Course

MTH-5163-2

Geometric Representation in an Applied Context 2

Mathematics

INTRODUCTION

The goal of the *Geometric Representation in an Applied Context 2* course is to enable adult learners to deal with situations that involve representing and describing two-dimensional geometric transformations of an object or a geometric locus in an applied context.

In this course, adult learners broaden their network of concepts to include equivalent figures, metric relations in circles and trigonometry in triangles. They are introduced to new mathematical knowledge related to vectors and geometric representation, which help them make connections with the sciences. They find the resultant and make connections with a composition of translations, triangles and parallelograms. Trigonometric relations are used in situations involving the orthogonal projection of a vector. As with vectors, studying the relative position of two circles and constructing the segment representing the distance from a point to a circle or an ellipse allows adult learners to apply the concept of distance to other situations. Mathematical concepts related to loci and relative positions are introduced intuitively. Adult learners continue to develop this mathematical knowledge through exploration and observation activities that involve finding the figure that corresponds to the description of a locus. Conversely, they describe the locus corresponding to a given figure. The emphasis is on describing a geometric locus so as to identify the necessary and sufficient conditions that make it possible to understand and use it. When adult learners define a locus, they first describe it in terms of the concept of distance. They then use their understanding of algebraic expressions as well as familiar operations to modify the expression without changing its meaning. Adult learners formulate and validate conjectures relating to a locus, i.e. the possible position of a set of points that meets specific conditions. They construct loci using properties and devising mechanisms or procedures; they draw or modify loci using geometric transformations. Constructing knowledge related to the concept of geometric locus involves exploring several different loci and recognizing that a given locus can be generated in different ways. Connections with the sciences and with vocational and technical training can readily be made through the study of this concept.

By the end of this course, adult learners will be able to use trigonometric relations, the properties of equivalent figures and metric relations in circles to describe and represent geometric transformations. Used with matrices, analytic geometry makes it possible to algebraically model certain geometric transformations of objects. Adult learners will also be able to describe, represent and generalize certain characteristics of geometric loci in the Cartesian plane using vectors in accordance with the mathematical rules and conventions employed in geometry.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

PROCESS AND STRATEGIES	
REPRESENTATION	
- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. They use strategies essential to inductive reasoning. - They conceive of probable or plausible relationships that can then be expressed as a formal conjecture. To do this, they use strategies to clarify the different patterns and invariants. - They distinguish between the mathematical and everyday meanings of the terms used so that they can understand what is meant by focus, ellipse, vertex, arc and so on. They use different sources of information to provide a correct representation of the problem.	
Examples of strategies	• Using an estimate, a scale plan or a literal description to determine the nature of the task involved • Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to find measurements or provide a spatial representation • Representing the situational problem mentally or in writing • Describing the situational problem in their own words to show that they have understood it
PLANNING	
- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient. - They use reasoning when they employ different registers of representation to highlight the properties of geometric transformations. - They use a literal expression to describe the image of an object resulting from a geometric transformation, taking into account the elements of mathematical language.	
Examples of strategies	• Dividing the situational problem into subproblems, for example, in order to find a measurement using the metric relations in a circle • Using lists, tables, diagrams, concrete materials or drawings to plan their solution
ACTIVATION	
- When dealing with a situational problem, adult learners make rigorous use of the elements of mathematical language and, to avoid confusion, they use the symbols, terms and notation in accordance with their meaning. - When using vectors to prove a geometric proposition, they identify patterns by exploring different cases related to the properties of vectors.	
Examples of strategies	• Using the characteristics of a conic, making a sketch to predict results • Solving certain situational problems by working backwards when the solution consists of several steps or when there is insufficient information • Analyzing the effects of a geometric transformation on a plane figure to properly understand its relationship with the parameters of the algebraic or matrix rule, for example
REFLECTION	
- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution. - They make conjectures about particular or special cases involving any triangle in order to validate certain results using reasoning. - They make sure that their message is clear and that they have observed the relevant codes and conventions.	
Examples of strategies	• Checking their solution by means of examples or counterexamples • Determining the strategies for dealing with situational problems in geometry (e.g. applying a rule, referring to a geometry principle, using a formula)

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Measurement and spatial representation*. Two of these are considered particularly relevant to this course: *Uses information* and *Uses information and communications technologies*.

Intellectual Competency

In a situation related to architecture or urban planning, adult learners could be asked to find information on conics related to these contexts. They would have to find and assess this information and organize it in accordance with the constraints of the problem. In this way, adult learners would be developing the competency *Uses information*.

Methodological Competency

By studying the movements of a computer animation, adult learners might decide to try their hand at this type of work using specialized software. They would first have to become familiar with a new computer environment so that they could perform geometric transformations involving simple objects. They would then use their knowledge of this medium to produce a more complex animation. In this way, adult learners would be able to develop the competency *Uses information and communications technologies*.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of geometry and algebra. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following three integrative processes:

- **describing geometric transformations of two-dimensional objects and representing them graphically and algebraically**
- **describing geometric loci and representing them graphically and algebraically**
- **using vectors to generalize geometry principles**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover all three processes.

