Course

MTH-5160-2

Optimization in an Applied Context

Mathematics

INTRODUCTION

The goal of the *Optimization in an Applied Context* course is to enable adult learners to use linear programming to deal with situations that involve optimization in an applied context.

In this course, adult learners solve situational problems that enable them to enhance their repertoire of strategies. They learn how to conduct case studies. They make comparisons, suggest corrections, propose favourable or optimal solutions, or issue recommendations. They formulate constructive criticism and make informed decisions concerning problems in various areas, including technical fields (e.g. graphics, biology, physics, administration). They apply their knowledge of arithmetic and algebra to different situational problems involving specific constraints which, in fact, represent limitations related to real-life situations involving optimization. In addition, they apply their knowledge of systems of first-degree inequalities in order to solve situational problems using linear programming. They use the simplex method to develop networks of cognitive resources.

By the end of this course, adult learners will be able to use linear programming to solve situational problems involving optimization. They will also be able to distinguish between explicit and implicit information, plan their solution based on the steps in the simplex method, apply their solution (process and outcome) taking constraints into account, and validate it according to the context of the situation.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

PROCESS AND STRATEGIES	
REPRESENTATION	
- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. - They use observational and representational strategies that are essential to inductive reasoning. - In attempting to understand the context and the problem, they also use deductive reasoning.	
Examples of strategies	• Determining the nature of the task involved and presenting the information in a table • Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to determine the economic or technical constraints involved in mathematizing the problem • Describing the situation in their own words and comparing their understanding of the problem with that of their classmates and teacher • Describing the characteristics of the situation • Determining questions about the situation
PLANNING	
- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient. - Planning the solution correctly involves decoding the elements of mathematical knowledge such as the meaning of symbols, terms and notation.	
Examples of strategies	• Finding an algebraic rule that takes into account the best relationship between the constraints and consequences associated with the situational problem: determining the relevant parameters of the scanning line or the economic function • Sketching the Cartesian coordinate graph
ACTIVATION	
- When dealing with a situational problem, adult learners use mathematical reasoning to graph the half-planes resulting from the constraints. - They deduce the scale of the axes by analyzing the maximum and minimum values of the variables. - They make rigorous use of mathematical language and, to avoid confusion, they use the symbols, terms and notation in accordance with their meaning.	
Examples of strategies	• Proceeding by trial and error to mathematize certain constraints • Referring to previously solved situational problems in order to graph the half-planes resulting from the constraints • Constructing tables of values in order to find two points to represent the boundary lines of the polygon of constraints
REFLECTION	
- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made. - They go back and forth between the graph and the economic function when the solutions are integers. - They express their ideas in accordance with mathematical codes and conventions, taking the constraints of the situation into account in formulating their messages.	
Examples of strategies	• Comparing their results with the expected results and those of others • Checking their solution by, for example, comparing the number of possible solutions for a system of equations with the number of solutions found, or using their intuition to make sure that the coordinates of the points they have found are those of the vertices of the polygon of constraints • Identifying the strategies used to solve the situational problem

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Optimizing solutions*. Two of these are considered particularly relevant to this course: *Achieves his/her potential* and *Adopts effective work methods*.

Social Competency

Achieving career goals is a daily concern for adults who go back to school. This course offers a range of learning situations that help adult learners fulfill their potential. Exploring trades and occupations gives adult learners the opportunity to reflect on and implement their career plan. More and more occupations related to engineering are open to them. Meaningful and contextualized learning situations allow them to identify abilities that can open up new avenues. Operational research and linear programming are valuable assets in the process of discovering certain technical trades and occupations: adults learn to recognize what is possible for them. Having the opportunity to utilize their personal resources enables them to develop the competency *Achieves his/her potential* because they are then able to develop self-knowledge and the motivation to use their abilities to the fullest.

Methodological Competency

Through the study of linear programming, this course gives adult learners the opportunity to use the competency *Adopts effective work methods*. This type of programming provides a very sequential method of optimizing linear functions of the form $z = ax + by$ subject to several constraints. This course helps adult learners consider all aspects of a task, examine numerous possibilities and find ways of doing things or methods that are best suited to a particular situation or context. They can then adjust their actions as required.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of arithmetic and algebra. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following integrative process:

- **optimizing a situation using linear programming**

This process, which is applied in the learning situations in this course, fosters the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve this integrative process.

Mathematical Knowledge	Restrictions and Clarifications
Linear programming	
• System of first-degree inequalities in two variables	
• Representing constraints and the function to be optimized (objective or economic function)	Constraints can be represented algebraically or graphically. In this course, the function to be optimized is expressed solely as an equation of the form $Ax + By + C = Z$, where A, B and C are rational numbers.
• Determining and interpreting the vertices and the feasible region (bounded or unbounded)	
• Changing the conditions associated with the situation to provide a more optimal solution	

Cultural References

Swiss mathematician Leonhard Euler (1707-1783), a pioneer in pure and applied mathematics, left his mark on numerous fields, including the theory of numbers, geometry, optics and astronomy. The origin of mathematical optimization lies in the principle of least action (i.e. to explain the world in terms of optimization), which is one of the boldest ideas in science.

