Course

MTH-5154-2

Financial Mathematics in a General Context

Mathematics

INTRODUCTION

The goal of the course *Financial Mathematics in a General Context* is to enable adult learners to deal effectively with situations that involve evaluating investment proposals and creating various financing scenarios in a general context.

Adult learners who take the course will learn how to analyze and interpret financial situations, and how to develop financing scenarios to meet their needs. Whether the goal is to manage personal, family or business assets, learners will become familiar with a number of financial concepts, including *future* or *earned value*, *fixed interest rate* and *compound interest rate*, so that they will be able to make informed decisions. They will use timelines to solve problems involving annuities, which will require them to evaluate the performance of financial plans over time and to take retroactive measures if necessary.

Since this is an introductory course, all the required formulas are provided for the calculations that need to be performed. Learners calculate the balance of a fund to which successive deposits are made; determine the value of a debt on a given date based on the payments due; or estimate the present value and the future value (earned value) of a series of equal payments using formulas or electronic spreadsheets. They learn how to calculate equivalent rates to determine the value of an annuity or the number of payments required. In addition, they will learn to analyze the behaviour of financial functions over different time periods and to compare the graphic representations of situations under study.

Finally, the course provides the opportunity to explore concepts related to periodic payments and the number of payments required when the amount of the regular payment is set in advance. The concepts of residual value, interest, and capital amortization on a given date are introduced. In addition, an overview of supplementary payments under a repayment annuity is given.

At the end of this course, adult learners will be able to understand financial language and use calculation tools related to financial mathematics to evaluate different investment proposals and choose the most suitable one. They will also be able to create a financial plan tailored to a particular situation.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly while observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

PROCESS AND STRATEGIES

REPRESENTATION

- Adult learners examine the situational problem to identify the context, the problem and the task to be performed.
- They use observational and representational strategies that are essential to inductive reasoning.
- In attempting to understand the context and the problem, they use deductive reasoning, particularly in situations that involve implicit data.
- They switch from one register of representation to another when they use spreadsheets, software, Web applications or any other technological tool considered useful.

Examples of strategies	• Describing the situation in their own words and making timelines to illustrate their understanding of the problem • Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to identify a dependency relationship and determine the variables in the situation • Determining the types of relationships that exist between the variables in the situation by using numerical values to illustrate these relationships • Describing the mathematical characteristics of the situation in order to identify the limitations and real constraints of the problem • Formulating questions with respect to the problem • Making false assumptions to identify an inconsistency or an absurdity to corroborate their perceptions or call them into question

PLANNING

- Adult learners look for ways of approaching the problem and choose those that seem the most efficient.
- They develop a plan, taking into account the different registers of representation and elements of mathematical language (symbols, terms and notation used).
- They establish organized and functional relationships among different aspects of their knowledge, thus expanding their networks of cognitive resources.

Examples of strategies	• Drawing a concept map showing the different steps in the solution • Listing the elements needed to represent a function graphically or algebraically • Referring to a list of elements to be considered in consolidating their work plan

ACTIVATION

- Adult learners establish organized and functional relationships between the future value and current value of amounts invested, thus expanding their networks of mathematical cognitive resources.
- By drawing on their knowledge of the properties of financial models, they are able to deduce certain relationships.
- They use the appropriate scale so that the graph they draw in solving the situational problem makes sense in light of the context.

Examples of strategies	• Systematically determining the algebraic rule of a function by manipulating the different variables • Finding combinations of simple variables (linear, quadratic or exponential) in order to determine the rule of a function • Changing perspective and observing the change in the function when the *y*-axis (instead of the *x*-axis) is considered to be its domain

REFLECTION

- Adult learners use a reflective approach throughout the process of solving the situational problem.
- They always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
- A reflective approach allows them to reject extrapolations that would yield nonsensical results.
- When decoding mathematical elements, adult learners make sure they can distinguish between the mathematical and everyday meanings of the terms used.

