Course

MTH-4153-2

Geometric Representation in a General Context 1

Mathematics

MTH-4153-2 Geometric Representation in a General Context 1

INTRODUCTION

The goal of the *Geometric Representation in a General Context 1* course is to enable adult learners to use trigonometry to deal with situations that involve the geometric representation of an object or a physical space in a general context.

In this course, adult learners encounter various situational problems that enable them to expand their knowledge of geometry, and trigonometry in particular. They use and improve their knowledge of plane figures, solids, isometries, similarity transformations and projections, and apply and develop their ability to represent and construct figures or determine and deduce measurements. In some situational problems, they must find unknown measurements (length, area, volume) using different metric or trigonometric relations and the properties of right triangles or congruent, similar or decomposable figures. In addition, they validate certain conjectures through simple deductive reasoning based on their knowledge of geometry. They justify their choices and the steps in their process. Lastly, in communicating using mathematical language, they identify and interpret different measurements that they themselves or others have taken, or information contained in drawings and geometric constructions. In geometric contexts, they decode the information in a geometric figure or the elements of the construction of an object based on a two-dimensional representation. They also use their spatial sense, as well as their knowledge of measurement and proportionality, to describe this representation and interpret geometric figures in contexts involving the concepts of similarity or trigonometry.

By the end of this course, adult learners will be able to use the properties of congruent or similar figures and trigonometric relations to represent and describe an object or a physical space. They will also be able to use different strategies and types of reasoning to manage various situations in accordance with the mathematical rules and conventions used in geometry.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

PROCESS AND STRATEGIES	
REPRESENTATION	
- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. They use observational and representational strategies that are essential to inductive reasoning. - In describing the characteristics of the situational problem, they organize the elements that will enable them to plan the main steps in the deductive reasoning they will use to determine whether or not they are dealing with a case of similarity, and master the elements of mathematical language.	
Examples of strategies	• Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to identify a dependency relationship and determine the variables in the situation • Representing the situational problem mentally or in writing • Listing their geometry-related strategies and knowledge pertaining to the situation • Describing the characteristics of the situation • Gathering relevant information
PLANNING	
- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient. - They develop a plan, taking into account the elements of mathematical language (key elements, subject of the message, overall meaning of the situation). - They use different registers of representation to illustrate certain properties of trigonometric ratios.	
Examples of strategies	• Dividing the situational problem into subproblems • Using lists, tables, diagrams, concrete materials or drawings to plan their solution
ACTIVATION	
- When dealing with a situational problem, adult learners identify patterns by exploring different figures. - They also observe mathematical codes, symbols and rules.	
Examples of strategies	• Solving certain situational problems by working backwards when the solution consists of several steps or when there is insufficient information • Analyzing the parameters of a right triangle, for example, to properly understand how they are related to the parameters of any given triangle
REFLECTION	
- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution. - They validate their message using new mathematical symbols to describe the organization or representation of their physical environment, consulting different references in the process. - Validating certain results could involve making conjectures about particular or special cases involving any triangle in order to see the effect on the area or the perimeter of a triangular shape when the angle varies in the cosine law or the sine law formula.	
Examples of strategies	• Checking their solution by means of examples or counterexamples, particularly by using the Pythagorean theorem to validate the lengths of the sides of a triangle to be able to conclude that it is, in fact, a right triangle • Making sure that their solution makes sense, for example, by using the sine law to determine whether an angle of any given triangle is acute or obtuse • Using a calculator or geometric modelling software to validate their work

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Measurement and spatial representation*. Two of these are considered particularly relevant to this course: *Uses creativity* and *Adopts effective work methods*.

Intellectual Competency

Creative individuals find imaginative ways of using the resources and materials available to them. Representing physical spaces and organizing a layout often call for varied and personal approaches, which could lead adult learners to express their ideas and use their intuition. For example, they could determine how they will go about drawing a scale diagram of a room or a piece of land. Since objects are sometimes out of reach and therefore difficult to measure, adult learners must find an innovative and feasible way of measuring them. In this course, the competency *Uses creativity* enables them to explore situational problems in original ways.

