***Processing data*** family of learning situations
An adult learner realizes he has put on weight over the last few years because of his sedentary lifestyle. He decides to sign up at a gym in order to follow a structured training program. Before starting the program, he must undergo a physical fitness test to determine his endurance, muscle strength, flexibility, weight and height.	**Integrative process:** *Comparing collections of data* In carrying out the four phases in the problem-solving process, adult learners could: Representation • Decide which elements are relevant to their analysis • Make conjectures (e.g. Should there be a correlation between endurance and length of training, or between muscle strength and the type of exercise chosen?) Planning • Determine the information to be gathered (e.g. endurance, muscle strength, weight) • Periodically gather information about different exercises in order to conduct further analysis • Choose the most appropriate register of representation (e.g. the scatter plot)

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Processing data* family of learning situations
After defining his goals (e.g. to build muscle mass, increase endurance or lose weight), he decides to determine which of the suggested exercises will be the most effective in helping him achieve his objectives.	Activation • For a given exercise, check and describe the correlation between, for example, the following variables: - VO_2 max (maximum oxygen consumption) or number of heartbeats and the number of days spent training (variables related to endurance) - the maximum load and the number of days spent training (variables related to muscle strength) - the measurement of flexibility and the number of days spent training (variables related to flexibility) - weight and the number of days spent training (variables related to weight loss) • Use another register of representation for the data to better illustrate their conclusions Reflection • Establish organized and functional relationships between such concepts as the intensity and sign of the correlation coefficient and determine the correlation line • Compare their results with those of their classmates to determine other factors that could have been taken into account • Make sure that their solution makes sense

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Processing data*, adult learners collect, compare and interpret data resulting from a random experiment. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems, Uses mathematical reasoning* and *Communicates by using mathematical language*.

To collect data, adult learners use problem-solving strategies to define the problem and identify the tasks involved. They determine the important elements of information and the obstacles to be overcome in order to study two-variable statistical distributions. In working out their solution, they establish and carry out a plan involving the previously validated steps: data collection and processing (interpretation and analysis). This last step requires that they use mathematical reasoning, explore the problem in question and identify patterns. Adult learners make conjectures using a correlation line or curves in order to make decisions in the medium and long term. They draw conclusions based on laws or rules related to the properties of the functions in question. Lastly, they produce mathematical messages, using the appropriate register of representation given the constraints of the situational problem. In other words, they choose the functional model best suited to the situation.

To compare collections of data, adult learners interpret mathematical messages by making connections between the elements of the message, determining its overall meaning, or associating pictures, objects or knowledge with mathematical terms and symbols. In addition, they use mathematical reasoning by developing and using networks of cognitive resources in order to compare trends, for example the rate of change, the rate of growth, the correlation coefficient, or any other characteristic of the functions covered in this course.

To interpret data resulting from an experiment in order to study the correlation between two quantities, adult learners decode the elements of mathematical language, distinguishing between the mathematical and everyday meanings of terms. In addition, they interpret mathematical messages, distinguishing between elements that are relevant and those that are not, and recognizing the purpose of the message. They use mathematical reasoning by developing and using networks of cognitive resources, such as a correlation line or a functional model adapted to the situation in the case of a two-variable statistical distribution. They generalize, derive laws and rules, and deduce propositions to help them make informed decisions.

Throughout the problem-solving process, adult learners apply their knowledge of functions to their study of two-variable statistical distributions. Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

  * The solution includes a procedure, strategies and a final answer.
  ** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*

Course

MTH-4273-2

Geometric Representation in a Fundamental Context 1

Mathematics

INTRODUCTION

The goal of the *Geometric Representation in a Fundamental Context 1* course is to enable adult learners to use trigonometry to deal with situations that involve the geometric representation of an object or a physical space in a fundamental context.

In this course, adult learners encounter various situational problems that enable them to expand their knowledge of geometry, and trigonometry in particular. In solving situational problems that involve trigonometric concepts, adult learners use inductive reasoning to derive properties of triangles and deductive reasoning to find measurements. They derive different metric relations in right triangles by using proportional and geometric reasoning, as well as concepts related to similar triangles. The geometry principles studied should ideally emerge as conclusions related to the exploratory activities carried out by adult learners. These principles help adult learners justify the steps in their work when they solve a situational problem. Thus, adult learners use the different relations associated with geometric figures, as well as proportional and geometric reasoning or trigonometry, to find unknown measurements based on congruent, similar or equivalent figures. These lines of reasoning also enable adult learners to deduce unknown measurements in geometric figures, which may or may not result from similarity transformations, in order to validate or refute a conjecture. Adult learners use definitions, properties, relations and theorems to prove other conjectures. At times, they identify the structure of someone else’s line of reasoning, then analyze it, evaluate it and reformulate it in their own words. Lastly, they describe the relationship between the different measurements of a figure in terms of metric or trigonometric relations.

By the end of this course, adult learners will be able to use various metric or trigonometric relations to represent and describe an object or a physical space in accordance with the mathematical notation and conventions used in geometry. They will also be able to use different strategies and types of reasoning to organize a physical space in accordance with certain constraints.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the notation and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

<table>
<tr><th colspan="2">PROCESS AND STRATEGIES</th></tr>
<tr><th colspan="2">REPRESENTATION</th></tr>
<tr><td colspan="2">- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. They use observational and representational strategies that are essential to inductive reasoning.
- They organize the elements that make it possible to plan the main steps in a line of deductive reasoning related to the concept of similarity.
- They distinguish between the mathematical and everyday meanings of the terms used to show that they understand the concepts: angle of depression, angle of inclination, adjacent side and so on.</td></tr>
<tr><td>Examples of strategies</td><td>• Describing the situational problem in their own words to show that they have understood it
• Representing the situational problem mentally or in writing
• Listing their geometry-related strategies and the metric relations pertaining to the situation
• Describing the characteristics of the situation
• Determining questions about the situation</td></tr>
<tr><th colspan="2">PLANNING</th></tr>
<tr><td colspan="2">- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient.
- Mathematical reasoning allows them to use different registers of representation to illustrate certain properties of trigonometric ratios.
- By making connections between the elements of the message and by giving a literal description of the ratios of the corresponding sides of two plane figures, adult learners are able to construct a figure based on their description.</td></tr>
<tr><td>Examples of strategies</td><td>• Dividing the situational problem into subproblems
• Using lists, tables, diagrams, concrete materials or drawings to work out their solution</td></tr>
<tr><th colspan="2">ACTIVATION</th></tr>
<tr><td colspan="2">- In developing a line of mathematical reasoning to prove geometry principles pertaining to right triangles, adult learners give several examples before drawing conclusions.
- In producing the plan of an architectural structure, they take into account the proportions indicated by the scale and use the related symbols and conventions.</td></tr>
<tr><td>Examples of strategies</td><td>• Simplifying the situational problem by comparing it with a similar problem that has already been solved and using it as a point of departure for solving a more complex problem
• Using the parameters of a function to make a sketch in order to predict results
• Comparing the parameters of a right triangle with those of any given triangle in order to make connections or formulate laws such as the cosine law</td></tr>
<tr><th colspan="2">REFLECTION</th></tr>
<tr><td colspan="2">- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
- Reasoning helps them make conjectures about particular or special cases involving triangles in order to validate certain results. Reasoning also enables them to reject extrapolations that would yield nonsensical results.
- They use different sources of information to validate their mathematical messages.</td></tr>
<tr><td>Examples of strategies</td><td>• Checking their solution by means of examples or counterexamples
• Determining the strategies for dealing with situational problems in geometry (e.g. applying a rule, referring to a theorem)
• Using a calculator or geometric modelling software to validate their work</td></tr>
</table>

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Measurement and spatial representation*. Two of these are considered particularly relevant to this course: *Uses creativity* and *Uses information and communications technologies*.

Intellectual Competency

In this course, adult learners have many opportunities to use the competency *Uses creativity*. When dealing with a learning situation that involves analyzing and meeting certain technical challenges in order to build a structure, adult learners look for original solutions to the problems at hand. They could also be required to prove the Pythagorean theorem. Is there still room for innovation given that this theorem has been proved in countless ways, each one different from the others? Often, creativity has less to do with adding new resources than with using the available resources in a new way. Adult learners are encouraged to allow themselves to be guided by both their intuition and their logic.

Methodological Competency

The competency *Uses information and communications technologies* could help adult learners deal with situations that involve representing objects or physical spaces. Geometry software makes it easier to manipulate figures and makes it possible to create isometries or dilatations, modify the angles and validate trigonometric relations through proofs. With time, adult learners will want to use these technologies for a variety of tasks.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of geometry. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following two integrative processes:

- **organizing a physical space**
- **describing an object or a physical space and representing it in two or three dimensions**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover both processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Metric and trigonometric relations in triangles	
• Representing and interpreting situations using triangles	*The trigonometric ratios studied are sine, cosine and tangent.* *The sine law and the cosine law are also studied in this course.* *Hero's formula is optional in the Science option.* *The other metric and trigonometric relations are listed in the Principles table that comes after this table.*
• Justifying their solution using the properties of trigonometric ratios	Adult learners use the properties of trigonometric ratios in a formal manner to justify the steps in their solution.

<table>
<tr><th>Mathematical Knowledge</th><th>Restrictions and Clarifications</th></tr>
<tr><td>Metric and trigonometric relations in triangles (cont.)</td><td></td></tr>
<tr><td>Determining the slope, measurements and positions using metric and trigonometric relations in triangles</td><td>In this course, the concept of distance and the properties of congruent, similar or equivalent figures are used to find measurements and positions pertaining to:
the angles in triangles or in figures that can be split into triangles
the altitude to the hypotenuse, the orthogonal projection of the legs on the hypotenuse
the sides of a triangle
the areas and volumes of figures
the length of a segment resulting from an isometry or a similarity transformation
the distance between two points</td></tr>
<tr><td>Similar and congruent triangles</td><td></td></tr>
<tr><td>Determining the minimum conditions required to conclude that triangles are congruent or similar</td><td>Adult learners use the properties of congruent or similar figures in a formal manner to justify the steps in their solution. They may be required to prove these properties.

These conditions are listed in the Principles table that comes after this table.</td></tr>
<tr><td>Equivalent figures (plane figures or solids)</td><td></td></tr>
<tr><td>Finding measurements:
lengths of segments
areas
volumes
capacities</td><td>These unknown measurements are found by applying the properties of congruent, similar or equivalent figures.

The other relations associated with equivalent figures are listed in the Principles table that comes after this table.</td></tr>
</table>

Principles

Adult learners must master the following compulsory principles, which may be used in a proof:

P1. If the corresponding sides of two triangles are congruent, then the triangles are congruent.

P2. If two sides and the contained angle of one triangle are congruent to the corresponding two sides and contained angle of another triangle, then the triangles are congruent.

P3. If two angles and the contained side of one triangle are congruent to the corresponding two angles and contained side of another triangle, then the triangles are congruent.

P4. Plane figures are congruent if and only if all of their corresponding sides and angles are congruent.

P5. If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the triangles are similar.

P6. If the lengths of the corresponding sides of two triangles are in proportion, then the triangles are similar.

P7. If the lengths of two sides of one triangle are proportional to the lengths of the two corresponding sides of another triangle and the contained angles are congruent, then the triangles are similar.

P8. Transversals intersected by parallel lines are divided into segments of proportional lengths.

P9. The midpoint of the hypotenuse of a right triangle is equidistant from the three vertices.

P10. The lengths of the sides of any triangle are proportional to the sines of the angles opposite these sides.

P11. The segment joining the midpoints of two sides of a triangle is parallel to the third side and its length is one-half the length of the third side.

P12. The length of a leg of a right triangle is the geometric mean between the length of its projection on the hypotenuse and the length of the hypotenuse.

P13. The length of the altitude to the hypotenuse of a right triangle is the geometric mean between the lengths of the segments of the hypotenuse.

P14. The product of the lengths of the legs of a right triangle is equal to the product of the length of the hypotenuse and the length of the altitude to the hypotenuse.

P15. The square of the length of a side of any triangle is equal to the sum of the squares of the lengths of the other two sides, minus twice the product of the lengths of the other two sides multiplied by the cosine of the contained angle.

P16. The segment joining the midpoints of the nonparallel sides of a trapezoid is parallel to the bases and its length is one-half the sum of the lengths of the bases.

