Course

MTH-5170-2

Optimization in a Fundamental Context

Mathematics

INTRODUCTION

The goal of the *Optimization in a Fundamental Context* course is to enable adult learners to use linear programming to deal with situations that involve optimization in a fundamental context.

In this course, adult learners find an optimal solution to a situation involving specific constraints. They take these constraints into account, representing them using a system of inequalities in two variables, and define the function to be optimized. They graph the situation, which enables them to study the polygon of constraints or identify the feasible region in order to solve the system graphically or algebraically.

In situational problems, adult learners optimize a linear relation that could be economic or objective. They are required to take the constraints into account, and their analysis of the situation enables them to determine the best solution. They learn to illustrate their reasoning and explain their solution. They show how they interpreted the feasible region and the vertices of the polygon. It is therefore important that they make sure that their result is plausible in the given context and that they specify the degree of precision they took into account. If they see that their result is improbable, they suggest changes, a new solution or ways of making it more efficient.

By the end of this course, adult learners will be able to represent optimization situations. They will produce clear and accurate work in accordance with the rules and conventions of mathematics. Their analysis will address limiting solutions as well as solutions that are integers when the situation refers to a discrete case or when one of the limits is a point on the grid.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

PROCESS AND STRATEGIES

REPRESENTATION

- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. They use observational and representational strategies that are essential to inductive reasoning.
- In attempting to understand the context and the problem, they also use deductive reasoning.
- They have suitably mastered the elements of mathematical language.

Examples of strategies	• Using a table, determining the nature of the task involved • Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to determine the economic or technological constraints involved in mathematizing the problem • Determining questions about the situation • Gathering relevant information (maximum, minimum, economic function)

PLANNING

- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient.
- Planning the solution correctly involves decoding the elements of mathematical language such as the meaning of symbols, terms and notation.

Examples of strategies	• Finding an algebraic rule that takes into account the best relationship between the constraints and the consequences associated with the situational problem: determining the relevant parameters of the scanning line or the economic function • Using their intuition to sketch the boundaries (rectangle parallel to the axes of the Cartesian plane) of the solution set

ACTIVATION

- When dealing with a situational problem, adult learners use mathematical reasoning to graph the half-planes resulting from the constraints.
- They deduce the scale of the axes by analyzing the maximum and minimum values of the variables.
- They use rigorous mathematical language and, to avoid confusion, they use symbols, terms and notation in accordance with their meaning.

Examples of strategies	• Proceeding by trial and error to mathematize certain constraints • Constructing tables of values in order to find two points to represent the boundary lines of the polygon of constraints • Proceeding step by step in solving the inequalities

REFLECTION

- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made.
- They go back and forth between the graph and the economic function when the solutions are integers.
- They express their ideas in accordance with mathematical codes and conventions and take the constraints of the situation into account in their message.

Examples of strategies	• Checking their solution by, for example, comparing the number of possible solutions for a system of equations with the number of solutions found, or using their intuition to make sure that the coordinates of the points they have found are those of the vertices of the polygon of constraints • Using graphing software to validate their work

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Optimizing solutions*. Two of these are considered particularly relevant to this course: *Solves problems* and *Uses information*.

Intellectual Competency

Although the cross-curricular competency *Solves problems* is closely related to mathematics, it can be developed in a broader context through a learning situation in which mathematics can be an invaluable resource. Through trial and error and by reformulating a problem, one can often work out a solution that is satisfactory but is not the only possible one. In order to draw on and develop this competency, adult learners may be asked to devise their own learning situations based on their own areas of interest. Linear programming often provides the flexibility needed to model and represent the observable world through linearization.

Intellectual Competency

A learning situation related to humanitarian aid could enable adult learners to draw on and develop the cross-curricular competency *Uses information*. They must not only know how to find this information and assess its value, but also learn to organize it. Dealing with issues such as sending the military into Afghanistan or East Timor would provide adult learners with an interesting opportunity to organize information found on the Web in order to work out and optimize a plan for deploying military personnel.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of arithmetic and algebra. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following integrative process:

- **optimizing a situation using linear programming**

This process, which is applied in the learning situations in this course, fosters the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve this integrative process.

Mathematical Knowledge	Restrictions and Clarifications
Linear programming	
• System of first-degree inequalities in two variables	
• Representing the constraints and the function to be optimized (objective or economic function)	Constraints can be represented algebraically or graphically. In this course, the function to be optimized is expressed solely as an equation of the form $Ax + By + C = Z$, where A, B and C are rational numbers.
• Determining and interpreting the vertices and the feasible region (bounded or unbounded)	
• Changing the conditions associated with the situation to provide a more optimal solution	

Cultural References

Linear programming results from the work of the mathematicians Joseph Fourrier (1768-1830) and Georges Dantzig (1914-2005). While in the United States Air Force during the Second World War, Dantzig developed a technique for solving the army’s logistical problems at a minimum cost. This method, which combines power and flexibility, was adopted in other fields to solve a variety of economic problems.

In the field of health care, certain decisions can be controversial because they are often made with financial interests in mind. Linear programming, which is a branch of optimization, is very useful in guiding decision making in this field and in solving optimization problems in numerous other fields. Consider the conflicts of interest that threaten the environment. For example, in order to feed a population or provide it with energy, to what extent can we exploit a given area without destroying it? Adult learners could study how certain scientists have been able to answer this question and use linear programming to find an equilibrium point.

By making it possible to perform calculations and process data, computers have made it considerably easier to find optimal solutions. This is one of the reasons that optimization is now used in numerous fields of activity. Once again, adult learners cannot help but notice the key role that mathematics plays in the search for optimal solutions.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Optimizing solutions* involve problems that can be solved in part through optimization using linear programming. *The Optimization in a Fundamental Context* course gives adult learners an opportunity to learn how to maximize a profit, a process or a number of objects or people, and to minimize costs or losses.