Methodological Competency

The competency *Adopts effective work methods* can be developed in dealing with a situational problem that involves the organization of a physical space. For example, a great deal of rigour and precision is required to design a scale model that accurately represents the object in question. In this regard, it is crucial to choose the right measuring instruments. Adult learners take into account all the constraints and analyze their consequences. To see their project through to completion, they carefully plan it and manage their time efficiently. In this way, they use a methodical approach suited to the given context.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of geometry. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following two integrative processes:

- **organizing a physical space**
- **describing an object or a physical space and representing it in two or three dimensions**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover both processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Metric and trigonometric relations in triangles	
• Representing and interpreting situations using triangles	The trigonometric ratios studied are: sine, cosine and tangent. The sine law and Hero's formula are also studied in this course. The other metric and trigonometric relations are listed in the Principles table that comes after this table.
• Describing the properties of trigonometric ratios	Adult learners intuitively use the properties of trigonometric ratios to justify the steps in their solution, but they are not required to prove these properties.
• Determining the slope, measurements and positions using metric and trigonometric relations in triangles	The measurements and positions studied in this course pertain to: • the angles in a triangle • the altitude to the hypotenuse, the orthogonal projection of the legs on the hypotenuse • the sides of a triangle • the area of a triangle and a quadrilateral • the coordinates of a point (point of division) • the length of a segment • distance (between two points)

Mathematical Knowledge	Restrictions and Clarifications
Similar and congruent triangles	
• Determining the minimum conditions required to conclude that triangles are congruent or similar	These conditions are listed in the Principles table that comes after this table.

Principles
Adult learners must master the following compulsory principles, which may be used in a proof:
P1. If the corresponding sides of two triangles are congruent, then the triangles are congruent.
P2. If two sides and the contained angle of one triangle are congruent to the corresponding two sides and contained angle of another triangle, then the triangles are congruent.
P3. If two angles and the contained side of one triangle are congruent to the corresponding two angles and contained side of another triangle, then the triangles are congruent.
P4. If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the triangles are similar.
P5. If the lengths of the corresponding sides of two triangles are in proportion, then the triangles are similar.
P6. If the lengths of two sides of one triangle are proportional to the lengths of the two corresponding sides of another triangle and the contained angles are congruent, then the triangles are similar.
P7. In a right triangle, the length of the side opposite an angle of 30° is equal to half the length of the hypotenuse.
P8. The lengths of the sides of any triangle are proportional to the sines of the angles opposite these sides: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ (sine law).
P9. The area *A* of a triangle whose sides measure a, b, and c is: $A = \sqrt{p(p-a)(p-b)(p-c)}$ where p is half the perimeter of the triangle (Hero's formula).
P10. The length of a leg of a right triangle is the geometric mean between the length of its projection on the hypotenuse and the length of the hypotenuse.
P11. The length of the altitude to the hypotenuse of a right triangle is the geometric mean between the lengths of the segments of the hypotenuse.
P12. The product of the lengths of the legs of a right triangle is equal to the product of the length of the hypotenuse and the length of the altitude to the hypotenuse.

Cultural References

Geometric figures are found everywhere in our environment, both in human creations (e.g. works of art, various objects, fabrics, wallpaper, architecture, structures) as well as in nature (e.g. crystallography, trajectories).

A number of mathematicians, such as Archimedes, Hero of Alexandria, Galileo and Leonardo da Vinci, designed machines, tools and measuring instruments, some of which are still used today. Adult learners can identify properties of measuring instruments used in drawing, navigation, geodesy or astronomy. They can appreciate how a number of instruments (e.g. balance, odometer, global positioning system, compass, sextant, quadrant) used today or in the past have helped solve real-world problems. Furthermore, surveying equipment, the mirror and shadow technique, the pantograph, the proportional compasses, and Jacob's and Gerbert's staffs can help adult learners develop their understanding of the concept of similarity.

Different branches of geometry have been developed to address various questions and needs. One of the most recent types of geometry to emerge is fractal geometry, which is used to model a variety of things including different natural occurrences such as atmospheric phenomena, floral patterns and geographical features. It is used in the arts and in digital imaging. Examples given during the course will enable adult learners to understand its importance.