P17. Regular polygons have the smallest perimeter of all equivalent polygons with n sides.

P18. Of two equivalent regular convex polygons, the polygon with the most sides will have the smaller perimeter. (Ultimately, an equivalent circle will have the smaller perimeter.)

P19. Cubes have the largest volume of all rectangular prisms with the same total surface area.

P20. Spheres have the largest volume of all solids with the same total surface area.

P21. Cubes have the smallest total surface area of all rectangular prisms with the same volume.

Cultural References

Geometry has a rich history. The Ancient Greek thinkers were geometers first and foremost. They worked on abstract objects and organized geometry deductively. Adults learning how to deal with abstraction and how to apply the principles of deduction may be interested in learning about the major contributions of these mathematicians and how their ideas evolved. Whether it be Thales of Miletus, Euclid or Archimedes, numerous thinkers built on the knowledge of their time by making connections with other disciplines such as mechanics and astronomy. By the end of the 16th century, after a long period during which it was almost exclusively associated with astronomy, trigonometry finally started to be used in other fields such as surveying.

Today, trigonometry and geometry are no longer called into question. The study of symmetry and shapes can also be applied in chemistry in order to understand the structure of molecules and crystals. Architects also use geometric concepts in developing plans.

The various aspects of geometric representation can be examined in many different contexts. Depending on their interests, adult learners could study the works of artists like Escher or Reutersvärd, global positioning systems (GPS), or the principles of mechanics or even astronomy in order to discover the usefulness of geometry in interpreting reality.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Measurement and spatial representation* involve problems that can be solved in part through the geometric description or representation of an object or a physical space. The *Geometric Representation in a Fundamental Context 1* course provides adult learners with an opportunity to develop their spatial representation skills.

In the situational problems in this course, adult learners make a table of values or draw a graph in the Cartesian plane, solve certain situational problems by working backwards when the solution consists of several steps or when there is insufficient information, and produce a scale plan of an architectural structure, taking into account the proportions indicated by the scale and using the related symbols and conventions.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Environmental Awareness and Consumer Rights and Responsibilities, and Career Planning and Entrepreneurship.

Environmental Awareness and Consumer Rights and Responsibilities

The concepts of trigonometry studied in this course could be used to compare the operation of two telecommunications networks: one land-based and the other satellite-based. A comparative analysis will help adult learners recognize the impact of technological advances on economic development, in contrast to an ecological project more focused on the development of a sustainable environment. This could make adult learners aware of the issues involved and encourage them to develop an active relationship with their environment. This type of study is closely related to the educational aim of this broad area of learning.

Career Planning and Entrepreneurship

Adult learners who are interested in, or simply curious about, architecture could learn about architectural concepts by studying some of the more remarkable urban structures produced by human engineering. For example, they could draw the Millau Viaduct. The different mathematical calculations that go into drawing the structure could help them better understand architectural and civil engineering work, which ties in with the focuses of development of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Environmental Awareness and Consumer Rights and Responsibilities
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Measurement and spatial representation
Targeted cross-curricular competency • Is developed at the same time and in the same context as the subject-specific competencies.	• Uses information and communications technologies
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Measurement and spatial representation* family of learning situations
Deducing distances by means of triangulation is practical for determining lengths that are difficult to measure. In land surveying, for example, triangulation is useful when physical obstacles such as a body of water or a wooded area make it impossible to measure certain distances. Adult learners are required to prove the validity of the principle of triangulation in the plane. Working with a satellite photo, they must first determine the actual distance between two points separated by a body of water. They must then show that triangulation would make it possible to deduce this distance using one of these points and a third point, both of which are	**Integrative process:** *Describing an object or a physical space and representing it in two or three dimensions* In carrying out the four phases in the problem-solving process, adult learners could: Representation • Become familiar with elements of a satellite photo • Sketch a triangle joining the three points mentioned • Determine the task to be carried out: show that triangulation produces the same result as that obtained when the distance between two points in a plane is measured directly Planning • Determine the steps involved in working out the proof: draw the triangle joining the three points in the situation, make a connection between the situation and their mathematical knowledge, use appropriate mathematical language to formulate the proof and make sure the solution is plausible • State the mathematical concepts needed for the proof: concept of distance, properties of congruent figures, principles of trigonometry, sine and cosine laws

<table>
<tr><th>Situational problem</th><th colspan="2">Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the Measurement and spatial representation family of learning situations</th></tr>
<tr><td>located on the same side of the body of water.</td><td>Activation</td><td>Draw the triangle joining points A, B and C on the satellite photo
Determine the measures of angles and the distances between these three points
Determine the actual distances between the three points, using the scale provided
Determine whether the triangle is a scalene triangle or a right triangle
Use the given information to determine the applicable law: sine or cosine law
Use the sine law to calculate the distance between points A and C located on either side of the body of water
Show that this value is very close to that found by means of the satellite photo, using an appropriate type of representation and correct mathematical language</td></tr>
<tr><td></td><td>Reflection</td><td>Make conjectures explaining why there may be a slight difference between the result obtained using the sine law and the measurement in the plane: inherent limitations with regard to the precision of the angular and distance measurements on the satellite photo
Determine in which other situation this method could be used: navigation, GPS, astronomy
Determine when to apply the sine law rather than the cosine law</td></tr>
</table>

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Measurement and spatial representation*, adult learners describe an object or a physical space and represent it in two or three dimensions, and organize a physical space. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems, Uses mathematical reasoning* and *Communicates by using mathematical language*.

To describe and represent an object or a physical space, adult learners interpret and produce sketches, drawings or plans by using complex figures that can be broken down into right triangles or other types of triangles. They identify the key elements of mathematical language (e.g. dimensions, perimeter, area) and associate pictures, objects or knowledge with mathematical terms and symbols. In addition, they apply newly acquired mathematical knowledge such as the sine or cosine law, which enables them to determine unknown measurements in unusual situations.

To organize a physical space, adult learners use a variety of strategies: making a sketch or drawing, dividing the task into subtasks, etc. They apply their knowledge of trigonometry to carry out a complex process that includes everything from representing the problem to validating the solution. They use the concept of triangulation to organize a physical space and validate every step against the theorems covered in the course. They deduce unknown measurements, determine results through inductive reasoning and draw conclusions based on their study of the theorems. When these conclusions involve the properties of certain figures, they prove their accuracy by developing a formal proof.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (trigonometric and metric relations in triangles, similar and congruent triangles, and equivalent figures). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

* The solution includes a procedure, strategies and a final answer.
** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*

6.4.2 Secondary V Courses

Course

MTH-5170-2

Optimization in a Fundamental Context

Mathematics

INTRODUCTION

The goal of the *Optimization in a Fundamental Context* course is to enable adult learners to use linear programming to deal with situations that involve optimization in a fundamental context.

In this course, adult learners find an optimal solution to a situation involving specific constraints. They take these constraints into account, representing them using a system of inequalities in two variables, and define the function to be optimized. They graph the situation, which enables them to study the polygon of constraints or identify the feasible region in order to solve the system graphically or algebraically.

In situational problems, adult learners optimize a linear relation that could be economic or objective. They are required to take the constraints into account, and their analysis of the situation enables them to determine the best solution. They learn to illustrate their reasoning and explain their solution. They show how they interpreted the feasible region and the vertices of the polygon. It is therefore important that they make sure that their result is plausible in the given context and that they specify the degree of precision they took into account. If they see that their result is improbable, they suggest changes, a new solution or ways of making it more efficient.

By the end of this course, adult learners will be able to represent optimization situations. They will produce clear and accurate work in accordance with the rules and conventions of mathematics. Their analysis will address limiting solutions as well as solutions that are integers when the situation refers to a discrete case or when one of the limits is a point on the grid.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

<table>
<tr><th colspan="2">PROCESS AND STRATEGIES</th></tr>
<tr><th colspan="2">REPRESENTATION</th></tr>
<tr><td colspan="2">- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. They use observational and representational strategies that are essential to inductive reasoning.
- In attempting to understand the context and the problem, they also use deductive reasoning.
- They have suitably mastered the elements of mathematical language.</td></tr>
<tr><td>Examples of strategies</td><td>• Using a table, determining the nature of the task involved
• Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to determine the economic or technological constraints involved in mathematizing the problem
• Determining questions about the situation
• Gathering relevant information (maximum, minimum, economic function)</td></tr>
<tr><th colspan="2">PLANNING</th></tr>
<tr><td colspan="2">- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient.
- Planning the solution correctly involves decoding the elements of mathematical language such as the meaning of symbols, terms and notation.</td></tr>
<tr><td>Examples of strategies</td><td>• Finding an algebraic rule that takes into account the best relationship between the constraints and the consequences associated with the situational problem: determining the relevant parameters of the scanning line or the economic function
• Using their intuition to sketch the boundaries (rectangle parallel to the axes of the Cartesian plane) of the solution set</td></tr>
<tr><th colspan="2">ACTIVATION</th></tr>
<tr><td colspan="2">- When dealing with a situational problem, adult learners use mathematical reasoning to graph the half-planes resulting from the constraints.
- They deduce the scale of the axes by analyzing the maximum and minimum values of the variables.
- They use rigorous mathematical language and, to avoid confusion, they use symbols, terms and notation in accordance with their meaning.</td></tr>
<tr><td>Examples of strategies</td><td>• Proceeding by trial and error to mathematize certain constraints
• Constructing tables of values in order to find two points to represent the boundary lines of the polygon of constraints
• Proceeding step by step in solving the inequalities</td></tr>
<tr><th colspan="2">REFLECTION</th></tr>
<tr><td colspan="2">- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
- They go back and forth between the graph and the economic function when the solutions are integers.
- They express their ideas in accordance with mathematical codes and conventions and take the constraints of the situation into account in their message.</td></tr>
<tr><td>Examples of strategies</td><td>• Checking their solution by, for example, comparing the number of possible solutions for a system of equations with the number of solutions found, or using their intuition to make sure that the coordinates of the points they have found are those of the vertices of the polygon of constraints
• Using graphing software to validate their work</td></tr>
</table>

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Optimizing solutions*. Two of these are considered particularly relevant to this course: *Solves problems* and *Uses information*.

Intellectual Competency

Although the cross-curricular competency *Solves problems* is closely related to mathematics, it can be developed in a broader context through a learning situation in which mathematics can be an invaluable resource. Through trial and error and by reformulating a problem, one can often work out a solution that is satisfactory but is not the only possible one. In order to draw on and develop this competency, adult learners may be asked to devise their own learning situations based on their own areas of interest. Linear programming often provides the flexibility needed to model and represent the observable world through linearization.

Intellectual Competency

A learning situation related to humanitarian aid could enable adult learners to draw on and develop the cross-curricular competency *Uses information*. They must not only know how to find this information and assess its value, but also learn to organize it. Dealing with issues such as sending the military into Afghanistan or East Timor would provide adult learners with an interesting opportunity to organize information found on the Web in order to work out and optimize a plan for deploying military personnel.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of arithmetic and algebra. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following integrative process:

- **optimizing a situation using linear programming**

This process, which is applied in the learning situations in this course, fosters the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve this integrative process. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Linear programming	
• System of first-degree inequalities in two variables	
• Representing the constraints and the function to be optimized (objective or economic function)	Constraints can be represented algebraically or graphically. In this course, the function to be optimized is expressed solely as an equation of the form $Ax + By + C = Z$, where A, B and C are rational numbers.
• Determining and interpreting the vertices and the feasible region (bounded or unbounded)	
• Changing the conditions associated with the situation to provide a more optimal solution	

Cultural References

Linear programming results from the work of the mathematicians Joseph Fourrier (1768-1830) and Georges Dantzig (1914-2005). While in the United States Air Force during the Second World War, Dantzig developed a technique for solving the army's logistical problems at a minimum cost. This method, which combines power and flexibility, was adopted in other fields to solve a variety of economic problems.

In the field of health care, certain decisions can be controversial because they are often made with financial interests in mind. Linear programming, which is a branch of optimization, is very useful in guiding decision making in this field and in solving optimization problems in numerous other fields. Consider the conflicts of interest that threaten the environment. For example, in order to feed a population or provide it with energy, to what extent can we exploit a given area without destroying it? Adult learners could study how certain scientists have been able to answer this question and use linear programming to find an equilibrium point.

