Course

MTH-3051-2

Algebraic and Graphical Modelling

Mathematics

INTRODUCTION

The goal of the *Algebraic and Graphical Modelling* course is to enable adult learners to deal with situations that involve using an algebraic or graphical model to represent a relationship between quantities.

In this course, adult learners deal with various situational problems to expand their knowledge of algebra. By solving problems, they become familiar with algebra as a generalization tool used to represent relationships between quantities on the basis of observed patterns. In these situational problems, adult learners must identify relevant information given verbally, algebraically, graphically or in a table of values. They must also interpolate or extrapolate from a model, using the functions covered in the course. Interpreting and representing a situation sometimes involves producing inverse models. In the case of first-degree polynomial functions and rational functions, adult learners compare the rules, graphs and verbal descriptions of the dependency relationship in question. Studying functions is an important part of the modelling process. In examining the graphic representation of an experiment, adult learners come to realize that, because of handling or measurement errors or the level of precision of the instrument used, the resulting data do not always form a curve that corresponds exactly to a given mathematical model.

By the end of this course, adult learners will be able to use algebra to represent concrete situations in accordance with the rules and conventions of mathematics. By algebraically or graphically representing a situation using a first-degree function or a rational function, they will be able to deduce results through interpolation or extrapolation. In addition, they will use different registers of representation to generalize a model so it can be applied to a range of situations.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly while observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

<table>
<tr><th colspan="2">PROCESS AND STRATEGIES</th></tr>
<tr><th colspan="2">REPRESENTATION</th></tr>
<tr><td colspan="2">– Adult learners examine the situational problem and represent it adequately by switching from one register to another.
– In attempting to understand the context and the problem, they use deductive reasoning, particularly in situations that involve implicit data.</td></tr>
<tr><td>Examples of strategies</td><td>• Identifying relevant information given verbally, algebraically, graphically or in a table of values
• Determining the nature of the task involved (e.g. instructions, expected results, goals, time allotted)
• Describing the situation in their own words and comparing their understanding of the problem with that of their classmates and teacher
• Determining the mathematical characteristics of the relationship described in the situation (e.g. x-intercept, y-intercept)</td></tr>
<tr><th colspan="2">PLANNING</th></tr>
<tr><td colspan="2">– Adult learners set priorities and identify the relevant resources.
– They look for ways of approaching the problem and choose those that seem the most efficient.
– They develop a plan, taking into account the elements of mathematical language (key elements, subject of the message, overall meaning of the situation).</td></tr>
<tr><td>Examples of strategies</td><td>• Using brainstorming techniques
• Dividing the situational problem into subproblems
• Systematically determining the functional model best suited to the situation, while taking into account the limitations of the model
• Finding an algebraic rule that reflects the best relationship between the constraints and possible consequences of the situational problem</td></tr>
<tr><th colspan="2">ACTIVATION</th></tr>
<tr><td colspan="2">– Adult learners follow the plan they have worked out and take into account the constraints involved.
– In developing their reasoning, adult learners propose probable or plausible ideas, anticipate the implications of these ideas and use examples to find invariants.</td></tr>
<tr><td>Examples of strategies</td><td>• Proceeding by trial and error to determine certain properties of the relation
• Dividing the situational problem into subproblems to work out a solution
• Making connections between the algebraic and graphical representation of a situation
• Illustrating a correlation graphically in order to confirm their intuition</td></tr>
<tr><th colspan="2">REFLECTION</th></tr>
<tr><td colspan="2">– Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
– Reflection helps them hone their ability to use exact mathematical language.</td></tr>
<tr><td>Examples of strategies</td><td>• Comparing their results with the expected results and those of others
• Finding examples and counterexamples
• Checking their solution by making sure that the resulting values satisfy the range of the function, and by substituting the values of the variables in the algebraic expression in order to validate a graphical interpolation or extrapolation</td></tr>
</table>

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Relationship between quantities*. Two cross-curricular competencies are considered particularly relevant to this course: *Adopts effective work methods* and *Communicates appropriately*.

Methodological Competency

The competency *Adopts effective work methods* is an essential tool for adult learners when they use an algebraic model to represent a situation. For example, being organized and using a structured approach can help them compare the costs of purchasing or leasing a car. In this regard, adult learners represent the costs of each option as accurately as possible and adapt their work methods to the type of information collected.

Communication-Related Competency

The competency *Communicates appropriately* is often used in establishing models. Modelling involves using mathematical language to represent a phenomenon or experiment and, by virtue of that fact, is a valuable tool for helping adult learners illustrate their ideas on a specific topic.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of arithmetic and algebra. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following three integrative processes:

- **using an algebraic or graphical model to represent a situation**
- **interpolating or extrapolating from an algebraic or graphical model**
- **using an algebraic or graphical model to generalize a set of situations**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover all three processes. The learning situations may be purely mathematical or based on everyday events.