In the situational problems in this course, adult learners recognize and decode the meaning of symbols, terms and notation, distinguish between the mathematical and everyday meanings of the terms used, and deduce the optimal solution by substituting the coordinates of the vertices of the polygon of constraints into the equation of the economic function.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Health and Well-Being, and Environmental Awareness and Consumer Rights and Responsibilities.

Health and Well-Being

Some of the proposed learning situations make adult learners aware of health issues. The course could involve learning situations in which health-related concerns are set aside in favour of financial interests. When dealing with learning situations that illustrate the delicate balance between health concerns and profit considerations, and when reviewing their work in this regard, adult learners may become aware of the types of decisions certain companies make. The course focuses on problems whose solution must take into account health-related needs as well as cost-cutting needs. In this way, adult learners are required to take responsibility for adopting good living habits, which is the educational aim of this broad area of learning.

Environmental Awareness and Consumer Rights and Responsibilities

Some learning situations on oil refining techniques may motivate adult learners to develop an active relationship with their environment, while maintaining a critical attitude toward consumption and the exploitation of the environment. Calculating the optimum preheating temperature of crude oil and feedstocks and determining the best “vapour-electricity” balance of a refinery are two examples of optimization that involve environmental issues. A presentation on these issues could encourage adult learners to gather information that they can use to make future decisions.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning – Helps contextualize learning and makes it meaningful.	• Environmental Awareness and Consumer Rights and Responsibilities
Prescribed subject-specific competencies – Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations – Consists of real-life situations applicable to a given course. – Helps adult learners acquire mathematical knowledge.	• Optimizing solutions
Targeted cross-curricular competencies – Are developed at the same time and in the same context as the subject-specific competencies.	• Solves problems • Uses information
Prescribed essential knowledge – Refers to mathematical knowledge and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Optimizing solutions* family of learning situations
The trucking industry is a vital part of the Canadian economy. Even though it is thriving, this industry faces many challenges such as competition and rising fuel prices. As a result, truck fleets must be carefully managed in order to allow trucking companies to maximize their use before increasing the number of vehicles. Adult learners are asked to optimize the use of a truck fleet, taking into account certain constraints such as the maximum number of consecutive hours of work permitted by law, the minimum operating cost per kilometre, etc. They are required to identify the limiting solutions.	**Integrative process:** *Optimizing a situation using linear programming* In carrying out the four phases in the problem-solving process, adult learners could: Representation - Write literal expressions to represent the elements of the situation that seem relevant - Determine the variables involved such as the number of drivers and the number of kilometres travelled - Break down the complex situational problem into subproblems to identify the relationships between the constraints of the situation and the problem: the company’s operating costs, its expected revenue, maximization of the company’s profitability, analysis of the system of inequalities as a function of the scanning line

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Optimizing solutions* family of learning situations	
In addition, the scanning line is parallel to one of the sides of the polygon of constraints, and adult learners are asked to show that this situation may involve several optimal values.	Planning	- Use brainstorming to look for possible solutions if the work is carried out in teams - Refer to the solution of a similar situational problem to carry out their plan - List the mathematical knowledge needed to deal with the situation: choosing the variables, determining the constraints, establishing a system of first-degree inequalities in two variables, graphing the feasible region, comparing the slopes of the sides of the polygon of constraints and the slope of the scanning line in order to determine the parallel lines, etc.
	Activation	- Select the variables: number of drivers, number of kilometres travelled - Mathematize the constraints of the situation through trial and error - Construct tables of values to draw the boundary lines of the polygon of constraints - Determine the vertex of the polygon of constraints that optimizes the profitability of the truck fleet - Calculate the minimum operating cost - Determine the side parallel to the scanning line (economic function) and then show that there may be several solutions
	Reflection	- Make sure that the solution makes sense by using any point to check that the feasible region has been correctly determined - Compare their solution and results with those of their classmates to identify the strengths and weaknesses of the proposed model - Determine if there is an easier way of finding the vertices of the polygon of constraints (e.g. by comparison rather than by substitution) - Examine the role of the parameters of the scanning line

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Optimizing solutions*, adult learners optimize a situation using linear programming. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To use linear programming to solve situational problems, adult learners apply different mathematical models and different types of strategies, combining reasoning and creativity to overcome obstacles. They decode relevant information in order to find an optimal solution. They translate the different constraints using a system of inequalities in two variables, and give an algebraic definition of the function to be optimized. They graph the polygon of constraints and the feasible region. They use algebra to determine the coordinates of the vertices, or approximate an answer using a graph. To prove a conjecture, they use structured deductive reasoning and correctly use the codified form required for their proof. They illustrate, explain or justify their arguments. To develop a proof (e.g. *reductio ad absurdum*, proof by contrapositive or by induction), they use different types of reasoning, including proof by exhaustion. They observe real-life situations and generalize their observations. Lastly, in some situations, they analyze data in order to identify the necessary and sufficient conditions for drawing a conclusion, make decisions and determine how best to approach, optimize or adjust a situation.

When conducting case studies, synthesizing information, constructing proofs or making presentations in order to deal with situational problems related to linear programming, adult learners must accurately identify the purpose of the mathematical messages to be conveyed or interpreted. They select the medium, the type of discourse and the register of representation best suited to the audience and the purpose of the message. They switch easily from one register to another. They use a wide range of communication strategies that enable them to regulate the transmission of a message based on the specific reactions of the audience or to take new requirements into account. They adopt language that appropriately combines everyday, mathematical, scientific and technical terms.

Throughout the problem-solving process, adult learners apply their knowledge of linear programming. Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

* The solution includes a procedure, strategies and a final answer.
** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, rules and conventions of mathematics, and suited to the context*