By making it possible to perform calculations and process data, computers have made it considerably easier to find optimal solutions. This is one of the reasons that optimization is now used in numerous fields of activity. Once again, adult learners cannot help but notice the key role that mathematics plays in the search for optimal solutions.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Optimizing solutions* involve problems that can be solved in part through optimization using linear programming. *The Optimization in a Fundamental Context* course gives adult learners an opportunity to learn how to maximize a profit, a process or a number of objects or people, and to minimize costs or losses.

In the situational problems in this course, adult learners recognize and decode the meaning of symbols, terms and notation, distinguish between the mathematical and everyday meanings of the terms used, and deduce the optimal solution by substituting the coordinates of the vertices of the polygon of constraints into the equation of the economic function.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Health and Well-Being, and Environmental Awareness and Consumer Rights and Responsibilities.

Health and Well-Being

Some of the proposed learning situations make adult learners aware of health issues. The course could involve learning situations in which health-related concerns are set aside in favour of financial interests. When dealing with learning situations that illustrate the delicate balance between health concerns and profit considerations, and when reviewing their work in this regard, adult learners may become aware of the types of decisions certain companies make. The course focuses on problems whose solution must take into account health-related needs as well as cost-cutting needs. In this way, adult learners are required to take responsibility for adopting good living habits, which is the educational aim of this broad area of learning.

Environmental Awareness and Consumer Rights and Responsibilities

Some learning situations on oil refining techniques may motivate adult learners to develop an active relationship with their environment, while maintaining a critical attitude toward consumption and the exploitation of the environment. Calculating the optimum preheating temperature of crude oil and feedstocks and determining the best “vapour-electricity” balance of a refinery are two examples of optimization that involve environmental issues. A presentation on these issues could encourage adult learners to gather information that they can use to make future decisions.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Environmental Awareness and Consumer Rights and Responsibilities
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Optimizing solutions
Targeted cross-curricular competencies • Are developed at the same time and in the same context as the subject-specific competencies.	• Solves problems • Uses information
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

<table>
<tr><th>Situational problem</th><th colspan="2">Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the Optimizing solutions family of learning situations</th></tr>
<tr><td rowspan="2">The trucking industry is a vital part of the Canadian economy. Even though it is thriving, this industry faces many challenges such as competition and rising fuel prices. As a result, truck fleets must be carefully managed in order to allow trucking companies to maximize their use before increasing the number of vehicles.

Adult learners are asked to optimize the use of a truck fleet, taking into account certain constraints such as the maximum number of consecutive hours of work permitted by law, the minimum operating cost per kilometre, etc. They are required to identify the limiting solutions.</td><td colspan="2">Integrative process: Optimizing a situation using linear programming

In carrying out the four phases in the problem-solving process, adult learners could:</td></tr>
<tr><td>Representation</td><td>• Write literal expressions to represent the elements of the situation that seem relevant
• Determine the variables involved such as the number of drivers and the number of kilometres travelled
• Break down the complex situational problem into subproblems to identify the relationships between the constraints of the situation and the problem: the company’s operating costs, its expected revenue, maximization of the company’s profitability, analysis of the system of inequalities as a function of the scanning line</td></tr>
</table>

<table>
<tr><th>Situational problem</th><th colspan="2">Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the Optimizing solutions family of learning situations</th></tr>
<tr><td rowspan="3">In addition, the scanning line is parallel to one of the sides of the polygon of constraints, and adult learners are asked to show that this situation may involve several optimal values.</td><td>Planning</td><td>• Use brainstorming to look for possible solutions if the work is carried out in teams
• Refer to the solution of a similar situational problem to carry out their plan
• List the mathematical knowledge needed to deal with the situation: choosing the variables, determining the constraints, establishing a system of first-degree inequalities in two variables, graphing the feasible region, comparing the slopes of the sides of the polygon of constraints and the slope of the scanning line in order to determine the parallel lines, etc.</td></tr>
<tr><td>Activation</td><td>• Select the variables: number of drivers, number of kilometres travelled
• Mathematize the constraints of the situation through trial and error
• Construct tables of values to draw the boundary lines of the polygon of constraints
• Determine the vertex of the polygon of constraints that optimizes the profitability of the truck fleet
• Calculate the minimum operating cost
Determine the side parallel to the scanning line (economic function) and then show that there may be several solutions</td></tr>
<tr><td>Reflection</td><td>• Make sure that the solution makes sense by using any point to check that the feasible region has been correctly determined
• Compare their solution and results with those of their classmates to identify the strengths and weaknesses of the proposed model
• Determine if there is an easier way of finding the vertices of the polygon of constraints (e.g. by comparison rather than by substitution)
• Examine the role of the parameters of the scanning line</td></tr>
</table>

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Optimizing solutions*, adult learners optimize a situation using linear programming. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To use linear programming to solve situational problems, adult learners apply different mathematical models and different types of strategies, combining reasoning and creativity to overcome obstacles. They decode relevant information in order to find an optimal solution. They translate the different constraints using a system of inequalities in two variables, and give an algebraic definition of the function to be optimized. They graph the polygon of constraints and the feasible region. They use algebra to determine the coordinates of the vertices, or approximate an answer using a graph. To prove a conjecture, they use structured deductive reasoning and correctly use the codified form required for their proof. They illustrate, explain or justify their arguments. To develop a proof (e.g. *reductio ad absurdum*, proof by contrapositive or by induction), they use different types of reasoning, including proof by exhaustion. They observe real-life situations and generalize their observations. Lastly, in some situations, they analyze data in order to identify the necessary and sufficient conditions for drawing a conclusion, make decisions and determine how best to approach, optimize or adjust a situation.

When conducting case studies, synthesizing information, constructing proofs or making presentations in order to deal with situational problems related to linear programming, adult learners must accurately identify the purpose of the mathematical messages to be conveyed or interpreted. They select the medium, the type of discourse and the register of representation best suited to the audience and the purpose of the message. They switch easily from one register to another. They use a wide range of communication strategies that enable them to regulate the transmission of a message based on the specific reactions of the audience or to take new requirements into account. They adopt language that appropriately combines everyday, mathematical, scientific and technical terms.

Throughout the problem-solving process, adult learners apply their knowledge of linear programming. Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps*** *in the solution*

* The solution includes a procedure, strategies and a final answer.
** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*

Course

MTH-5171-2

Algebraic and Graphical Modelling in a Fundamental Context 2

Mathematics

INTRODUCTION

The goal of the *Algebraic and Graphical Modelling in a Fundamental Context 2* course is to enable adult learners to deal with situations that involve using an algebraic or graphical model to represent a dependency relationship between quantities in a fundamental context.

In this course, adult learners encounter several situations involving periodic functions. The standard unit circle provides a context for analyzing the specific case of trigonometric functions. This circle enables adult learners to visualize the periodic nature of trigonometric functions and trigonometric lines, to derive properties and to prove certain identities. For example, the use of piecewise functions makes it possible to analyze a variety of situations, such as remuneration during and after regular working hours. A rule is determined and defined for each interval of the domain. The concept of continuity then comes into play and can be used to interpret the variation in the rate of change.

Operations on functions are studied in practical contexts that involve, for example, calculating the total income tax payable (addition) or sales tax (composition). The study of these operations must not be an end in itself, but should be part of the process of analyzing situations and developing corresponding models. The concepts of infinity and continuity allow adult learners to understand the asymptotes of functions, and vice versa. The definition of the concept of limit is introduced intuitively (without referring to symbols) in order to clarify certain situations. The study of rational, tangent, exponential or logarithmic functions can also give rise to a discussion of these concepts.

By the end of this course, adult learners will be able to use algebra to represent concrete situations. They will produce clear and accurate work in accordance with the rules and conventions of mathematics. By algebraically or graphically representing a situation using real functions and their inverse, they will be able to employ inductive or deductive reasoning to obtain results through interpolation or extrapolation. In addition, adult learners will use different registers of representation to generalize results and extend them to other situations.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how the three subject-specific competencies are used to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

<table>
<tr><th colspan="2">PROCESS AND STRATEGIES</th></tr>
<tr><th colspan="2">REPRESENTATION</th></tr>
<tr><td colspan="2">- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. They use observational and representational strategies that are essential to inductive reasoning.
- By becoming more familiar with the symbols and notation used in the course, they develop their ability to communicate using mathematical language.</td></tr>
<tr><td>Examples of strategies</td><td>• Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to identify a dependency relationship and determine the variables in the situation
• Using examples involving numbers, determining the type of relationship that exists between the variables of the situation
• Sketching a Cartesian coordinate graph to represent the dependency relationship between the variables
• Gathering relevant information in order to describe their understanding of the dependency relationship between the variables</td></tr>
<tr><th colspan="2">PLANNING</th></tr>
<tr><td colspan="2">- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient.
- They attempt to extrapolate results using an algebraic rule or graph, thus expanding their networks of cognitive resources.
- To correctly plan their solution, they decode the elements of mathematical language such as the meaning of the symbols, terms and notation used, as well as the different registers of representation.</td></tr>
<tr><td>Examples of strategies</td><td>• Drawing a concept map showing the different steps in the solution
• Listing the elements needed to represent a function graphically or algebraically
• Systematically asking questions to consolidate their work plan, for example, What should the scale of the axes be?</td></tr>
<tr><th colspan="2">ACTIVATION</th></tr>
<tr><td colspan="2">- When dealing with a situational problem, adult learners make connections between the algebraic and graphical form in order to derive and generalize the rules.
- They use the appropriate scale so that the graph they draw in solving the situational problem makes sense in light of the context.</td></tr>
<tr><td>Examples of strategies</td><td>• Changing perspective
• Systematically determining the general form of the algebraic rule of a function
• Finding combinations in order to determine the rule of a quadratic function</td></tr>
<tr><th colspan="2">REFLECTION</th></tr>
<tr><td colspan="2">- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
- Reasoning helps them make conjectures about particular or special cases in order to validate certain results, and enables them to reject extrapolations that would yield nonsensical results.
- They use different strategies to make sure that the dependent and independent variables are properly defined, that the axes are correctly scaled, that no unit of measure has been omitted and that the data have been correctly transcribed.</td></tr>
</table>

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Relationship between quantities*. Two of these are considered particularly relevant to this course: *Exercises critical judgment* and *Communicates appropriately*.

Intellectual Competency

Representing a situation by means of an algebraic model could provide adult learners with an opportunity to use the competency *Exercises critical judgment.* Adult learners must be discerning when analyzing demographic data in order to determine a population growth model. They must exclude superfluous data from their analysis and select the functional model that applies to the situation and that will differ according to the population growth pattern in question. Adult learners use their judgment to qualify the importance of the margin of error. They question the future growth trends, knowing that the population cannot increase indefinitely.