Mathematical Knowledge	Restrictions and Clarifications
Inequality	
• Inequality relation	The relations studied are: • $a \leq b$, $a \geq b$, $a > b$ and $a < b$ such that a and b belong to the set of real numbers.
• Solving first-degree equations and inequalities in one variable	The inequalities studied are of the form: • $ax + b \leq cx + d$ • $ax + b \geq cx + d$ • $ax + b > cx + d$ • $ax + b < cx + d$ such that a and b belong to the set of real numbers.
Relation	
• Observing, describing, interpreting and representing the dependency relationship between the variables of a situation	The dependency relationship between the variables can be described and interpreted using the following registers of representation: • literal or verbal expression • algebraic rule • graph • table of values

Mathematical Knowledge	Restrictions and Clarifications
Relation (cont.)	
• Functions and inverse functions	Only polynomial functions of degrees 0 or 1 and rational functions are studied in this course: • constant function f(x) = b • linear functions f(x) = ax f(x) = ax + b • rational function of the form $f(x)=\frac{k}{x}$, where $k \in \mathbb{Q}_+$ • piecewise function (*In Secondary III, adult learners are informally introduced to this function.*)
• Drawing a scatter plot representing an experiment or a statistical study	*It should be noted that a scatter plot is used only to illustrate the relationship between the variables and that adult learners could use a linear or rational function to represent any dependency relationships. The scatter plot is only an approximation, since in this course adult learners are not required to determine the correlation coefficient or the linear regression line.*
• Representing and interpreting the inverse of a function	The inverse of a function (linear or reciprocal) can be represented or interpreted using the following registers of representation: • literal or verbal expression • algebraic rule • graph • table of values
▪ Determining the rule of correspondence	The following information could be used to derive the rule: • an ordered pair and the rate of change • two ordered pairs Certain values will be determined graphically or by using the rule, with the degree of precision required by the context.

Mathematical Knowledge	Restrictions and Clarifications
Relation (cont.)	
▪ Describing the properties of a function in context	The properties of functions studied are: • domain and codomain (range) • increasing and decreasing intervals • extrema • sign • x- and y-intercepts Adult learners derive the properties in an informal manner and always in relation to the context.
• Providing a qualitative description of how the graph is affected by a change in the value of a parameter of a linear function	*Parameters* a *and* b *are never changed at the same time. In this course, adult learners analyze how a change in parameter* a *or parameter* b *of the linear function affects the graph, but each parameter is studied separately.* *In Secondary III, adult learners are informally introduced to the study of properties.*
System	
• Solving systems of first-degree equations in two variables	The equations must be of the form $y = ax + b$ and may be solved: • using a table of values • graphically • algebraically (using the comparison method)

Cultural References

Algebra is an ancient discipline that dates back to the Babylonians, but it only came into its own in the West during the Renaissance. Adult learners could discover that the word “algebra” originated in the ninth century and comes from the Arab mathematician Al-Khwarizmi who used the term “al-djabr” to refer to a calculation that involved adding the same number to both members of an equality. His work on the decimal number system and on solving first- and second-degree equations led to the development of the algebraic processes used today.

Algebra is widely used in the modern world. Adult learners could learn that simple yet ancient algebraic processes like proportional reasoning are commonly used in fields such as health care and construction. With time, algebraic symbolism became standardized and more complex. In addition, algebra has become an intrinsic part of such branches of mathematics as geometry, function theory and logic. When observing and analyzing different phenomena before making decisions, financial experts and demographers use algebra, among other tools. In the modern world, algebra has become indispensable in a number of fields.

Professionals in the field of pharmacology have long applied proportional reasoning to determine the dosage for different types of medication on the basis of a patient’s age or weight. Adult learners interested in public health issues and, more specifically, in the safe consumption of medication, could study how to apply proportional reasoning to determine, for example, the number of tablets that a person may take to treat certain symptoms.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Relationship between quantities* involve problems that can be solved in part by using an algebraic or graphical model to represent a relationship between quantities. The *Algebraic and Graphical Modelling* course provides adult learners with an opportunity to establish connections or dependency relationships between quantities.

In the situational problems in this course, adult learners select relevant information in order to establish relationships between two elements; they represent, graphically, algebraically or using a table of values, the inverse of a previously determined function; and they describe how a change in one of the parameters will affect this graph.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Environmental Awareness and Consumer Rights and Responsibilities, and Health and Well-Being.

Environmental Awareness and Consumer Rights and Responsibilities

Integrating the broad area of learning Environmental Awareness and Consumer Rights and Responsibilities can prove useful, especially in situations in which adult learners must compare types of investments and loans, as well as purchase or lease options for goods and services. Using a model to represent their finances, adult learners could make certain extrapolations based on their model in order to make informed consumer choices.

