Course

MTH-4151-1

Algebraic and Graphical Modelling in a General Context 1

Mathematics

INTRODUCTION

The goal of the *Algebraic and Graphical Modelling in a General Context* course is to enable adult learners to deal with situations that involve using an algebraic or graphical model to represent a dependency relationship between quantities in a general context.

In this course, adult learners deal with various situational problems to expand their knowledge of algebra. In studying real functions, they learn to characterize the different types of dependency relationships between two quantities. They explore situations that do not necessarily involve linear relationships, such as exponential, periodic, quadratic and step models. They observe patterns and distinguish between linear growth (arithmetic progression) and exponential growth (geometric progression) in situations involving population growth, for example. In addition, they make conjectures based on functions, and validate them using different types of reasoning involving direct or indirect proofs, or refute them using counterexamples. In situational problems, adult learners identify and use relevant information. For example, in a situation that involves planning the purchase of goods and services, this information can be presented verbally, algebraically, graphically or in a table of values. Adult learners must analyze different possibilities and make decisions. They must also use their number and operation sense and proportional reasoning to validate their solutions. In addition, they must solve systems of linear equations to compare and analyze phenomena in order to make choices. They can interpolate or extrapolate from models, using real functions expressed in different forms. In terms of communication, they analyze graphs and tables of values in order to identify specific information and draw conclusions. In other situations, they transpose a verbal or written description using one or more algebraic expressions (equation, inequality, system of linear equations or function). They use numbers written in different notation, taking the units into account where applicable. They may have to describe a situation expressed as a graph or table of values.

By the end of this course, adult learners will be able to use algebra to represent concrete situations. They will produce clear and accurate work in accordance with the rules and conventions of mathematics. By algebraically or graphically representing a situation using real functions and their inverse, they will be able to deduce results through interpolation or extrapolation. In addition, adult learners will use different registers of representation to generalize a model so it can be applied to a range of situations.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

PROCESS AND STRATEGIES	
REPRESENTATION	
- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. - They use observational and representational strategies that are essential to inductive reasoning. - In attempting to understand the context and the problem, they use deductive reasoning, particularly in situations that involve implicit data.	
Examples of strategies	• Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to identify a dependency relationship and determine the variables in the situation • Using examples involving numbers, determining the types of relationship that exist between the variables in the situation • Listing their algebraic strategies and knowledge pertaining to the situation • Describing the characteristics of the situation
PLANNING	
- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient. - They develop a plan, taking into account the elements of mathematical language (symbols, terms and notation used, and the different registers of representation). - Through reasoning, they establish organized and functional relationships among different aspects of their knowledge, thus expanding their networks of cognitive resources.	
Examples of strategies	• Systematically determining the functional model best suited to the situation, bearing in mind the limitations regarding the model's precision • Finding an algebraic rule that reflects the best relationship between the constraints and possible consequences of the situational problem
ACTIVATION	
- By making connections between algebraic and graphical representations in studying a system of equations, adult learners can derive rules and conditions that determine the number of solutions for the system and make generalizations. - By drawing on their knowledge of the properties of functions in dealing with a situation, they are able to deduce certain relationships (e.g. between the extrema and the optimal value, between the increasing interval of the function and the growth of the company).	
Examples of strategies	• Carrying out a simulation using concrete objects or technology to determine a relationship • Using technology (e.g. spreadsheet program, graphing calculator) to analyze the role of the different parameters of a function • Using the parameters of a function to make a sketch in order to predict results
REFLECTION	
- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution. - Reasoning can be used to reject extrapolations that would yield nonsensical results.	
Examples of strategies	• Comparing their results with the expected results and those of others • Checking their solution by, for example, making sure that the resulting values satisfy the range of the function • Using a set of metacognitive questions such as, Why did I proceed in this way? What would I change and why? • Using a calculator to validate their work.

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Relationship between quantities*. Two of these are considered particularly relevant to this course: *Exercises critical judgment* and *Uses information and communications technologies*.

Intellectual Competency

Many companies seek to attract customers and develop their loyalty by offering plans based on a contract that lasts one, two, three or more years. These offers sometimes seem unbeatable, and adult learners could rush to benefit from these plans for fear of missing out on a unique opportunity. By learning to use an algebraic model to extrapolate in this situation, they would be better able to critically assess the choices offered. In rigorously analyzing and comparing the available options, they could become more objective and more realistic about the long-term costs involved. By developing the competency *Exercises critical judgment*, adult learners could take a more thoughtful approach before signing a contract that will lock them in for many months.