Communication-Related Competency

The need to make extrapolations or provide proof and justifications could motivate adult learners to develop the competency *Communicates appropriately.* Providing proof requires that adult learners organize their thinking, formulate arguments using the correct vocabulary, show respect for others and be open to their ideas.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of arithmetic and algebra. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following three integrative processes:

- **using an algebraic or graphical model of a function to represent a situation**
- **interpolating or extrapolating from an algebraic or graphical model**
- **using an algebraic or graphical model of a function to generalize a set of situations**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover all three processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Numerical and algebraic expressions	
• Real numbers	The expressions involve the properties of: • absolute values • radicals • exponents • logarithms
• Manipulating arithmetic and algebraic expressions	Working with these expressions enables adult learners to expand their knowledge of the laws of exponents, deduce the different properties of radicals: • $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$ • $\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$ • $(\sqrt{a})^2=a$ • $\frac{a}{\sqrt{b}}=\frac{a\sqrt{b}}{b}$ • $\frac{1}{\sqrt{a}+\sqrt{b}}=\frac{\sqrt{a}-\sqrt{b}}{a-b}$ and deduce the following equivalences: • $a^b=c \Leftrightarrow \log_a c=b$ • $\log_a c^n=n\log_a c$ • $\log_a c=\frac{\ln c}{\ln a}=\frac{\log c}{\log a}$

Mathematical Knowledge	Restrictions and Clarifications
Relation, function and inverse	
• Experimenting with real functions and their inverse as well as observing, interpreting describing and representing them	Functions can be represented: • verbally • using a table of values • algebraically • graphically The following real functions will be studied in this course: • exponential $f(x)=ac^{b(x-h)}+k$ • logarithmic $f(x)=a\log_c b(x-h)+k$ • rational $f(x)=a\left(\frac{1}{b(x-h)}\right)+k$ • square root $f(x)=a\sqrt{b(x-h)}+k$ • sinusoidal $f(x)=a\sin b(x-h)+k$ $f(x)=a\cos b(x-h)+k$ • tangent $f(x)=a\tan b(x-h)+k$ • piecewise • absolute value $f(x)=a\lvert b(x-h)\rvert+k$ The concept of inverse is mainly associated with logarithmic, rational, exponential and square root functions. Experimental data are modelled by using curves related to the functional models under study and associating them with scatter plots.
• Operations on functions	*The four operations are studied in addition to the composition of functions.*
• Finding the rule of a function or its inverse, depending on the context	
• Describing and interpreting the properties of a function	The properties of real functions covered in this course are: • domain and codomain (range) • increasing and decreasing intervals • extrema • sign • x- and y-intercepts

<table>
<tr><th>Mathematical Knowledge</th><th>Restrictions and Clarifications</th></tr>
<tr><td>Relation, function and inverse (cont.)</td><td></td></tr>
<tr><td>Interpreting the multiplicative and additive parameters</td><td></td></tr>
<tr><td>Determining the type of dependency relationship, using the curve of best fit, with or without the help of technology</td><td></td></tr>
<tr><td>Solving equations and inequalities in one variable</td><td>The following equations and inequalities will be studied in this course:
trigonometric equations and inequalities of the first degree containing a sine, a cosine or a tangent
square root equations and inequalities
rational equations and inequalities
exponential and logarithmic equations and inequalities that involve applying the properties of exponents and logarithms
absolute value
The concepts of arcsine, arccosine and arctangent are studied mainly as inverse operations involved in solving equations or inequalities. The same is true for the concepts of square root and the logarithm introduced in previous courses.

The study of exponential and logarithmic functions should focus on bases 2, 10 and e.</td></tr>
</table>

Cultural References

In modern societies, overall population growth and the increase in the number of people that fall into each age group are among the major factors (urban, social and economic) to be taken into account for different types of planning. It is important to know the number of children who will be attending school in five years, the number of people who will need a doctor and the number of cars that will be on the road in the years to come. In the 19th century, Benjamin Gompertz was one of the first mathematicians to model population trends. His work and mathematical equations are used today to represent numerous progressions that follow a logistic curve. The increase in the sales of new products, the change in the size of tumours, the evolution of epidemics and the growth of a bacterial or animal population are often represented by the Gompertz model.

The study of functions in concrete situations can provide adult learners with the opportunity to study this type of curve since, in reality, exponential growth is not infinite: the curve enters a different phase when limiting factors are taken into account. It is often by understanding such factors that adult learners can determine the real meaning of the relation represented and understand its progression.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Relationship between quantities* involve problems that can be solved in part by using an algebraic or graphical model of a function to represent a relationship between quantities. The *Algebraic and Graphical Modelling in a Fundamental Context 2* course provides adult learners with an opportunity to express a connection or a dependency relationship between quantities.

In the situational problems in this course, adult learners recognize how a change in a parameter affects the graph of a function, use trial and error to determine the algebraic rule of a function, and deduce certain relationships such as the maximum value of a rational function when the values of the abscissas tend toward infinity by approaching the value of the horizontal asymptote.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Citizenship and Community Life, and Environmental Awareness and Consumer Rights and Responsibilities.

Citizenship and Community Life

People can find themselves in a crowd in different circumstances: during a public demonstration, during national holidays or even when school lets out. Everyone knows that a crowd does not behave like each individual in it and that the number of people in a crowd cannot be determined exactly. To become familiar with crowd phenomena, adult learners could compare different situations that arise in their centre. They could calculate the time it takes for a certain number of people to evacuate a given space and modify the parameters in order to find a rule that would make it possible to use the available space more efficiently. They could then make extrapolations involving movie theatres or office buildings. Studying crowd behaviour and designing circulation space to meet specific needs could help adult learners understand certain rules of social conduct and give them the opportunity to improve relationships among citizens. The promotion of the rules of social conduct is one of the focuses of development of this broad area of learning.

Environmental Awareness and Consumer Rights and Responsibilities

Some adult learners struggle with their income, expenses, investments or debts. This is why financial institutions offer a variety of investment opportunities or loans with variable interest rates, cumulative loans and loans with fixed terms. How can adult learners make informed choices and learn to better manage their money? Studying the factors involved in calculating interest rates (rate of change, vertex, initial value) and solving the resulting functions can help them better understand the concepts of budget, investment and credit. In this way, they can gain more control over their finances, which ties in with one of the focuses of development of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Citizenship and Community Life
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Relationship between quantities
Targeted cross-curricular competencies • Are developed at the same time and in the same context as the subject-specific competencies.	• Exercises critical judgment • Communicates appropriately
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Relationship between quantities* family of learning situations
Governments consult demographic studies before allowing a new school to be built in a growing city. Specialists also analyze recent demographic data. They derive an equation that makes it possible to estimate, by extrapolation, the size of a future population, with the smallest possible margin of error. Using fictitious demographic data, adult learners are required to determine the equation that could describe the typical change in the size of the population of a growing city and to analyze the influence of the margin of error on the parameters of this type of equation.	**Integrative process:** *Using an algebraic or graphical model to represent a situation* In carrying out the four phases in the problem-solving process, adult learners could: Representation • Describe the situation in their own words by considering the relevant elements • Make conjectures (e.g. that the function grows to infinity) • Exclude superfluous demographic data, such as average family income Planning • Refer to similar situations studied previously • List the types of information needed for the graph: population, year or period covered, margin of error • Draw a graph of the situation to be able to better choose the applicable functional model (e.g. quadratic function, exponential function)

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Relationship between quantities* family of learning situations	
	Activation	• Draw on the mathematical knowledge needed to deal with the current situation: suggest a probable algebraic rule, then check whether it applies • Use a spreadsheet program to determine the equation in light of the data provided • Decide on a realistic margin of error • Modify the data in accordance with the chosen margin of error • Find the new applicable algebraic rule • Analyze the different algebraic rules to determine which parameters were affected by the margin of error • Clearly formulate the conclusions of this analysis using the appropriate mathematical language
	Reflection	• Compare their solution and results with those of their classmates or teacher in order to determine whether the theoretical models are appropriate • Review the model they initially chose • Determine whether the chosen equation would be valid in the long term (concept of infinity) if the population growth remains constant

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Relationship between quantities*, adult learners represent a situation, interpolate or extrapolate, and generalize a set of situations using an algebraic or graphical model. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To represent a situational problem using an algebraic or graphical model, adult learners describe, symbolize, code, decode, explain or illustrate the information contained in tables of values or algebraic rules. They combine different registers of representation as needed to produce a message in accordance with the notation, rules and conventions associated with mathematical language. They use problem-solving strategies to make comparisons, propose corrections, present favourable or optimal solutions, or issue recommendations. They formulate constructive criticism and make informed decisions about issues in a variety of fields, including technical fields (e.g. graphics, biology, physics, administration).

To interpolate or extrapolate results from an algebraic or graphical model in order to make decisions, adult learners use different types of functional models and strategies, combining reasoning and creativity to overcome obstacles. They use structured deductive reasoning and become familiar with the codified form required for their proof. They illustrate, explain or justify their arguments. They use different types of proofs and different lines of reasoning, including proof by exhaustion. The latter is used in particular in analyzing or conducting case studies, or in applying a generalization process leading to the validation of a conjecture. Adult learners also observe specific real-life cases and generalize their observations.

To generalize a set of situations using an algebraic or graphical model, adult learners specify the purpose of their communication and switch from one register to another as needed. They demonstrate their understanding of the problems in question using a wide range of communication strategies, which enables them to regulate their transmission of a message based on the specific reaction of the audience or to take new requirements into account. They learn and correctly use language that appropriately combines everyday, mathematical, scientific and technical terms. They deduce new algebraic rules by combining the different operations on functions they have mastered, and prove them, justifying all the steps in their procedure. In addition, they make effective use of the parameters of the functions to illustrate generalities about a set of functions.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (relations, functions, inverse and operations on functions). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

  * The solution includes a procedure, strategies and a final answer.
  ** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*

Course

MTH-5173-2

Geometric Representation in a Fundamental Context 2

Mathematics

INTRODUCTION

The goal of the Geometric Representation in a Fundamental Context 2 course is to enable adult learners to deal with situations that involve describing geometric loci in a fundamental context.

Adult learners enhance their knowledge of the relationships between geometry and algebra by using trigonometric identities and studying conics, among other things. With respect to trigonometry, they use their understanding of equivalence relations and their ability to work with algebraic expressions to prove identities involving trigonometric expressions and to solve trigonometric equations. In studying conics, they discover other applications, in particular with regard to telecommunications systems. They examine conics based on a cross-section of a cone or through various hands-on activities (folding, play of light and shadows, construction). They observe patterns and attempt to define the different conics. They determine the equations associated with them and describe each region using inequalities. They find the coordinates of points of intersection and different measurements using algebraic manipulations and changing variables when necessary.

In studying the concept of vectors, adult learners build on what they learned about linearity in the previous cycle. Vectors make it possible to take a new approach to certain situations by using geometry and can be related to different concepts such as proportionality, linear functions, first-degree equations and geometric transformations associated with displacement. Adult learners can then compare the properties of real numbers with those of vectors. When performing vector operations, they use the Chasles relation, among other things. Depending on the situations involved, adult learners can also work with different linear combinations or determine the coordinates of a point of division using the product of a vector and a scalar. Vectors are studied in both the Euclidean and the Cartesian planes.

By the end of this course, adult learners will be able to use various vector and conic relations to represent and describe an object or a physical space in accordance with the mathematical notation and conventions used in geometry. They will also be able to use different strategies and types of reasoning to organize a physical space, taking into account different constraints.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the notation and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

<table>
<tr><th colspan="2">PROCESS AND STRATEGIES</th></tr>
<tr><th colspan="2">REPRESENTATION</th></tr>
<tr><td colspan="2">- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. They use strategies that are essential to inductive reasoning.
- They distinguish between the mathematical and everyday meanings of the terms used so that they can understand what is meant by focus, angle, vertex, arc, etc.</td></tr>
<tr><td>Examples of strategies</td><td>• Using an estimate, a scale plan or a literal description to determine the nature of the task involved
• Illustrating their understanding of the situational problem by drawing on relevant mathematical knowledge related to conics
• Describing the situation in their own words in order to show that they understand the situational problem, and comparing their understanding of it with that of their classmates or teacher</td></tr>
<tr><th colspan="2">PLANNING</th></tr>
<tr><td colspan="2">- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem most efficient.
- They use different registers of representation to illustrate certain properties of vectors and conics.
- They make connections between the elements of the message, for example, to determine if the algebraic rule accurately reflects the situation.</td></tr>
<tr><td>Examples of strategies</td><td>• Dividing the situational problem into subproblems, for example, in order to find a measurement using the metric relations in a circle
• Using lists, tables, diagrams, concrete materials or drawings to work out their solution</td></tr>
<tr><th colspan="2">ACTIVATION</th></tr>
<tr><td colspan="2">- In dealing with a situational problem, adult learners use mathematical reasoning to prove geometry principles pertaining to metric relations in a circle, giving several examples before drawing conclusions.
- They use exact mathematical language to ensure that the various steps involved in working out the solution are carried out efficiently.</td></tr>
<tr><td>Examples of strategies</td><td>• Using the characteristics of a conic to make a sketch in order to predict results
• Simplifying the situational problem by comparing it with a similar problem that has already been solved
• Predicting the possible solutions for a system of second-degree equations involving conics in order to be able to understand, for example, the relationship between the degree of an equation and the maximum number of possible solutions</td></tr>
<tr><th colspan="2">REFLECTION</th></tr>
<tr><td colspan="2">- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
- Reasoning helps them make conjectures about particular or special cases involving any triangle in order to validate certain results.
- They consult different reference documents to validate their mathematical message when they use new mathematical symbols to describe the organization or representation of their physical environment by means of conics and vectors.</td></tr>
<tr><td>Examples of strategies</td><td>• Checking their solution by means of examples or counterexamples
• Determining the strategies for dealing with situational problems in geometry (e.g. making a drawing, changing perspective)
• Using a calculator or geometric modelling software to validate their work</td></tr>
</table>

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Measurement and spatial representation*. Two of these are considered particularly relevant to this course: *Uses information and communications technologies and Solves problems*.