Health and Well-Being

The algebraic concepts studied in this course could help adult learners reflect on healthy lifestyle habits. In analyzing situational problems that focus on specific relationships between various aspects of health, adult learners can become aware of certain factors that will harm or benefit their health. They can then anticipate the health impact of certain decisions regarding nutrition and physical activity. This awareness is consistent with the educational aim of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning • Helps contextualize learning and makes it meaningful.	• Environmental Awareness and Consumer Rights and Responsibilities
Prescribed subject-specific competencies • Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations • Consists of real-life situations applicable to a given course. • Helps adult learners acquire mathematical knowledge.	• Relationships between quantities
Targeted cross-curricular competencies • Are developed at the same time and in the same context as the subject-specific competencies.	• Adopts effective work methods • Communicates appropriately
Prescribed essential knowledge • Refers to knowledge to be applied and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the ***Relationship between quantities*** family of learning situations
Upon completing her education, an adult learner must reimburse a student loan. She wants to estimate the monthly amount to be set aside to pay back that debt, taking into account possible interest rates and the term over which the loan will be reimbursed. Using a linear function, she may have to provide a graphical or algebraic justification for the amortization term chosen.	**Integrative process:** *Using an algebraic or graphical model to represent a situation* In carrying out the four phases in the problem-solving process, adult learners could: Representation • Select relevant information to determine the relationship between the amount set aside to pay back the debt (dependent variable) and the interest rate (independent variable) • Describe the characteristics of the situational problem orally or in writing in order to determine the independent variable (interest rate) and the dependent variable (amount set aside each month to pay back the debt)

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Relationship between quantities* family of learning situations	
The situation could be modified by including constraints (e.g. limiting the amount of the monthly payment). In this case, the adult learner could be asked to use extrapolation to make a conjecture about the term over which the loan will be repaid.	Planning	• Find out about current interest rates and determine the amount to be paid back each month by reading newspapers or using Web-based personal loan calculators
	Activation	• Illustrate graphically, through linear approximation, the dependency relationship between the variables; for example, if the interest rate increases, the monthly payments will also increase • Establish the rule of linear correspondence using two points on the graph • Determine the interest rate corresponding to a specific monthly amount; in other words determine, graphically, algebraically or using a table of values, the inverse of the previously determined function
	Reflection	• Describe how a change in one of the parameters (e.g. the initial debt amount) will affect this graph

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Relationships between quantities*, adult learners represent a situation, interpolate or extrapolate, and generalize a set of situations using an algebraic or graphical model. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To represent a situational problem using an algebraic or graphical model, adult learners adopt various strategies to identify the problem. They determine the mathematical characteristics of the relationship described in the situation: y-intercept, x-intercept, increasing or decreasing intervals, sign, etc. They choose the most accurate representation, aware that it does not necessarily reflect what they have observed, but that it is the best choice given the functions studied in the course. They systematically determine the functional model best suited to the situation, bearing in mind the limitations regarding the model's precision: $f(x) = b$ or $f(x) = ax$ or $f(x) = ax + b$. They produce mathematical messages, using recognized codes and conventions in order to effectively communicate their intention: rate, y-intercept, x-intercept, increasing interval, decreasing interval, etc. They choose the register of representation best suited to the situation (table of values, Cartesian coordinate graph or algebraic rule).

To interpolate or extrapolate results using an algebraic or graphical model with a view to making a decision, adult learners interpret the algebraic or graphical model presented and distinguish between the elements that are relevant and those that are not. In addition, they use mathematical reasoning to explore the situational problem and to determine questions about the issue involved, and gather relevant information in order to draw a conclusion. They deduce the rate of change of the relationship and determine the y-intercept in accordance with actual data: initial value, value of the function at time zero, quantity at the beginning of the experiment, etc.

To generalize results in order to obtain a family of linear functions or a system of linear relations, adult learners deduce similar properties by observing a variety of situations. They identify the parameters at play: rate of change, y-intercepts, increasing function, etc. They use inductive reasoning to determine the type of relationship between the variables (linear or rational function). Using graphical or algebraic representation, they confirm that the parameterized model $f(x) = ax + b$ does indeed correspond to a set of situations. In addition, when the situation involves a system of linear relations, they use a system of two first-degree relations in two variables to represent it and then solve the system algebraically (using the comparison method) or graphically, in accordance with the limitations imposed by the context. They validate their solution by substituting the values in the algebraic expression associated with the system in question.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (inequality, relation and system). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

  * The solution includes a procedure, strategies and a final answer.
  ** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, notation and conventions of mathematics, and suited to the context*