Examples of strategies	• Checking their solution:
	  • by confirming, for example, that the *y*-axis represents the change in the value of a loan or investment
	  • by making sure that the resulting values satisfy the range of the function
	  • by comparing the amount invested, the interest earned and the investment gain
	  • by substituting the values of the variables in the algebraic expression in order to validate a graphical interpolation or extrapolation

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Relationship between quantities*. Two cross-curricular competencies are considered particularly relevant to this course: *Adopts effective work methods* and *Exercises critical judgment*.

Methodological Competency

When adult learners work with situational problems in the family *Relationship between quantities*, they deal with data derived from financial statements, statistical reports, balance sheets or the results of studies. To carry out their task, adult learners must develop the competency *Adopts effective work methods.* They therefore use timelines and tables to represent a situational problem, organize and analyze data, facilitate enumeration, calculate financial values and produce graphs illustrating the change in the value of an investment or loan. They exchange information with their peers regarding the solution by explaining their approach, choice of registers, decisions, recommendations or conclusions. In gauging the reactions of their peers, they look for ways of evaluating the effectiveness of their solution or the reliability of the study carried out.

Intellectual Competency

When adult learners use iterative reasoning, they explore and compare different possibilities, and justify their choices. They identify various relationships and, depending on their goals, use interpolation, extrapolation or optimization processes by drawing on their understanding of dependency relationships and the concepts of function and inverse (compounding and discounting). They use algebraic processes to identify laws, rules and properties which, in turn, serve to validate conjectures, for example, when adult learners use deductive reasoning to show that two interest rates are equivalent. By developing the competency *Exercises critical judgment*, adult learners may take a more thoughtful approach before signing a contract that will lock them in for many months.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of arithmetic and algebra. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following two integrative processes:

- **evaluating investment proposals tailored to a particular situation**
- **creating a financial plan tailored to a particular situation**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover both processes**.**

Mathematical Knowledge	Restrictions and Clarifications
Performing financial calculations related to an investment or a loan • Determining the interest period, future value and current value • Determining the interest rate[1]: o simple, o compound, o equivalent.	The formulas studied are used to determine: • discounting (current value): $C_0 = C_n(1 + i)^{-n}$ • compounding (future value): $C_n = C_0(1 + i)^n$ • the interest period: $n = \frac{\log\left(C_n / C_0\right)}{\log(1 + i)}$ • the interest rate (i) in situations involving compound interest: $i = \left(\frac{C_n}{C_0}\right)^{1/n} - 1$ *Compound interest is presented using graphs or compiled data tables.*

[1] Use of a spreadsheet is recommended.

Mathematical Knowledge	Restrictions and Clarifications
Analyzing a series of equal or unequal payments • Producing and analyzing a statement outlining the current situation regarding of an investment • Interpreting calculations in a financial plan	Timelines are used to show the change in the value of an investment or loan. Using formulas or software, analyzing and calculating: • the current value of a series of equal or unequal payments • the future value of a series of equal or unequal payments *A withdrawal is regarded as a negative payment.*
Analyzing annuities (repaying a loan by constant annuity) • Interpreting annuity calculations • Interpreting calculations of current value and future value	In this course, the study of annuities is limited to determining: • the annuity period • the current value of a series of annuities • the future value of a series of annuities
Analyzing amortization • Interpreting amortization tables or calculations	In this course, the study of amortization (progressive or unique) is limited to the calculation of: • the payment • the number of payments • the amortization • the residual debt • the interest portion and the capital portion of a payment

Cultural References

Financial mathematics are rich in cultural references, ranging from the history of money to major financial crises and the different theories regarding credit, insurance, real estate, and the stock market, among other things. This course enables adult learners to gain a thorough understanding of the different phases in history that have led societies to develop financial models. Adult learners may also discover how, in developing these models, mathematicians have been influenced by Brownian motion, the heat transfer equation or the random walk.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Relationship between quantities* involve purely mathematical problems that can be solved in part by evaluating an investment proposal or creating a financial plan tailored to a particular situation, in a general context.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Citizenship and Community Life and Environmental Awareness and Consumer Rights and Responsibilities.