Methodological Competency

Simulation makes it considerably easier to understand optical phenomena. The use of specialized geometry and virtual laboratory software simplifies the representation of different phenomena. Adult learners could also use a spreadsheet to perform different calculations involving trigonometric ratios. In developing the competency *Uses information and communications technologies*, adult learners could come to regard specialized software as a valuable tool for representing reality.

Intellectual Competency

Analyzing problems with a view to explaining the mathematical phenomena in different situations helps adult learners develop the competency *Solves problems*. If warranted by the degree of complexity involved, adult learners use strategies to examine and select the key elements needed to solve the problem. They make appropriate use of the mathematical concepts they have learned (e.g. conics and vectors). It should be noted that dealing with a situation involves exploring avenues that may not lead to the correct solution. Because of the variety of situations they encounter, adult learners are able to hone their problem-solving skills.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of geometry and algebra. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following three integrative processes:

- **describing an object or a physical space and representing it in two or three dimensions**
- **describing geometric loci and representing them graphically and algebraically**
- **using vectors to generalize geometry principles**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover all three processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Geometric loci	
• Describing, representing and constructing geometric loci ○ conics studied: ▪ parabola (centred at the origin and translated) ▪ circle (centred at the origin) ▪ ellipse (centred at the origin) ▪ hyperbola (centred at the origin)	Conics are the only geometric loci studied in this option. The elements described are: • the radius • the axes • the directrix • the vertices • the foci • the asymptotes • the regions *When equations are used to describe conics, only the standard form is used.* *Inequalities related to conics are studied in this option.*

Mathematical Knowledge	Restrictions and Clarifications
Geometric loci (cont.)	
• Solving a system of second-degree equations with respect to conics	
• Determining the coordinates of points of intersection of a straight line and a conic or of a parabola and another conic	
Standard unit circle	
• Finding measurements: ○ arcs or angles (radians)	
• Finding the coordinates of points associated with important angles	Important angles are multiples of $\frac{\pi}{6}$, $\frac{\pi}{4}$ and $\frac{\pi}{2}$. The properties of periodicity and symmetry are studied in this course.
Trigonometric identities	
• Manipulating simple trigonometric expressions using definitions	*Only the Pythagorean identities and the properties of periodicity and symmetry are studied in this course.* *Formulas for finding the sum or difference of two angles are studied in this option.*

Mathematical Knowledge	Restrictions and Clarifications
Vectors	
• Resultant and projection	
• Operations on vectors	Geometric and free vectors are studied in this course. Operations on vectors are limited to the following: • adding and subtracting vectors • multiplying a vector by a scalar • the scalar product of two vectors • the properties of the scalar product of two vectors ○ commutativity of the scalar product $\vec{u} \bullet \vec{v} = \vec{v} \bullet \vec{u}$ ○ distributivity over vector addition $\vec{u} \bullet (\vec{v} + \vec{w}) = \vec{u} \bullet \vec{v} + \vec{u} \bullet \vec{w}$ ○ associativity of scalars $k_1\vec{u} \bullet k_2\vec{v} = k_1 k_2(\vec{u} \bullet \vec{v})$ • linear combination • properties of vectors ○ associativity $k_1(k_2\vec{u}) = (k_1 k_2)\vec{u}$ ○ existence of a scalar acting as an identity element $1\vec{u} = \vec{u}$ ○ existence of a zero scalar and an absorbing element $\left(k\vec{u} = \vec{0}\right) \Leftrightarrow \left(k = 0 \vee \vec{u} = \vec{0}\right)$ ○ distributivity over vector addition $k(\vec{u} + \vec{v}) = k\vec{u} + k\vec{v}$ ○ distributivity over scalar addition $(k_1 + k_2)\vec{u} = k_1\vec{u} + k_2\vec{u}$
• Determining the coordinates of a point of division	The coordinates of a point of division are determined using the product of a vector and a scalar.

Principles
Adult learners must master the following compulsory principles, which may be used in a proof. ▪ Consider $\vec{u}$, $\vec{v}$, and $\vec{w}$, which are vectors in the plane, as well as scalars $\mathbf{r}$ and $\mathbf{s}$. **P22.** Vector $\mathbf{r}\vec{u} = \vec{0}$ if, and only if, $\mathbf{r} = 0$ or $\vec{u} = \vec{0}$. $\left(\mathbf{r}\vec{u} = \vec{0}\right) \Leftrightarrow \left(r = 0 \vee \vec{u} = \vec{0}\right)$ **P23.** If vectors $\vec{u}$ and $\vec{v}$ are non-collinear, then $\mathbf{r}\vec{u} = \mathbf{s}\vec{v}$ if, and only if, $\mathbf{r} = 0$ and $\mathbf{s} = 0$. If $\vec{u}$ and $\vec{v}$ are non-collinear vectors, then $(\mathbf{r}\vec{u} = \mathbf{s}\vec{v}) \Leftrightarrow (\mathbf{r} = \mathbf{s} = 0)$. **P24.** Vectors $\vec{u}$ and $\vec{w}$ are collinear if, and only if, there is a real number $\mathbf{r}$ that is not equal to zero such that $\vec{w} = \mathbf{r}\vec{u}$. ($\vec{w}$ is collinear with $\vec{u}$) $\Leftrightarrow$ $(\exists!\,\mathbf{r} \in \mathbb{R} : \vec{w} = \mathbf{r}\vec{u})$ **P25.** Vectors $\vec{u}$ and $\vec{v}$ are non-collinear if, and only if, for any vector $\vec{w}$, there are two real numbers $\mathbf{r}$ and $\mathbf{s}$ such that $\vec{w} = \mathbf{r}\vec{u} + \mathbf{s}\vec{v}$. ($\vec{u}$ and $\vec{v}$ are non-collinear) $\Leftrightarrow$ $(\forall\vec{w},\ \exists!\,\mathbf{r} \in \mathbb{R},\ \exists!\,\mathbf{s} \in \mathbb{R} : \vec{w} = \mathbf{r}\vec{u} + \mathbf{s}\vec{v})$ **P26.** Vectors $\vec{u}$ and $\vec{v}$ are orthogonal if, and only if, their scalar product is equal to zero. $(\vec{u} \perp \vec{v}) \Leftrightarrow (\vec{u} \bullet \vec{v}) = 0$

Cultural References

Before becoming a science in its own right, classical mechanics was a branch of mathematics. Up until the end of the 18th century, the field of mechanics was used to test mathematical laws and theories. Mechanics and mathematics are still closely linked today. The need to model experimental data prompted mathematicians to develop theories related to geometry or differential equations. Many great mathematicians, such as Euler, Cauchy and Lagrange, made key contributions to this area.

In this course, adult learners could also explore three-dimensional representation. Human beings have spatial visualization abilities as a result of the positioning of their eyes and the space between them. Each eye sees a slightly different image of an object. The brain processes these differences and makes it possible not only to construct a 3-D object, but also to gauge the distance between the object and a person. This is the idea behind 3-D IMAX films, as the distance between the two cameras used is equivalent to the average distance between human eyes. The use of stereoscopy to create topographic maps is based on the same principle. When aerial surveys are conducted, two photographs of the same place are taken at different times (and therefore from different points of view). Binocular vision can therefore be simulated from these two photographs. However, since the distance between the two images is greater than the space between the eyes, the resulting 3-D effect is exaggerated, thereby making it possible to create more accurate topographical maps. A project that involves representing a three-dimensional object could help adult learners better understand the related mathematical concepts.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Measurement and spatial representation* involve problems that can be solved in part through the mathematical description or representation of geometric objects or geometric loci. The *Geometric Representation in a Fundamental Context 2* course provides adult learners with an opportunity to develop their spatial representation skills.

In the situational problems in this course, adult learners examine conics based on a cross-section of a cone or through various hands-on activities, find the coordinates of points of intersection and different measurements by using algebra and changing variables where necessary, and compare the properties of real numbers with those of vectors.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: *Health and Well-Being*, and *Citizenship and Community Life*.

Health and Well-Being

Many people must wear corrective lenses because they are either shortsighted or farsighted, but how are vision problems corrected? Starting with this question, adult learners can explore the basic concepts of optics as they relate to vision, including mirrors and light. Many mathematical concepts can be used to explain these phenomena. For example, adult learners could use their knowledge of conics to understand the concept of the focus of a lens (e.g. magnifying glass) or a parabolic mirror. They could also do research on the possible applications of the principles of conics in everyday technology (e.g. rotating lights on emergency vehicles). The study of optics can help adult learners adopt a safer lifestyle because they will have a better understanding of how light waves work and will therefore be able to use them without jeopardizing their health, which ties in with the focuses of development of this broad area of learning.

Citizenship and Community Life

Games have been an integral part of societies throughout history. Some games help develop social skills, while others call more upon individual qualities (e.g. games that test thinking abilities, intelligence, strength and dexterity). The latter provide useful material for the study of mathematics, in terms of how they relate to kinematics and conics. Adult learners could be asked to calculate the parabolic trajectory of a projectile (e.g. javelin, dart), or the velocity resulting from the impact between two objects in games or sports such as billiards, curling or other, older games. The study of games and sports in different countries throughout history helps adult learners gain a broader perspective of the world, which ties in with the educational aim of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Health and Well-Being
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Measurement and spatial representation
Targeted cross-curricular competencies • Are developed at the same time and in the same context as the subject-specific competencies.	• Uses information and communications technologies • Solves problems
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

<table>
<tr><th>Situational problem</th><th colspan="2">Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the Measurement and spatial representation family of learning situations</th></tr>
<tr><td rowspan="4">The headlights on a car have two light bulbs and a concave mirror that acts as a reflector. As part of a science fair, an adult learner decides to give a presentation illustrating the operating principle of these headlights and to explain the difference between high beams and low beams. In addition to defining concepts such as radius of curvature and focus, the adult learner must draw graphs to illustrate how the position of the light bulb affects the amount of space illuminated in front of the car, and do so using appropriate mathematical language.</td><td colspan="2">Integrative process: Describing geometric loci and representing them graphically and algebraically

In carrying out the four phases in the problem-solving process, adult learners could:</td></tr>
<tr><td>Representation</td><td>Determine the key elements in this situation (e.g. the radius of curvature of the mirror, the position of the light bulbs with respect to the mirror)
Draw a diagram of the situation by sketching the light rays emitted by the bulbs and reflected by the curved mirror</td></tr>
<tr><td>Planning</td><td>Look for possible solutions through brainstorming
Use concrete materials to work out the solution</td></tr>
<tr><td>Activation</td><td>Draw on the mathematical knowledge needed to deal with the situation: line tangent to a circle, perpendicular lines, normal, measures of angles, etc.
Predict how the light rays are likely to behave in the two situations, then use examples to check this prediction</td></tr>
</table>

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Measurement and spatial representation* family of learning situations	
Adult learners will use concepts related to optics, but this learning situation could be studied in science courses at the same time.	Reflection	• Draw a graph showing how the behaviour of the light rays will change, depending on the position of the light source • Compare their solution and results with those of their classmates or teacher to determine whether the theoretical models are appropriate • Examine the effect of changing a parameter such as the radius of curvature or the position of the focus

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Measurement and spatial representation*, adult learners describe geometric loci, represent them graphically and algebraically, and use vectors to generalize geometry principles. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To describe and represent an object or a physical space, adult learners interpret and produce drawings, plans or graphs using complex figures that can be broken down into all types of triangles. They identify the key elements of mathematical language (e.g. radian, unit circle, arc length). In addition, they apply newly acquired mathematical knowledge such as conics and vectors, which enables them to determine unknown measurements.

