Course

MTH-4272-2

Data Collection in a Fundamental Context

Mathematics

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INTRODUCTION

The goal of the *Data Collection in a Fundamental Context* course is to enable adult learners to deal with situations that involve collecting or processing data pertaining to a one- or two-variable distribution in a fundamental context.

In this course, adult learners continue to develop their statistical thinking skills by applying prior learning as well as the concept of dispersion to the study of two-variable statistical distributions. In some situations, they learn about measurement and measurement errors by collecting data related to a two-variable distribution. Elsewhere, they compare the data collected in different situations. To determine the rule that best corresponds to the analysis of their results, they represent the data in a contingency table or a scatter plot, and use the concept of linear correlation to verify the strength of the relationship between two quantities. Analyzing the strength of the relationship makes it possible to describe and characterize the correlation in a qualitative fashion (perfect, strong, weak, zero, positive, negative). In situational problems, adult learners must validate and correct the solutions as needed, and organize and interpret statistical data in order to be able to represent it using linear correlation or second-degree polynomial functions. They become aware that handling and measurement errors affect the results of experiments, and that the resulting graphs are not always “perfect” curves. By analyzing various situations or conducting experiments, they learn that a mathematical model, such as a function, can be associated with a scatter plot. Situations involving the concept of correlation help adult learners develop a line of reasoning which, supported by an understanding of dependency relationships and a capacity for abstraction, lead them to recognize cause-and-effect relationships.

By the end of this course, adult learners will be able to collect and compare data pertaining to one- or two-variable distributions in order to answer a question related to a problem that they themselves have defined. They will present the results of their analysis in accordance with the rules and conventions of mathematics. They will use problem-solving strategies to determine the most efficient solution. In addition, by conducting an experiment using technological tools, they will be able to test the way that they use statistical analysis to deal with a situation.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult

learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

PROCESS AND STRATEGIES

REPRESENTATION

- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. They use observational and representational strategies that are essential to inductive reasoning.
- In attempting to understand the context and the problem, they use deductive reasoning, particularly in situations that involve implicit data.
- When they study a case of correlation using a particular model, they determine the parameters needed to find the algebraic rule or to draw the graph.

Examples of strategies	• Describing the situation in their own words and comparing their understanding of the problem with that of their classmates or teacher • Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to find a relationship between the variables when looking for a correlation using the model best suited to the problem • Listing their statistical and probability-related strategies and knowledge pertaining to the situation • Describing the characteristics of the situation • Gathering relevant information

PLANNING

- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient.
- Reasoning enables them to establish organized and functional relationships among different aspects of their knowledge when they attempt to extrapolate results using an algebraic rule or a correlation graph.
- They find the elements needed to transpose data from one register of representation to another, for instance when switching from a statistical distribution to a stem-and-leaf plot, and vice versa.

Examples of strategies	• Systematically determining the correlation model best suited to the situation, bearing in mind the limitations regarding the model's precision • Finding an effective proof method to compare two correlation models

ACTIVATION

- When dealing with a situational problem, adult learners establish organized and functional relationships among different aspects of their knowledge, for example, when checking and describing the correlation between two variables.
- They use different strategies by associating pictures, objects or concepts with mathematical terms and symbols and by switching from one register of representation to another.

Examples of strategies	• Using a table to connect the properties of the correlation • Using relevant data, drawing the functional model best suited to the situation • Using technology to analyze the role of the different parameters of the rule of the correlation line or another model

REFLECTION

- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
- Reasoning can be used to reject extrapolations that would yield nonsensical results.
- Adult learners hone their ability to use exact mathematical language, especially when producing a message.

Examples of strategies	• Comparing their results with the expected results and those of others • Checking their solution by making sure, for example, that the resulting values satisfy the range of the function in the case of a correlation • Determining the strategies used to deal with the situation

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Processing data*. Two of these are considered particularly relevant to this course: *Adopts effective work methods* and *Exercises critical judgment*.