Methodological Competency

Representing a functional model involving any kind of complexity could be made easier through the use of special software. In order to see their task through to completion, adult learners could develop the competency *Uses information and communications technologies* to create and manipulate graphs by modifying certain parameters. For example, it would be easier to compare different cooking methods, since simulation would save on the time involved in trial-and-error experimentation. Using technology, adult learners could find a model more quickly and focus on analyzing and justifying it rather than on performing algebraic calculations.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of arithmetic and algebra. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following three integrative processes:

- **using an algebraic or graphical model to represent a situation**
- **interpolating or extrapolating from an algebraic or graphical model**
- **using an algebraic or graphical model to generalize a set of situations**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover all three processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Relation, function and inverse	
• Experimenting with real functions as well as observing, interpreting, describing and representing them	The real functions studied are: • second-degree polynomial function $f(x) = ax^2$ • exponential function $f(x) = ab^x$, where $a \neq 0$ and $b > 0$ • periodic function • step function • piecewise function Functions can be represented using: • a table of values • an algebraic rule • a graph, with or without the use of technology *For periodic, piecewise and step functions, graphical representations of the context are emphasized even if, in some cases, the symbolic register could be used.* *In determining the value of the exponent in an exponential function, adult learners use a graph, a table of values or technological tools.*

Mathematical Knowledge	Restrictions and Clarifications
Relation, function and inverse (cont.)	
• Describing and interpreting the properties of real functions using a graph	The properties of real functions studied are: • domain and codomain (range) • increasing and decreasing intervals • extrema • sign • x- and y-intercepts The properties of functions must be studied in context.
System	
• Representing a situation using straight lines	The properties of the following lines are studied: • parallel lines • intersecting lines • coincident lines • perpendicular lines The equation of the line in standard form: • f(x) = ax + b *The symmetric and general forms of the equation of a line are not covered in the* Cultural, Social and Technical *option.*
• Solving systems of first-degree equations in two variables	Systems of equations may be solved by means of: • a table of values • an algebraic method chosen by the adult learner • a graphical method, with or without the use of technology

Cultural References

Proportional reasoning is used in everyday life and in different occupations in fields such as construction, the arts, health care, tourism and business administration. Observations of the dependency relationship between two quantities contributed to the development of the concept of function, which is used in such areas as navigation, astronomy and ballistics. Adult learners will have the opportunity to discover its importance and to appreciate how different mathematicians, such as Oresme, Descartes and Fermat, contributed to the development of this concept.

More recently, Thomas Malthus analyzed arithmetic and geometric progressions in his work on population growth and the growth of the food supply. With this idea in mind, adult learners could be asked to make observations, comparisons or decisions regarding different instances of growth and decline in fields such as demography and finance.

Today, scuba divers quickly learn the principle of decompression stops in order to prevent any nitrogen that accumulated in the body during the dive from forming bubbles in their blood if they come up to the surface too rapidly. Divers learn their decompression tables by heart. Adult learners who are interested in this sport and in safety could study the related principles through modelling and graph the duration of the decompression stops as a function of the depth of the dive.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Relationship between quantities* involve problems that can be solved in part by using an algebraic or graphical model to represent a relationship between quantities in a general context. The *Algebraic and Graphical Modelling in a General Context 1* course gives adult learners an opportunity to express a connection or a dependency relationship between quantities.

In the situational problems in this course, adult learners derive rules and conditions that determine the number of solutions for the system and make generalizations, they find connections between algebraic and graphical representations in studying a system of equations, and they attempt to extrapolate results using an algebraic rule or a graph.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Environmental Awareness and Consumer Rights and Responsibilities, and Career Planning and Entrepreneurship.

Environmental Awareness and Consumer Rights and Responsibilities

Studying certain functions could help adult learners calculate the real cost of a cell phone and select the best plan, depending on how they intend to use their phone. The course content could help them make better decisions based on their needs and budget. Adult learners could therefore learn to critically assess the choices they make as consumers, which ties in with the educational aim of this broad area of learning.