To describe geometric loci and represent them graphically and algebraically, adult learners use different mathematical models and types of strategies, combining reasoning and creativity to overcome obstacles. They use structured deductive reasoning and become familiar with the codified form required for their proof. They use different types of proofs and different lines of reasoning, including proof by exhaustion. The latter is used in particular in analyzing or conducting case studies, or in applying a generalization process leading to the validation of a conjecture. Adult learners observe specific real-life cases and generalize some of their observations. They experiment with certain situations that involve analyzing data in order to identify the necessary and sufficient conditions for drawing a conclusion, making decisions and determining how best to approach, optimize or adjust a situation. In addition, they base their reasoning on a Euclidean or Cartesian plane in order to determine measurements in conics by using their knowledge of such concepts as the directrix, the vertex, the foci and the asymptotes.

To prove a theorem using vectors, adult learners translate the hypotheses and thesis into vector expressions and construct an equality. They develop the equality and use the Chasles relation to reduce the initial equality to its simplest form. As needed, they apply the properties of the scalar product of two vectors. They make connections between vector notation and the properties of geometric figures. In addition, they justify all the steps in their solution, applying the properties of vectors so as to communicate clearly and concisely.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (geometric loci, unit circle, trigonometric identities and vectors). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

* The solution includes a procedure, strategies and a final answer.
** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*

6.5 Secondary V Optional Courses

The Secondary V optional courses provide adults with tools that will help them increase their capacity for analysis, consider different possibilities, make informed decisions, support their reasoning and take a position with respect to various issues. The objective of these courses is to help adult learners continue to develop their sense of citizenship and to better equip them for post-secondary studies. The context associated with the first optional course involves general mathematics while that associated with the second course involves applied mathematics.

In both of these optional courses, adult learners hone their competencies in a number of ways: they compare their solutions with those of their peers or their teacher; consider various points of view; exercise their critical judgment when validating a solution or conjecture; look for the causes of a problem, for mistakes or for anomalies in solutions or algorithms; and issue recommendations with a view to taking corrective measures or making their actions more efficient.

Besides continuing to develop their mathematical competencies and familiarizing themselves with new concepts and processes, adult learners further their understanding of previously learned concepts. It is important, therefore, to allow them to use what they already know and to approach the learning content in a way that illustrates how mathematical ideas build on one another. Emphasis is placed on consolidating and integrating knowledge in a variety of activities: hands-on activities, exploration activities, simulations, games, presentations, meetings with resource persons, and so on. Throughout both of these optional courses, adult learners have the opportunity to use or develop their observational, design, managerial, optimization, decision-making, argumentative and other skills. They are generally required to carry out concrete and practical activities. Adult learners will also use technology to represent or process large amounts of data and to simplify tedious calculations.

The suggested cultural references in these courses present the historical and social elements that have influenced the development of mathematics. They also allow adult learners to appreciate the role of mathematics in daily life and work situations, as well as the contributions of numerous people to the development of this subject.

Course

MTH-5154-2

Financial Mathematics in a General Context

Mathematics

INTRODUCTION

The goal of the course *Financial Mathematics in a General Context* is to enable adult learners to deal effectively with situations that involve evaluating investment proposals and creating various financing scenarios in a general context.

Adult learners who take the course will learn how to analyze and interpret financial situations, and how to develop financing scenarios to meet their needs. Whether the goal is to manage personal, family or business assets, learners will become familiar with a number of financial concepts, including *future* or *earned value*, *fixed interest rate* and *compound interest rate*, so that they will be able to make informed decisions. They will use timelines to solve problems involving annuities, which will require them to evaluate the performance of financial plans over time and to take retroactive measures if necessary.

Since this is an introductory course, all the required formulas are provided for the calculations that need to be performed. Learners calculate the balance of a fund to which successive deposits are made; determine the value of a debt on a given date based on the payments due; or estimate the present value and the future value (earned value) of a series of equal payments using formulas or electronic spreadsheets. They learn how to calculate equivalent rates to determine the value of an annuity or the number of payments required. In addition, they will learn to analyze the behaviour of financial functions over different time periods and to compare the graphic representations of situations under study.

Finally, the course provides the opportunity to explore concepts related to periodic payments and the number of payments required when the amount of the regular payment is set in advance. The concepts of residual value, interest, and capital amortization on a given date are introduced. In addition, an overview of supplementary payments under a repayment annuity is given.

At the end of this course, adult learners will be able to understand financial language and use calculation tools related to financial mathematics to evaluate different investment proposals and choose the most suitable one. They will also be able to create a financial plan tailored to a particular situation.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly while observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.
Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

<table>
<tr><th colspan="2">PROCESS AND STRATEGIES</th></tr>
<tr><th colspan="2">REPRESENTATION</th></tr>
<tr><td colspan="2">- Adult learners examine the situational problem to identify the context, the problem and the task to be performed.
- They use observational and representational strategies that are essential to inductive reasoning.
- In attempting to understand the context and the problem, they use deductive reasoning, particularly in situations that involve implicit data.
- They switch from one register of representation to another when they use spreadsheets, software, Web applications or any other technological tool considered useful.</td></tr>
<tr><td>Examples of strategies</td><td>• Describing the situation in their own words and making timelines to illustrate their understanding of the problem
• Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to identify a dependency relationship and determine the variables in the situation
• Determining the types of relationships that exist between the variables in the situation by using numerical values to illustrate these relationships
• Describing the mathematical characteristics of the situation in order to identify the limitations and real constraints of the problem
• Formulating questions with respect to the problem
• Making false assumptions to identify an inconsistency or an absurdity to corroborate their perceptions or call them into question</td></tr>
<tr><th colspan="2">PLANNING</th></tr>
<tr><td colspan="2">- Adult learners look for ways of approaching the problem and choose those that seem the most efficient.
- They develop a plan, taking into account the different registers of representation and elements of mathematical language (symbols, terms and notation used).
- They establish organized and functional relationships among different aspects of their knowledge, thus expanding their networks of cognitive resources.</td></tr>
<tr><td>Examples of strategies</td><td>• Drawing a concept map showing the different steps in the solution
• Listing the elements needed to represent a function graphically or algebraically
• Referring to a list of elements to be considered in consolidating their work plan</td></tr>
<tr><th colspan="2">ACTIVATION</th></tr>
<tr><td colspan="2">- Adult learners establish organized and functional relationships between the future value and current value of amounts invested, thus expanding their networks of mathematical cognitive resources.
- By drawing on their knowledge of the properties of financial models, they are able to deduce certain relationships.
- They use the appropriate scale so that the graph they draw in solving the situational problem makes sense in light of the context.</td></tr>
<tr><td>Examples of strategies</td><td>• Systematically determining the algebraic rule of a function by manipulating the different variables
• Finding combinations of simple variables (linear, quadratic or exponential) in order to determine the rule of a function
• Changing perspective and observing the change in the function when the y-axis (instead of the x-axis) is considered to be its domain</td></tr>
<tr><th colspan="2">REFLECTION</th></tr>
<tr><td colspan="2">- Adult learners use a reflective approach throughout the process of solving the situational problem and always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
- A reflective approach allows them to reject extrapolations that would yield nonsensical results.
- When decoding mathematical elements, adult learners make sure they can distinguish between the mathematical and everyday meanings of the terms used.</td></tr>
<tr><td>Examples of strategies</td><td>• Checking their solution:
  • by confirming, for example, that the y-axis represents the change in the value of a loan or investment
  • by making sure that the resulting values satisfy the range of the function
  • by comparing the amount invested, the interest earned and the investment gain
  • by substituting the values of the variables in the algebraic expression in order to validate a graphical interpolation or extrapolation</td></tr>
</table>

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Relationship between quantities*. Two cross-curricular competencies are considered particularly relevant to this course: *Adopts effective work methods* and *Exercises critical judgment*.

Methodological Competency

When adult learners work with situational problems in the family *Relationship between quantities*, they deal with data derived from financial statements, statistical reports, balance sheets or the results of studies. To carry out their task, adult learners must develop the competency *Adopts effective work methods.* They therefore use timelines and tables to represent a situational problem, organize and analyze data, facilitate enumeration, calculate financial values and produce graphs illustrating the change in the value of an investment or loan. They exchange information with their peers regarding the solution by explaining their approach, choice of registers, decisions, recommendations or conclusions. In gauging the reactions of their peers, they look for ways of evaluating the effectiveness of their solution or the reliability of the study carried out.

Intellectual Competency

When adult learners use iterative reasoning, they explore and compare different possibilities, and justify their choices. They identify various relationships and, depending on their goals, use interpolation, extrapolation or optimization processes by drawing on their understanding of dependency relationships and the concepts of function and inverse (compounding and discounting). They use algebraic processes to identify laws, rules and properties which, in turn, serve to validate conjectures, for example, when adult learners use deductive reasoning to show that two interest rates are equivalent. By developing the competency *Exercises critical judgment*, adult learners may take a more thoughtful approach before signing a contract that will lock them in for many months.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of arithmetic and algebra. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following two integrative processes:

- **evaluating investment proposals tailored to a particular situation**
- **creating a financial plan tailored to a particular situation**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover both processes**.** The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Performing financial calculations related to an investment or a loan	
• Determining the interest period, future value and current value	The formulas studied are used to determine: • discounting (current value): $C_0 = C_n(1 + i)^{-n}$ • compounding (future value): $C_n = C_0(1 + i)^n$ • the interest period: $n = \dfrac{\log\left({}^{C_n}/_{C_0}\right)}{\log(1 + i)}$ • the interest rate (i) in situations involving compound interest: $i = \left(\dfrac{C_n}{C_0}\right)^{1/n} - 1$
• Determining the interest rate[2]: ○ simple, ○ compound, ○ equivalent.	The formulas studied are used to determine: • discounting (current value): $C_0 = C_n(1 + i)^{-n}$ • compounding (future value): $C_n = C_0(1 + i)^n$ • the interest period: $n = \dfrac{\log\left({}^{C_n}/_{C_0}\right)}{\log(1 + i)}$ • the interest rate (i) in situations involving compound interest: $i = \left(\dfrac{C_n}{C_0}\right)^{1/n} - 1$

[2] Use of a spreadsheet is recommended.

Mathematical Knowledge	Restrictions and Clarifications
Analyzing a series of equal or unequal payments	
• Producing and analyzing a statement outlining the current situation regarding of an investment	
• Interpreting calculations in a financial plan	Compound interest is presented using graphs or compiled data tables. Timelines are used to show the change in the value of an investment or loan. Using formulas or software, analyzing and calculating: • the current value of a series of equal or unequal payments • the future value of a series of equal or unequal payments *A withdrawal is regarded as a negative payment.*
Analyzing annuities (repaying a loan by constant annuity)	
• Interpreting annuity calculations	In this course, the study of annuities is limited to determining: • the annuity period • the current value of a series of annuities • the future value of a series of annuities
• Interpreting calculations of current value and future value	
Analyzing amortization	
• Interpreting amortization tables or calculations	In this course, the study of amortization (progressive or unique) is limited to the calculation of: • the payment • the number of payments • the amortization • the residual debt • the interest portion and the capital portion of a payment

Cultural References

Financial mathematics are rich in cultural references, ranging from the history of money to major financial crises and the different theories regarding credit, insurance, real estate, and the stock market, among other things. This course enables adult learners to gain a thorough understanding of the different phases in history that have led societies to develop financial models. Adult learners may also discover how, in developing these models, mathematicians have been influenced by Brownian motion, the heat transfer equation or the random walk.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Relationship between quantities* involve purely mathematical problems that can be solved in part by evaluating an investment proposal or creating a financial plan tailored to a particular situation, in a general context.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Citizenship and Community Life and Environmental Awareness and Consumer Rights and Responsibilities.

Citizenship and Community Life

Adult learners are already aware of how important it is to start saving early. The financial concepts and calculations they learn in this course will enable them to analyze different financial strategies when a major change occurs in their lives that has an impact on their budget (e.g. birth of a child, going back to school, buying a home). They must be able to continue meeting the needs of each member of their family, while saving enough to be financially independent and maintain their lifestyle after retirement.