Methodological Competency

Presenting a statistical profile requires organized planning to avoid possible sources of bias. Adult learners must therefore conduct their study by observing the criteria for ensuring the validity of the data they collect. They must examine the data rigorously to ensure that their interpretation and analysis reflect the reality of the situation and are not influenced by prejudice or misconceptions. They must properly plan the tasks to be performed and carry them out in a logical order. They allot the time needed to complete these tasks. The competency *Adopts effective work methods* is very useful for this course, particularly for adult learners who wish to go on to study science or eventually go into research.

Intellectual Competency

The media discuss statistical studies and report on situations that must be analyzed with a certain degree of objectivity to be able to separate fact from opinion. Bearing in mind that a study is reliable only if it is objective, adult learners must determine the motivation behind the decision to promote one type of information over another. The competency *Exercises critical judgment* is very useful, for example, when interpreting the correlation between two variables and determining how they are actually related.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of statistics. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following three integrative processes:

- **collecting data**
- **comparing collections of data**
- **interpreting data resulting from an experiment**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover all three processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Two-variable distribution	
• Constructing and interpreting two-variable distributions	
• Drawing a scatter plot	
• Representing and determining the equation of the regression line	
• Interpolating or extrapolating using the regression line	
• Interpreting a correlation qualitatively and quantitatively	The characteristics of a correlation are: positive, negative or zero; perfect, strong, moderate or weak. Interpretation of the correlation is limited to cases involving linear correlations, which can be estimated using a graphical method (box method or ellipse). The exact value of the correlation coefficient is determined using technology.

Mathematical Knowledge	Restrictions and Clarifications
Two-variable distribution (cont.)	
• Interpolating and extrapolating using the functional model best suited to the situational problem	The functional models studied in this course are those covered in the *Algebraic and Graphical Modelling in a Fundamental Context 1* course.

Cultural References

From the first rudimentary livestock inventories recorded in the 23rd century B.C.E. in China and the 18th century B.C.E. in Egypt, statistics remained a simple data-collecting system until the 17th century C.E. It finally came into its own in the 19th century with the formulation of precise rules for collecting and interpreting data.

In the 20th century, industrial applications were developed first in the United States and then in Europe after the First World War. Computerization made it possible to cross-reference different types of data sequences. Methods became more sophisticated, and more and more studies were conducted. Today, statistics is a science in and of itself. Canada has become world-renowned for advances in this discipline and its applications in fields such as science, technology, business and public administration. Canada has played a leading role in this area thanks to the quality and commitment of its researchers. The decision to locate the headquarters of the UNESCO Institute for Statistics in Montréal was motivated by our country's excellent reputation in this field. This institute is responsible for collecting and publishing statistics related to science, technology and education for the United Nations.

Today, the world of statistics is no longer the preserve of a small group of experts: it is accessible to everyone thanks in large part to the Internet. Adult learners should now be able to understand statistics that are of public interest such as all the data that Statistics Canada produces annually for Canadian citizens. Depending on their interests and on the needs of the course, adult learners can check whether or not there are causal links between two variables in a given area.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Processing data* involve problems that can be solved in part by collecting or processing data in a fundamental context. The *Data Collection in a Fundamental Context* course provides adult learners with an opportunity to learn how to collect and compare data.

In the situational problems in this course, adult learners decode the meaning of mathematical symbols, terms and notation, interpret codes and rules to distinguish between the probability and odds of winning an amount of money in a game of chance, and correctly interpret the intensity and sign of the correlation coefficient.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Health and Well-Being, and Career Planning and Entrepreneurship.

Health and Well-Being

Many situations that involve statistics can be explored while bearing in mind the educational aim of this broad area of learning. For example, adult learners can monitor their own progress in a structured fitness program aimed at achieving optimal health. This exercise could motivate them to make sound decisions with respect to their health and to adopt healthy lifestyle habits.