Citizenship and Community Life

Adult learners are already aware of how important it is to start saving early. The financial concepts and calculations they learn in this course will enable them to analyze different financial strategies when a major change occurs in their lives that has an impact on their budget (e.g. birth of a child, going back to school, buying a home). They must be able to continue meeting the needs of each member of their family, while saving enough to be financially independent and maintain their lifestyle after retirement.

Environmental Awareness and Consumer Rights and Responsibilities

Adult learners will have to make major financial decisions throughout their lives. Whether it involves negotiating a mortgage, taking out life insurance or financing the purchase of a car, their ability to analyze different financing options will allow them to make an informed decision. This ties in with one of the focuses of development of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following pages.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning – Helps contextualize learning and makes it meaningful.	• Environmental Awareness and Consumer Rights and Responsibilities • Citizenship and Community Life
Prescribed subject-specific competencies – Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations – Consists of real-life situations applicable to a given course. – Helps adult learners acquire mathematical knowledge.	• Relationship between quantities
Targeted cross-curricular competencies – Are developed at the same time and in the same context as the subject-specific competencies.	• Exercises critical judgment • Adopts effective work methods
Prescribed essential knowledge – Refers to mathematical knowledge and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Relationship between quantities* family of learning situations
All parents worry about being able to afford a post-secondary education for their children. At some point in time, parents face certain choices. What type of savings vehicle should they choose to cover education costs: a registered education savings plan or a registered retirement savings plan?	**Integrative processes:** *Evaluating investment proposals tailored to a particular situation* *Creating a financial plan tailored to a particular situation* In carrying out the four phases in the problem-solving process, adult learners could: Representation - Select the most appropriate register of representation (algebraic or graphical) and determine the change in the value of several types of investments - Represent the evolution of a series of payments using a timeline Planning - Use algebraic representation to compare the costs and yields of different investments Activation - Establish the algebraic rule that relates the various elements of the situation (e.g. rates, interest, annuities, amortization payments) to be able to extrapolate the yield associated with each type of plan - Make a conjecture about the various aspects of an investment, and then verify the conjecture algebraically and graphically - Use a spreadsheet to analyze the change in the value of investments and determine which investment is the best, or determine the point at which one investment becomes more worthwhile than another Reflection - Conclude that graphical extrapolation is not very realistic in the case of extreme values - Validate a graphical extrapolation through algebraic calculations - Check the calculation of the future value using the discounted value formula - Decide which model is best suited to the household's financial situation

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Relationship between quantities*, adult learners evaluate investment proposals tailored to different situations or create a financial plan tailored to a particular situation. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

When evaluating investment proposals by using a graphical or algebraic model to represent the situational problem, adult learners employ various strategies to identify the problem. They reformulate the situational problem in their own words and determine the key elements and the obstacles to be overcome. They correctly identify the variables involved and determine those whose value they must calculate (rate, period, compounding, discounting, etc.) by using tables of values (sometimes actuarial tables) and timelines. They choose the most accurate representation, aware that it does not necessarily reflect what they have observed, but that it is the best choice given the functions studied in the course. They validate their representation, comparing their solution against known bank data. When explaining their conclusions after evaluating investment proposals, they produce their representation of the situational problem, determine the purpose of the message and observe mathematical codes and rules in order to effectively communicate their intention. They choose the register of representation best suited to the situation so that their message will be understood by as many people as possible.

When adult learners create a financial plan using an algebraic model or an amortization table, they interpret the model, making connections between the elements of the message and distinguishing between those that are relevant and those that are not. They recognize the purpose of the message and determine its overall meaning. In addition, they use mathematical reasoning to explore the situational problem and to determine questions about the issue. They gather relevant information in order to draw a conclusion. They make one or more conjectures, suggest probable or plausible ideas and, as required, anticipate annuities and amortization payments that are consistent with their financial plan.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

* The solution includes a procedure, strategies and a final answer.

** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, rules and conventions of mathematics, and suited to the context*