Environmental Awareness and Consumer Rights and Responsibilities

Adult learners will have to make major financial decisions throughout their lives. Whether it involves negotiating a mortgage, taking out life insurance or financing the purchase of a car, their ability to analyze different financing options will allow them to make an informed decision. This ties in with one of the focuses of development of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following pages.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Environmental Awareness and Consumer Rights and Responsibilities • Citizenship and Community Life
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Relationship between quantities
Targeted cross-curricular competencies • Are developed at the same time and in the same context as the subject-specific competencies.	• Exercises critical judgment • Adopts effective work methods
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Relationship between quantities* family of learning situations
All parents worry about being able to afford a post-secondary education for their children. At some point in time, parents face certain choices. What type of savings vehicle should they choose to cover education costs: a registered education savings plan or a registered retirement savings plan?	**Integrative processes:** *Evaluating investment proposals tailored to a particular situation* *Creating a financial plan tailored to a particular situation* In carrying out the four phases in the problem-solving process, adult learners could: Representation • Select the most appropriate register of representation (algebraic or graphical) and determine the change in the value of several types of investments • Represent the evolution of a series of payments using a timeline Planning • Use algebraic representation to compare the costs and yields of different investments Activation • Establish the algebraic rule that relates the various elements of the situation (e.g. rates, interest, annuities, amortization payments) to be able to extrapolate the yield associated with each type of plan

		• Make a conjecture about the various aspects of an investment, and then verify the conjecture algebraically and graphically • Use a spreadsheet to analyze the change in the value of investments and determine which investment is the best, or determine the point at which one investment becomes more worthwhile than another
	Reflection	• Conclude that graphical extrapolation is not very realistic in the case of extreme values • Validate a graphical extrapolation through algebraic calculations • Check the calculation of the future value using the discounted value formula • Decide which model is best suited to the household's financial situation

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Relationship between quantities*, adult learners evaluate investment proposals tailored to different situations or create a financial plan tailored to a particular situation. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

When evaluating investment proposals by using a graphical or algebraic model to represent the situational problem, adult learners employ various strategies to identify the problem. They reformulate the situational problem in their own words and determine the key elements and the obstacles to be overcome. They correctly identify the variables involved and determine those whose value they must calculate (rate, period, compounding, discounting, etc.) by using tables of values (sometimes actuarial tables) and timelines. They choose the most accurate representation, aware that it does not necessarily reflect what they have observed, but that it is the best choice given the functions studied in the course. They validate their representation, comparing their solution against known bank data. When explaining their conclusions after evaluating investment proposals, they produce their representation of the situational problem, determine the purpose of the message and observe mathematical codes and rules in order to effectively communicate their intention. They choose the register of representation best suited to the situation so that their message will be understood by as many people as possible.

When adult learners create a financial plan using an algebraic model or an amortization table, they interpret the model, making connections between the elements of the message and distinguishing between those that are relevant and those that are not. They recognize the purpose of the message and determine its overall meaning. In addition, they use mathematical reasoning to explore the situational problem and to determine questions about the issue. They gather relevant information in order to draw a conclusion. They make one or more conjectures, suggest probable or plausible ideas and, as required, anticipate annuities and amortization payments that are consistent with their financial plan.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

* The solution includes a procedure, strategies and a final answer.
** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*

Course

MTH-5164-2

Sequences and Series in an Applied Context

Mathematics

INTRODUCTION

The goal of the course *Sequences and Series in an Applied Context* is to enable adult learners to deal effectively with situations that involve using number sequences to represent a dependency relationship between quantities in an applied context.

The concept of arithmetic sequences is no doubt one of the cornerstones of modern mathematics, and its study provides a simpler representation of the direct relationship between two variables. When scientists study observable phenomena for the first time, their only resources are numbers that they compile in tables or graphs. Searching for a dependency relationship between the numbers can become a seemingly never ending task. A return to basic arithmetic often provides a better understanding of how variables are related. This course involves studying strategies that will make it possible to identify patterns that reflect a close relationship between variables. Different registers of representation (ordered or numbered sequences, tables of values, graphs or algebraic rules) can be used in the search for patterns.

This course presents different methods and strategies in order to help adult learners to distinguish between arithmetic and geometric sequences recursively and explicitly; to establish a direct relationship between geometric sequences and exponential functions; to deduce and apply formulas for the general term and the sum of a geometric sequence.

In addition, adult learners further explore mathematical formalism through the use of symbols for summation (Σ), logical connectors and set notation, in order to make algebraic manipulations less onerous. The concept of limit is introduced in an intuitive manner in order to determine the convergence of sequences and series.

At the end of this course, adult learners will be able to apply their knowledge of first- or second-degree polynomial functions, exponential functions and logarithmic functions, and to solve situational problems involving arithmetic or geometric sequences and series in accordance with the symbols and conventions of mathematics. By algebraically or graphically representing a situation using real functions, they will induce results through interpolation or extrapolation. They can interpolate or extrapolate using a table of values, a graph or algebra when the algebraic rule is given. Lastly, adult learners will use different registers of representation (tables of values, graphs or algebraic rules) to generalize a model so that it can be applied to a range of situations described by sequences and series.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly while observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

<table>
<tr><th colspan="2">PROCESS AND STRATEGIES</th></tr>
<tr><th colspan="2">REPRESENTATION</th></tr>
<tr><td colspan="2">- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. They use observational and representational strategies that are essential to inductive reasoning.
- They increase their knowledge of mathematical notation and symbols related to functions and inverse functions expressed in the general form.</td></tr>
<tr><td>Examples of strategies</td><td>• Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to identify a dependency relationship and determine the variables in the situation
• Using examples involving numbers, determining the types of relationship that exist between the variables in the situation
• Exploring a geometric sequence recursively</td></tr>
<tr><th colspan="2">PLANNING</th></tr>
<tr><td colspan="2">- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient.
- They attempt to extrapolate results using an algebraic rule or a graph, thus expanding their networks of cognitive resources.
- To correctly plan their solution, they decode elements of mathematical language, such as the meaning of the symbols, terms and notation used, as well as the different registers of representation.</td></tr>
<tr><td>Examples of strategies</td><td>• Drawing a concept map showing the different steps in the solution
• Referring to a list of elements to be considered in consolidating their work plan (e.g. the scale of the axes, the increasing and decreasing intervals, the maximum or the minimum, if any)
• Generating the first three terms in a geometric sequence
• Writing an exponential function given the corresponding geometric sequence</td></tr>
<tr><th colspan="2">ACTIVATION</th></tr>
<tr><td colspan="2">- When dealing with a situational problem, adult learners use reasoning to establish structured and functional relationships among different aspects of their knowledge, thus expanding their networks of mathematical cognitive resources.
- They use different strategies by associating pictures, objects or concepts with mathematical terms and symbols and by switching from one register of representation to another.</td></tr>
<tr><td>Examples of strategies</td><td>• Changing perspective
• Systematically determining the general form of the algebraic rule of a function
• Finding combinations in order to determine the rule of a quadratic function
• Creating a linear model from a non-linear one by replacing the values of the independent (X) or dependent (Y) variable, or both, with their logarithm
• Comparing sequences with known algebraic models
• Selecting the algebraic function that is best suited to the situation by determining the differences between the terms in the sequence and the differences in the y-values of different functions</td></tr>
<tr><th colspan="2">REFLECTION</th></tr>
<tr><td colspan="2">- Adult learners use a reflective approach throughout the situational problem and always review the steps in the problem-solving process and the choices made, with a view to validating the solution.
- Through reasoning, they can make conjectures about particular or special cases to validate certain results.
- They use different strategies to make sure that the dependent and independent variables are properly defined, that the axes are correctly scaled, that no unit of measure has been omitted and that the data have been correctly transcribed.</td></tr>
<tr><td>Examples of strategies</td><td>• Checking their solution by, for example, making sure that the resulting values satisfy the range of the function, or substituting the values of the variables in the algebraic expression in order to validate a graphical interpolation or extrapolation
• Exploring the limits of a formula deduced from the general term
• Validating their solution by means of a graph of the algebraic rule
• Determining the difference between the values in the situational problem and the corresponding model</td></tr>
</table>

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations Relationship between quantities. Two cross-curricular competencies are considered particularly relevant to this course: *Adopts effective work methods* and *Communicates appropriately*.

Methodological Competency

It is easier to interpret a situation if it is represented by a numerical sequence or series. Adult learners develop the competency *Adopts effective work methods* when they are required to analyze data derived from compiled observations (e.g. observations related to a complex biological phenomenon for which it is not easy to identify the variables involved).

Communication-Related Competency

The need to make extrapolations or provide proof and justifications could motivate adult learners to develop the competency *Communicates appropriately*. Providing proof requires that adult learners organize their thinking, formulate arguments using the correct vocabulary, show respect for others and be open to their ideas.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of arithmetic and algebra. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following three integrative processes:

- **using a sequence or series to represent a situation**
- **interpolating or extrapolating from an arithmetic, algebraic or graphical model**
- **using an algebraic or graphical model of a function to generalize a set of situations**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover all three processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Arithmetic and geometric sequences	

• Determining the general term, the convergence and the limits of a sequence	In this course, the study of arithmetic and geometric sequences is limited to:
	• $u_{n+1} = u_n+d$ (arithmetic sequence with common difference d)
	    ○ e.g.: sequence of odd numbers
	• $u_{n+1} = qu_n$ (geometric sequence)
	    ○ e.g.: Sierpinski triangle

• Experimenting, observing, interpreting, describing and representing situations using number sequences	The characteristics of the sequences studied in this course are: • increasing • decreasing • strictly increasing • strictly decreasing • monotonic • bounded from above • bounded from below • bounded

Mathematical Knowledge	Restrictions and Clarifications
Series	
• Determining the formula, convergence and limits of a series	
• Experimenting, observing, interpreting, describing and representing situations using number series	

Principles

Adult learners must master the following compulsory principles.

- In this course, the following statements are studied to determine the convergence (limit) of sequences.

E1. All monotonic sequences are bounded and convergent

E2. All convergent sequences are bounded

E3. If $\{u_n\}$ is a sequence converging toward a and $\{v_n\}$ is a sequence converging toward b, then: $\{u_n + v_n\}$ converges toward a + b

E4. If $\{u_n\}$ is a sequence converging toward a and $\{v_n\}$ is a sequence converging toward b, then: $\{u_n \cdot v_n\}$ converges toward a · b

E5. If $\{u_n\}$ is a sequence converging toward a and $\{v_n\}$ is a sequence converging toward b, then: $\left\{\frac{u_n}{v_n}\right\}$ converges toward $\frac{a}{b}$

E6. If $\{u_n\}$ is a sequence converging toward a and $\{v_n\}$ is a sequence converging toward b, then: $\{\lambda u_n\}$ converges toward λa

- In this course, the following tests and statement are studied to determine the convergence of series.

Comparison test

Given U = $\sum_{i=1}^{\infty} u_i$ and V = $\sum_{i=1}^{\infty} v_i$, series with positive terms:

V is converging $\wedge$ $u_i \leq v_i$, $\forall i$ $\Rightarrow$ U is converging

V is diverging $\wedge$ $u_i \geq v_i$, $\forall i$ $\Rightarrow$ U is diverging

Quotient test

Given U = $\sum_{i=1}^{\infty} u_i$ and $L = \lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right|$:

- o L < 1 ⇒ U converges absolutely
- o L > 1 ⇒ U diverges absolutely
- o L = 1 ⇒ no conclusion can be drawn

E7. A series is said to converge absolutely or diverge absolutely when the series of the absolute values of the terms is convergent or divergent respectively.

Cultural References

Sequences of real numbers have always been linked to experimental mathematics. Sequences and series were used in ancient civilizations, for instance, when Archimedes developed his recurrence algorithm, whose successive application provides an approximation of areas and a value for the constant π (pi). Sequences and series were also used in Egypt to compute a square root using Heron of Alexandria's method.

During WWII, advances in the field of informatics rekindled interest in sequences and series. Examples of these advances include the invention of a calculator that had SIN, COS, TAN, LOG and EXP keys which were programmed using the developments around convergent series.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Relationship between quantities* involve problems that can be solved in part by using an algebraic or graphical model of a function to represent a relationship between quantities. The *Sequences and Series in an Applied Context* course provides adult learners with an opportunity to learn how to express a connection or a dependency relationship between quantities.

In the situational problems in this course, adult learners become more familiar with the mathematical symbols and notation related to sequences and series. In addition to extrapolating results using a function or a graph, they use a scale appropriate to the context so that the graph they draw in solving the situational problem makes sense in the context.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Citizenship and Community Life and Career Planning and Entrepreneurship.