Career Planning and Entrepreneurship

Adult learners interested in scientific fields could conduct an experiment that would require them to interpret their results using the statistical methods and concepts learned in this course. The experimental and scientific approach involves compiling, analyzing and interpreting results. Drawing a scatter plot to represent data makes it possible to identify extreme or aberrant results and to find possible errors made in processing information. Exploring a situation related to their interests and aptitudes in order to master the strategies involved in carrying out a scientific experiment ties in with one of the focuses of development of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Health and Well-Being
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Processing data
Targeted cross-curricular competency • Is developed at the same time and in the same context as the subject-specific competencies.	• Exercises critical judgment
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	**Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Processing data* family of learning situations**
An adult learner realizes he has put on weight over the last few years because of his sedentary lifestyle. He decides to sign up at a gym in order to follow a structured training program. Before starting the program, he must undergo a physical fitness test to determine his endurance, muscle strength, flexibility, weight and height.	**Integrative process:** *Comparing collections of data* In carrying out the four phases in the problem-solving process, adult learners could: Representation • Decide which elements are relevant to their analysis • Make conjectures (e.g. Should there be a correlation between endurance and length of training, or between muscle strength and the type of exercise chosen?) Planning • Determine the information to be gathered (e.g. endurance, muscle strength, weight) • Periodically gather information about different exercises in order to conduct further analysis • Choose the most appropriate register of representation (e.g. the scatter plot)

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Processing data* family of learning situations	
After defining his goals (e.g. to build muscle mass, increase endurance or lose weight), he decides to determine which of the suggested exercises will be the most effective in helping him achieve his objectives.	Activation	• For a given exercise, check and describe the correlation between, for example, the following variables: - VO_2 max (maximum oxygen consumption) or number of heartbeats and the number of days spent training (variables related to endurance) - the maximum load and the number of days spent training (variables related to muscle strength) - the measurement of flexibility and the number of days spent training (variables related to flexibility) - weight and the number of days spent training (variables related to weight loss) • Use another register of representation for the data to better illustrate their conclusions
	Reflection	• Establish organized and functional relationships between such concepts as the intensity and sign of the correlation coefficient and determine the correlation line • Compare their results with those of their classmates to determine other factors that could have been taken into account • Make sure that their solution makes sense

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Processing data*, adult learners collect, compare and interpret data resulting from a random experiment. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems, Uses mathematical reasoning* and *Communicates by using mathematical language*.

To collect data, adult learners use problem-solving strategies to define the problem and identify the tasks involved. They determine the important elements of information and the obstacles to be overcome in order to study two-variable statistical distributions. In working out their solution, they establish and carry out a plan involving the previously validated steps: data collection and processing (interpretation and analysis). This last step requires that they use mathematical reasoning, explore the problem in question and identify patterns. Adult learners make conjectures using a correlation line or curves in order to make decisions in the medium and long term. They draw conclusions based on laws or rules related to the properties of the functions in question. Lastly, they produce mathematical messages, using the appropriate register of representation given the constraints of the situational problem. In other words, they choose the functional model best suited to the situation.

To compare collections of data, adult learners interpret mathematical messages by making connections between the elements of the message, determining its overall meaning, or associating pictures, objects or knowledge with mathematical terms and symbols. In addition, they use mathematical reasoning by developing and using networks of cognitive resources in order to compare trends, for example the rate of change, the rate of growth, the correlation coefficient, or any other characteristic of the functions covered in this course.

To interpret data resulting from an experiment in order to study the correlation between two quantities, adult learners decode the elements of mathematical language, distinguishing between the mathematical and everyday meanings of terms. In addition, they interpret mathematical messages, distinguishing between elements that are relevant and those that are not, and recognizing the purpose of the message. They use mathematical reasoning by developing and using networks of cognitive resources, such as a correlation line or a functional model adapted to the situation in the case of a two-variable statistical distribution. They generalize, derive laws and rules, and deduce propositions to help them make informed decisions.

Throughout the problem-solving process, adult learners apply their knowledge of functions to their study of two-variable statistical distributions. Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

* The solution includes a procedure, strategies and a final answer.
** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*