Mathematical Knowledge	Restrictions and Clarifications
Geometric transformations	
• Representing and describing geometric transformations	Geometric transformations are represented using: • algebraic rules • matrices *(the matrix form is introduced in order to enable adult learners to synthesize what they have learned)*
Finding measurements	
• Equivalent figures	
• Finding measurements of: – arcs or angles – lengths (segments, chords) – areas – volumes – capacities	These measurements are found by applying the properties of congruent, similar or equivalent figures, geometric transformations and metric or trigonometric relations. *The trigonometric relations studied in this course are:* • *the sine law* • *the cosine law* *The metric relations studied in this course are limited to those pertaining to circles. For more details, see the Principles table that comes after this table.*

Mathematical Knowledge	Restrictions and Clarifications
Geometric loci and relative position: plane loci involving lines or circles only, and conics • Describing and representing and constructing geometric loci (plane loci involving lines or circles only, and conics)	The plane loci studied involve line or circles only. The conics studied are: • the parabola (centred at the origin and translated) • the circle (centred at the origin and translated) • the ellipse (centred at the origin and translated) • the hyperbola (centred at the origin and translated) The elements described are: • the radius • the axes • the directrix • the vertices • the foci • the asymptotes • the regions *Conics are described using only the standard form of the algebraic rule.*
Trigonometric relations • Standard unit circle (radian and arc length) • Manipulating simple trigonometric expressions using definitions (sine, cosine, tangent, secant, cosecant and cotangent)	Only the Pythagorean identities and the properties of periodicity and symmetry are studied in this course.
Vectors • Resultant and projection • Operations on vectors	In this course, *vector* refers to a geometric or free vector. Operations on vectors are limited to the following: • adding and subtracting vectors • multiplying a vector by a scalar • the scalar product of two vectors

Principles
Adult learners must master the following compulsory principles, which may be used in a proof:
P13. The medians of a triangle determine six equivalent triangles.
P14. The midpoint of the hypotenuse of a right triangle is equidistant from the three vertices.
P15. The lengths of the sides of any triangle are proportional to the sines of the angles opposite these sides (*sine law*).
P16. The square of the length of a side of any triangle is equal to the sum of the squares of the lengths of the other two sides, minus twice the product of the lengths of the other two sides multiplied by the cosine of the contained angle (*cosine law*).
P17. Any diameter perpendicular to a chord divides that chord and each of the arcs that it subtends into two congruent parts.
P18. The measure of an inscribed angle is one-half the measure of its intercepted arc.
P19. If a line is perpendicular to a radius of a circle at the endpoint of the radius in the circle, the line is tangent to the circle. The converse is also true.
P20. In a circle or in congruent circles, two congruent chords are equidistant from the centre and vice versa.
P21. Two parallel lines, be they secants or tangents, intercept two congruent arcs of a circle.
P22. If point P is located outside circle O, and if segments PA and PB are tangents to that circle at points A and B respectively, then OP bisects angle APB and the length of segment PA is equal to the length of segment PB.
P23. The measure of an angle located between the circumference and the centre of a circle is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.
P24. The measure of an angle located outside a circle is one-half the difference of the measures of the intercepted arcs.
P25. If two chords of a circle intersect in its interior, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.
P26. If secants PAB and PCD of a circle have the same external endpoint P, then $m\overline{PA} \times m\overline{PB} = m\overline{PC} \times m\overline{PD}$

Cultural References

Although urban planning has probably existed since the advent of the first cities, it was raised to the status of a science in 1867 with the publication of Ildefons Cerdà's work entitled *General Theory of Urbanization.* However, this discipline was developed only in the 20th century through the creation of specialized organizations and schools. Moreover, it was in the 19th century that Baron Haussmann redeveloped a part of Paris not only to beautify it, but also to make it easier for pedestrians to get around and to improve the air quality. Today, urban planning deals with such matters as the organization of living space and the design of traffic circles that are similar to conics.

We can easily imagine the difficulty involved in managing a city the size of Montréal. Fortunately, specialized software helps engineers in their planning and representational tasks. An introduction to urban planning could encourage adult learners to carry out different projects suggested in the course.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Measurement and spatial representation* involve problems that can be solved in part through the mathematical description or representation of geometric transformations of objects or geometric loci. The *Geometric Representation in an Applied Context 2* course provides adult learners with an opportunity to develop their spatial representation skills.

In the situational problems in this course, adult learners prove geometric principles related to metric relations in a circle, giving several examples before drawing conclusions; apply the properties of geometric transformations; and validate their message by consulting different sources of information or by comparing their understanding of the message with that of their classmates.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Citizenship and Community Life, and Media Literacy.