Citizenship and Community Life

Many situations that involve a tenuous balance among several observable parameters highlight problems that have direct and indirect consequences on our economic life. Take, for example the problem of field mouse infestations, which wine producers know only too well. In order to familiarize themselves with certain phenomena stemming from the interplay of the parameters in this complex situation, adult learners could study these parameters by using sequences and series to simplify analysis and better identify the real-world implications that go far beyond the mathematics involved. This problem illustrates the close correlation between economic performance and the control of the reproduction of a species and the regulations that need to be enacted to deal with the situation. This ties in directly with the life of their society and is consistent with the educational aim of this broad area of learning.

Career Planning and Entrepreneurship

Adult learners faced with a learning situation that involves sequences and series used in biology or other branches of science and that also involves multiple variables may be required to determine the change in the birth rate of a species at the expense of another or to find a way of better controlling the prey-predator dynamic that could affect the balance of species. For example, effective management of the field mouse infestation problem could have positive economic spinoffs since a decrease in the number of field mice will lead to an increase in the harvests and therefore an increase in the related stock price. This ties in with one of the focuses of development of this broad area of learning, which deals with the exploration of plans for the future based on their interests and aptitudes.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Career Planning and Entrepreneurship
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Relationship between quantities
Targeted cross-curricular competencies • Are developed at the same time and in the same context as the subject-specific competencies.	• Adopts effective work methods • Communicates appropriately
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the ***Relationship between quantities*** family of learning situations
For a number of years, wine growers in the region have carried out a daily campaign against the field mouse infestation. These little rodents attack the vines and can destroy an entire crop in a single season. Several ecological and sustainable methods have been implemented, including bringing birds of prey into the region in order to stabilize the prey-predator ratio. To form a more accurate picture of the situation, wine growers have asked experts to carry out a study.	**Integrative process:** *Using an algebraic or graphical model of a function to generalize a set of situations* In carrying out the four phases in the problem-solving process, adult learners could: Representation • Select the relevant information (e.g. number of field mice counted in a given period, times at which measurements were taken, length of observation periods, temperature) and disregard superfluous information • List the different modes of representation that are best suited to address the situation Planning • Organize the information • Study the changes in the first and second elements in the sequence • Find a relationship between pairs of experimental parameters • List the elements needed to draw the graph

<table>
<tr><th>Situational problem</th><th colspan="2">Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the Relationship between quantities family of learning situations</th></tr>
<tr><td rowspan="2">Using the data that ecologists have compiled, the adult learner produces a report describing the situation.</td><td>Activation</td><td>Make a table of the data related to the situation, taking into account the limitations and precision of the measuring instruments used
Study the first and second differences in order to highlight the arithmetic or geometric nature of the sequence
Match an exponential function with a geometric sequence or a linear function with an arithmetic sequence or a quadratic function with the sum of the data values
Compare the coefficients of determination for each function
Choose the most appropriate one</td></tr>
<tr><td>Reflection</td><td>State the different conclusions associated with the different algebraic models
Highlight the differences between the y-values predicted by the functions and the y-values obtained through experimentation
Describe future scenarios using examples</td></tr>
</table>

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Relationship between quantities*, adult learners represent a situation, carry out interpolations and extrapolations and use an algebraic or graphical model to generalize a set of situations. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

When studying a situation using sequences and series, adult learners represent the situation by making a graph of the arithmetic data, which allows them to efficiently choose the type of function that best describes the situation. They combine different registers of representation as needed to produce a message in accordance with the notation, rules and conventions of mathematical language. They use problem-solving strategies to make comparisons, propose corrections, present favourable or optimal solutions, or issue recommendations. They formulate constructive criticism and make informed decisions concerning issues in a variety of fields, including technical fields (e.g. graphics, biology, physics, administration).

In interpolating or extrapolating results from an algebraic or graphical model, adult learners use their knowledge of different types of functions and strategies, combining reasoning and creativity to overcome obstacles and make decisions. They use structured deductive reasoning and become familiar with the codified form required for their proof. They use the properties of sequences and series to support their argument. They use illustrations, explanations or justifications to describe their conclusions.

To generalize a set of situations using an algebraic or graphical model derived from the preliminary study of sequences and series, adult learners specify the purpose of their communication and switch from one register to another as needed. They demonstrate their understanding of the mathematical concepts in question using a wide range of communication strategies, which enables them to take new requirements into account. They learn and correctly use language that appropriately combines, mathematical, technical and scientific and everyday terms. They deduce new algebraic rules by combining different operations on functions they have mastered. In addition, they use the parameters of the functions effectively to illustrate generalities about a set of functions.

Throughout the problem-solving process, adult learners make an effort to apply their mathematical knowledge (arithmetic and geometric sequences and series, functions, inverse and operations on functions). They make accurate use of symbols, terms and notation related to this knowledge, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps*** *in the solution*

* The solution includes a procedure, strategies and a final answer.

** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*

Appendix

Switching from one option to another

As shown in the tables in Chapter 4, courses in different options can have similar subject-specific content. This means that adult learners who decide to switch to another option do not have to take the courses whose content is deemed similar to that of courses they have already successfully completed. Furthermore, to ensure a smooth transition from one option to another, teachers may take the following clarifications into account.

Successfully completed course	Course with a similar content	Clarifications
From CST option	To TS option	Adult learners earn 12 credits in the TS option and 8 in the CST option.
MTH-4151-1	None	There is more prescribed knowledge in course MTH-4261-2.
MTH-4152-1	None	There is more prescribed knowledge in course MTH-4262-2.
MTH-4153-2	MTH-4263-2	The knowledge covered in course MTH-4263-2 is also covered in course MTH-4153-2, except the following knowledge related to **metric and trigonometric relations in triangles**: • **perpendicular bisector of a segment** • **areas of triangles, given the measure of an angle and the lengths of two sides or given the measures of two angles and the length of one side**
MTH-5150-2	MTH-5160-2	The knowledge covered in course MTH-5160-2 is also covered in course MTH-5150-2.
MTH-5151-1	None	There is more and different prescribed knowledge in course MAT-5161-2.
MTH-5152-1	None	There is no corresponding course in the TS option.
From CST option	To S option	Adult learners earn 12 credits in the S option and 8 in the CST option.
MTH-4151-1	None	There is more prescribed knowledge in course MTH-4271-2.
MTH-4152-1	None	There is more prescribed knowledge in course MTH-4272-2.
MTH-4153-2	MTH-4273-2	Knowledge covered in course MTH-4273-2 is also covered in course MTH-4153-2, except equivalent figures and the following knowledge related to **metric and trigonometric relations in triangles**: • **cosine law** • **angles in figures that can be split into triangles** • **area and volume of figures**
MTH-5150-2	MTH-5170-2	The knowledge covered in course MTH-5170-2 is also covered in course MTH-5150-2.
MTH-5151-1	None	There is more and different prescribed knowledge in course MAT-5171-2.
MTH-5152-1	None	There is no corresponding course in the S option.

Successfully completed course	Course with a similar content	Clarifications
From TS option	To CST option	Adult learners earn 12 credits in the TS option and 8 in the CST option.
MTH-4261-2	MTH-4151-1	The knowledge covered in course MTH-4151-1 is also covered in course MTH-4261-2.
MTH-4262-2	MTH-4152-1	Knowledge covered in course MTH-4152-1 is also covered in course MTH-4262-2, except knowledge related to **representing statistical data (stem-and-leaf diagram)**.
MTH-4263-2	MTH-4153-2	Knowledge covered in course MTH-4153-2 is also covered in course MTH-4263-2, except equivalent figures and the following knowledge related to **metric and trigonometric relations in triangles**: • **sine law** • **Hero's formula** • **area of a quadrilateral**
MTH-5160-2	MTH-5150-2	Knowledge covered in course MTH-5150-2 is also covered in course MTH-5160-2, except the knowledge related to **graph theory and solving first-degree inequalities in two variables.**
MTH-5161-2	MTH-5151-1[3]	There is more prescribed knowledge in course MTH-5161-2 than in course MTH-5151-1.
MTH-5163-2	None	There is no corresponding course in the CST option.
From TS option	To S option	Adult learners earn the same number of credits in both options.
MTH-4261-2	MTH-4271-2	Knowledge covered in course MTH-4271-2 is also covered in course MTH-4261-2, except the knowledge related to **solving systems composed of a first-degree equation and a second-degree equation in two variables,** and certain knowledge related to **functions (interpreting additive parameters** and **switching from one form to another in writing second-degree polynomial functions)** and **completing the square**.
MTH-4262-2	MTH-4272-2	The knowledge covered in course MTH-4272-2 is also covered in course MTH-4262-2.
MTH-4263-2	MTH-4273-2	Knowledge covered in course MTH-4273-2 is also covered in course MTH-4263-2, except the following knowledge related to **metric and trigonometric relations in triangles**: • **sine law** • **cosine law** • **area and volume of figures**
MTH-5160-2	MTH-5170-2	The knowledge covered in course MTH-5170-2 is also covered in course MTH-5160-2.
MTH-5161-2	MTH-5171-2	Knowledge covered in course MTH-5171-2 is also covered in course MTH-5161-2, except **real numbers (absolute values, radicals, exponents and logarithms**), **manipulating arithmetic and algebraic expressions (laws of exponents, properties of radicals and equivalence between logarithmic and exponential expressions),** certain **functions (piecewise and absolute value)** and **finding the rule of a function or its inverse**.
MTH-5163-2	MTH-5173-2	The knowledge covered in course MTH-5173-2 is also covered in course MTH-5163-2, but the knowledge related to **geometric loci**, **trigonometric relations** and **vectors** is covered in more detail.

[3] Although some knowledge covered in course MTH-5151-1 is not covered in course MTH-5161-2, as the latter covers more knowledge related to algebraic and graphical modeling, the adult learner is not obliged to take course MTH-5151-1.

Successfully completed course	Course with a similar content	Clarifications
From S option	To CST option	Adult learners earn 8 credits in the CST option and 12 in the S option.
MTH-4271-2	MTH-4151-1	The knowledge covered in course MTH-4151-1 is also covered in course MTH-4271-2.
MTH-4272-2	MTH-4152-1	Knowledge covered in course MTH-4152-1 is also covered in course MTH-4272-2, except **one-variable distributions**.
MTH-4273-2	MTH-4153-2	Knowledge covered in course MTH-4153-2 is also covered in course MTH-4273-2, except certain knowledge related to **metric and trigonometric relations in triangles** (**Hero's formula, area of a quadrilateral** and **coordinates of a point of division**).
MTH-5170-2	MTH-5150-2	Knowledge covered in course MTH-5150-2 is also covered in course MTH-5170-2, except the knowledge related to **graph theory and solving first-degree inequalities in two variables**.
MTH-5171-2	MTH-5151-1[4]	There is more prescribed knowledge covered in course MTH 5171-2 than in course MTH-5151-1.
MTH-5173-2	None	There is no corresponding course in the CST option.
From S option	To TS option	Adult learners earn the same number of credits in both options.
MTH-4271-2	MTH-4261-2	The knowledge covered in course MTH-4261-2 is also covered in course MTH-4271-2.
MTH-4272-2	MTH-4262-2	Knowledge covered in course MTH-4262-2 is also covered in course MTH-4272-2, except the knowledge related to **probability** and **one-variable distribution**.
MTH-4273-2	MTH-4263-2	Knowledge covered in course MTH-4263-2 is also covered in course MTH-4273-2, except the following knowledge related to **metric and trigonometric relations in triangles**: • **coordinates of a point of division** • **perpendicular bisector of a segment** • **areas of triangles, given the measure of an angle and the lengths of two sides or given the measures of two angles and the length of one side**
MTH-5170-2	MTH-5160-2	The knowledge covered in course MTH-5160-2 is also covered in course MTH-5170-2.
MTH-5171-2	MTH-5161-2	The knowledge covered in course MTH-5161-2 was covered in courses MTH-4271-2 and MTH-5171-2.
MTH-5173-2	MTH-5163-2	The knowledge covered in course MTH-5163-2 is also covered in courses MTH-4273-2 and MTH-5173-2.

[4] Although some knowledge covered in course MTH-5151-1 is not covered in course MTH-5171-2, as the latter covers more knowledge related to algebraic and graphical modeling, the adult learner is not obliged to take course MTH-5151-1.

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