FAMILY OF LEARNING SITUATIONS

The situations in the *Measurement and spatial representation* family of learning situations involve problems that can be solved in part through the geometric description or representation of an object or a physical space. The *Geometric Representation in a General Context 1* course provides adult learners with an opportunity to develop their spatial representation skills.

In the situational problems in this course, adult learners determine the corresponding sides and angles of two triangles, identify the corresponding sides of two similar triangles by recognizing common identification codes, and validate their message using new mathematical symbols to describe an organization or representation of their physical environment.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Citizenship and Community Life, and Health and Well-being.

Citizenship and Community Life

During a meeting of the board of their local community centre, adult learners who wish to be active in their community could present a development plan for a playground. The geometry concepts acquired in this course could help them complete this project, which ties in with one of the focuses of development of this broad area of learning.

Health and Well-Being

This course will prove useful for adult learners who spend long hours at their computer, whether it be for study, work or pleasure. Work space arrangement could help them adopt a better, more comfortable posture. Adults learn about certain ergonomic concepts related to computer use (e.g. proper posture in front of the computer, the right distance between the eyes and the screen, the proper angle for the arms and legs). To rearrange their work space and office, they determine how much distance there should be between themselves and their tools and where their tools should be positioned, depending on whether they are right-handed or left-handed. They also calculate the area of their work space and determine the type of lighting that will make it as easy as possible to read the screen. The geometry concepts studied in this course can help them plan the layout of the available space, which will prompt them to be more concerned with safety in accordance with one of the focuses of development of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Citizenship and Community Life
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Measurement and spatial representation
Targeted cross-curricular competencies • Are developed at the same time and in the same context as the subject-specific competencies.	• Uses creativity • Adopts effective work methods
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	**Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Measurement and spatial representation* family of learning situations**
A local community organization asked the parents of the children who attend the centre to find ways to decrease violent behaviour in the neighbourhood. Since an analysis of the situation showed that there are too few play areas in the neighbourhood, it was recommended that a playground be built.	**Integrative process:** *Describing a physical space and representing it in two dimensions* In carrying out the four phases in the problem-solving process, adult learners could: Representation • Make note of the relevant information about the space to be used for the playground: dimensions, irregular shapes, whether or not there is a fence, proximity to the street, etc. Planning • Describe the available space, make a sketch of the lot and use appropriate language • Choose the scale to be used to produce a clear and sufficiently detailed plan

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Measurement and spatial representation* family of learning situations	
The community organization has the funds to pay for materials, provided the work is done on a volunteer basis. Adult learners are asked to draw up the layout plan for the playground.	Activation	• Calculate the available surface areas for each play structure • Calculate the height of the structures, the distance between them, the angle of elevation of the slides, etc.
	Reflection	• Validate the resulting measurements by means of trigonometric ratios other than those used to find the measurements

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Measurement and spatial representation*, adult learners describe and represent objects and physical spaces in two or three dimensions, and organize a physical space. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To describe an object or a physical space and represent it in 2-D or 3-D, adult learners interpret and produce sketches, drawings or plans using different relations associated with geometric figures. In situational problems, they find measurements (length, area, volume) using different metric or trigonometric relations involving right triangles or congruent, similar or decomposable figures. They deduce properties based on their knowledge of geometry and validate their conjectures, justifying all the steps in their process. In addition, in producing a mathematical message, they identify the key elements of mathematical language (e.g. scale, dimensions, perimeter, area, volume) and associate images, objects or knowledge with mathematical terms and symbols. They apply their newly acquired knowledge (Hero's formula and the sine law), which enables them to find measurements in unusual situations.

To organize a physical space, adult learners use a variety of strategies such as producing a sketch or drawing, or dividing the task into subtasks. They use a complex process that includes representing the problem and validating their solution based on their knowledge of trigonometry. They use the concept of triangulation to organize a physical space and validate all of their steps using the theorems covered in the course. They find measurements using deductive reasoning, identify results using inductive reasoning, and draw conclusions based on their study of the theorems in question.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (trigonometric and metric relations in triangles, and similar and congruent triangles). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

* The solution includes a procedure, strategies and a final answer.
** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*