Citizenship and Community Life

Some learning situations could involve using a tracking system to locate an object or an individual, thereby helping adult learners become more aware of their surroundings and develop an attitude of openness to the world and respect for diversity. For example, imagine that a missing person with a cell phone must be found. By means of sensors, the person can be located using triangulation. In addition to dealing with the situation mathematically, adult learners must weigh the advantages and disadvantages of global positioning systems used for recreational, efficiency or security reasons. They will therefore be better able to critically assess the unintended consequences of such systems and to strike a balance between public security and individual freedom. This issue ties in with one of the focuses of development of this broad area of learning.

Media Literacy

Some learning situations could provide adult learners with the opportunity to learn how to produce media documents, including those that involve computer animation techniques. For example, a learning situation could require that adult learners create an animation whose movements would be programmed using the appropriate software. Producing this type of animated sequence would enable adult learners to integrate their knowledge of geometric transformations and matrices in linear programming. This approach ties in with one of the focuses of development of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning – Helps contextualize learning and makes it meaningful.	• Citizenship and Community Life
Prescribed subject-specific competencies – Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations – Consists of real-life situations applicable to a given course. – Helps adult learners acquire mathematical knowledge.	• Measurement and spatial representation
Targeted cross-curricular competencies – Are developed at the same time and in the same context as the subject-specific competencies.	• Uses information • Uses information and communications technologies
Prescribed essential knowledge – Refers to mathematical knowledge and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Measurement and spatial representation* family of learning situations
In some European countries, the movement of tectonic plates can produce a marked change in the local landscape. In Switzerland, for example, the land register of certain properties must be reviewed and corrected over time.	**Integrative process:** *Describing geometric transformations of two-dimensional objects and representing them graphically and algebraically* In carrying out the four phases in the problem-solving process, adult learners could: Representation - Decode the elements that can be processed mathematically - Illustrate the shift in the boundary points of a piece of land by sketching a Cartesian coordinate graph Planning - Review their knowledge of vectors and geometric transformations - Outline the different steps involved in working out their solution, taking into account the constraints of the situation

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Measurement and spatial representation* family of learning situations	
To understand this phenomenon, adult learners are required to use matrices and vectors to describe and represent the shift in the coordinates (*x, y*) of the points that define the boundaries of a piece of land. This shift is caused by natural disturbances. In addition, they must test their model by predicting the behaviour of the boundary points as a result of these disturbances if they are considered constant over time (e.g. a 2-m southward shift and a 3-m eastward shift every 125 years). A diagram with a legend explains the natural disturbances involved.	Activation Reflection	- Using vectors, calculate the position of the boundary points of the piece of land and determine the components of the matrix that models the disturbance described in the problem; these components can be rotations, dilatations or translations - Apply matrix rules and perform vector operations in solving the problem - Submit their solution for critical assessment by sharing it with their teacher and classmates so that it can be modified - Validate their predictions by having their classmates review their work - Consult reference materials in order to obtain background information that will help them better understand the problem

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Measurement and spatial representation*, adult learners describe geometric transformations of objects, describe geometric loci and represent them graphically and algebraically, and use vectors to generalize geometry principles. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To describe geometric transformations of objects and represent them graphically, adult learners use different strategies at every step of the problem-solving process. When they use reasoning, they employ the matrix form and the algebraic form of geometric transformations to compare information, propose corrections, present favourable or optimal solutions, or issue recommendations on drawings, specifications, machines, measuring instruments and so on. They formulate constructive criticism and make informed decisions concerning issues in a variety of fields, including technical fields (e.g. graphics, biology, physics, administration). They describe and illustrate the operation or use of various instruments using geometric transformations. In addition, they draw schematic diagrams of objects or figures using equivalent figures, geometric loci, distance and relative position. They determine the measurements needed for the solution based on their mathematical knowledge of metric or trigonometric relations in triangles and circles.

To describe geometric loci and represent them graphically and algebraically, adult learners use different mathematical models and different types of strategies, combining reasoning and creativity to overcome obstacles. Adult learners observe specific real-life cases and generalize their observations. They analyze data in order to identify the necessary and sufficient conditions for drawing a conclusion, making a decision, and determining how best to approach, optimize or adjust a situation. In addition, they base their reasoning on a Euclidean or Cartesian plane in order to determine measurements, optimize distances, construct geometric loci, represent the relative positions of figures or justify recommendations.

To prove a geometry theorem using vectors, adult learners translate the hypotheses and thesis using vectors and construct an equality. They develop the equality and use the Chasles relation to reduce the initial equality to its simplest form. As needed, they apply the properties of the scalar product of two vectors. They make connections between vector notation and the properties of geometric figures. In addition, they justify all the steps of their solution, applying the properties of vectors so as to communicate clearly and concisely.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (geometric transformations, geometric loci, trigonometric relations and vectors). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

* The solution includes a procedure, strategies and a final answer.
** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, rules and conventions of mathematics, and suited to the context*