Career Planning and Entrepreneurship

The content of this course could prove useful in conducting a feasibility study prior to organizing a student activity that involves a certain investment. An analysis of linear functions would make it easier to understand the concepts of break-even point (zeros of the function) and increasing and profitability interval (rate of change). Adult learners could use a greatest-integer function to graph economies of scale associated with purchasing or leasing materials. They could use the concepts learned in this course to master the strategies that apply to their project, which ties in with one of the focuses of development of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Environmental Awareness and Consumer Rights and Responsibilities
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Relationship between quantities
Targeted cross-curricular competency • Is developed at the same time and in the same context as the subject-specific competencies.	• Exercises critical judgment
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Relationship between quantities* family of learning situations
If an adult learner wants to have a cell phone, this course could enable him or her to compare different plans and determine the one that is most suitable or worthwhile if the phone is used for a certain number of minutes.	**Integrative processes:** *Using an algebraic or graphical model to represent a situation* *Interpolating or extrapolating from an algebraic or graphical model* In carrying out the four phases in the problem-solving process, adult learners could: Representation • Select the most appropriate model (algebraic or graphical representation) and determine the point when a plan, which seemed less attractive at the outset, becomes more worthwhile than another Planning • Use algebraic representation to compare the monthly cost of using a given set of telephone services, through interpolation or extrapolation, as applicable

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the ***Relationship between quantities*** family of learning situations	
	Activation	• Establish the algebraic rule that relates the various elements of the situation (the number of minutes being the independent variable and the rate of change corresponding to the rate per minute of use) to be able to extrapolate the cost of each plan for a given number of minutes of use • Make a conjecture about the various plans, taking into account the case when the time limits for each plan are exceeded, then verify the conjecture algebraically or graphically • Use a graphing calculator to compare plans and determine the best one, or determine the point at which the plan becomes worthwhile
	Reflection	• Conclude that, for extreme values, graphical extrapolation is not very realistic • Validate a graphical extrapolation through algebraic calculation • Decide that it is better not to buy a cell phone and justify their decision mathematically

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Relationship between quantities*, adult learners represent a situation, interpolate or extrapolate, and use an algebraic or graphical model to generalize a set of situations. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language.*

To represent a situational problem using an algebraic or graphical model, adult learners employ different strategies to identify the problem. They reformulate the situational problem in their own words and determine the key elements and the obstacles to be overcome. They find ways of illustrating the dependency relationship between two quantities, using tables of values, diagrams or Cartesian coordinate graphs. They choose the most accurate representation, aware that it does not necessarily reflect what they have observed, but that it is the best choice given the functions studied in the course. They validate their choice, comparing their solution against existing studies or experiments. For example, they may compare the mathematical model against the physical reality of the situation. In representing the situational problem, they determine the purpose of the message and observe mathematical codes and rules in order to effectively communicate their intention. They choose the register of representation best suited to the situation (e.g. table of values, Cartesian coordinate graph, algebraic equation, real functions covered in the course).

To make decisions, adult learners interpolate or extrapolate results from an algebraic or graphical model. They interpret the model, making connections between the elements of the message and distinguishing between those that are relevant and those that are not. They recognize the purpose of the message and determine its overall meaning. In addition, they use mathematical reasoning to explore the situational problem and to determine questions about the issue. They gather relevant information in order to draw a conclusion. They make one or more conjectures, suggesting probable or plausible ideas and anticipating the implications of these ideas as needed. They use examples to find invariants that will help them formulate their conjecture.

Mathematical reasoning involves using an algebraic model to generalize a set of situations. To do this, adult learners determine questions about the patterns observed. They gather relevant information about the relationships between the quantities (e.g. growth rate of exponential functions, height and length of the steps in a step function). They make conjectures, suggesting equations or formulas, or drawing sketches to identify invariants. They construct and use networks of cognitive resources in order to test their model. They draw conclusions and formulate them correctly, in accordance with the rules and conventions of mathematics. Their mathematical message is clear and accurate, and illustrates as effectively as possible the generalization of the set of situations using quadratic, exponential, step or other types of functions. They identify the weaknesses of the models, as well as the nuances between them and the reality observed.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (relation, function, inverse and systems of linear relations). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

* The solution includes a procedure, strategies and a final answer.

** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*