Linear programming, which is a branch of optimization, originated with the work of French mathematician Joseph Fourrier (1768-1830) on systems of inequalities, even though these systems have been attributed to American mathematician George Dantzig (1914-2005). While in the United States Air Force during the Second World War, Dantzig developed a technique for solving the army's logistical problems at a minimum cost, but he published his work only in 1947.

Adult learners could examine a fictitious logistical problem on a smaller scale. The optimal solution should take into account the constraints of the situation. In this way, adult learners will become aware of the difficulties involved in coming to the rescue of the victims of a natural disaster. They will better understand the importance of coordinated action to save as many lives as possible.

Linear programming, which combines power and flexibility, was soon adopted in business and industry. Businesses used it to solve major economic problems, while industry applied it to production management.

Since the 1970s, linear programming has been applied in a variety of fields such as health care, the environment, agriculture, communications, the oil industry, chemistry, computer science, energy, transportation, industrial production and finance. This breakthrough is the result of advancements in computer technology, which made it possible to deal with situations involving an astronomical number of calculations. Examples given during the course enable adult learners to understand the importance of linear programming in everyday life.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Optimizing solutions* involve problems that can be solved in part through optimization using linear programming. The *Optimization in an Applied Context* course provides adult learners with an opportunity to learn how to maximize a profit, a process or a number of objects or people, and to minimize costs and losses.

In the situational problems in this course, adult learners make connections between literal expressions and inequality symbols by using examples involving numbers, determine the half-planes that represent the constraints and their impact on the economic function, and deduce certain values of the points of intersection of the boundary lines using simple substitution.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Environmental Awareness and Consumer Rights and Responsibilities, and Career Planning and Entrepreneurship.

Environmental Awareness and Consumer Rights and Responsibilities

Adult learners could be required to optimize the use of a piece of farmland by taking into account the surface area allotted to certain crops and the cost of fertilizers and fungicides. Such a learning situation will help make them aware of the interdependence between the environment and human activity, which ties in directly with one of the focuses of development of this broad area of learning.

Career Planning and Entrepreneurship

New and increasingly challenging trades and occupations regularly emerge in the job market. This course could enable adult learners to investigate these emerging trades and occupations. For example, linear programming could help familiarize them with the organization and analysis involved in agricultural and food engineering. Learning situations that involve applying the rules of linear programming to agricultural and food engineering will give adult learners the opportunity to explore one aspect of this field: using exact mathematics to find solutions. This type of learning situation encourages adult learners to undertake and carry out plans designed to develop their potential and help them integrate into society, which is the educational aim of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning – Helps contextualize learning and makes it meaningful.	• Environmental Awareness and Consumer Rights and Responsibilities
Prescribed subject-specific competencies – Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations – Consists of real-life situations applicable to a given course. – Helps adult learners acquire mathematical knowledge.	• Optimizing solutions
Targeted cross-curricular competency – Is developed at the same time and in the same context as the subject-specific competencies.	• Adopts effective work methods
Prescribed essential knowledge – Refers to mathematical knowledge and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Optimizing solutions* family of situations
A farmer wants to plant corn and wheat on his land. He knows that in Québec the season is often too short to grow corn, unless nitrogen is added to the young plants to make them grow faster. Wheat must be treated to prevent such fungal diseases as leaf blotch, since up to 40% of the crop can be lost as a result of these diseases.	**Integrative process:** *Optimizing a situation using linear programming* In carrying out the four phases in the problem-solving process, adult learners could: Representation - Determine the key elements to be considered: the surface area of the farmer's land, the cost of such products as seeds, fungicide and nitrogen fertilizer Planning - Break down the problem into subproblems to identify the relationships between the constraints of the situation and the problem: corn production, wheat production, corn crop maintenance, wheat crop maintenance and fixed costs - State the constraints to be observed and what must be optimized

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Optimizing solutions* family of situations	
Adult learners are required to find the surface area that should be allotted to corn and to wheat in order to maximize the farmer's output. They will have to take into account the cost of nitrogen fertilizer and fungicide, as well as the number of hectares of land the farmer has.	Activation	- Mathematize the constraints related to farming and to corn and wheat crop maintenance - Draw a Cartesian coordinate graph of these equations to find the vertices - Find the optimal solution by determining what the surface area of the corn and wheat fields should be to provide the maximum output
	Reflection	- Make sure the maximum solution corresponds to the highest vertex of the polygon of constraints

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Optimizing solutions*, adult learners optimize a situation using linear programming. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To use linear programming to solve a situational problem, adult learners apply different mathematical models and different types of strategies, combining reasoning and creativity to overcome obstacles. They decode relevant information in order to find an optimal solution. They translate the different constraints using a system of inequalities in two variables, and give an algebraic definition of the function to be optimized. They graph the polygon of constraints and the feasible region. They use algebra to determine the coordinates of the vertices using matrices, or approximate an answer using a graph. To prove a conjecture, they use structured deductive reasoning and correctly use the codified form required for their proof. They illustrate, explain or justify their arguments. To develop a proof, they use different types of reasoning, including proof by exhaustion. They observe real-life situations and make generalizations. Lastly, in some situations, they analyze data in order to identify the necessary and sufficient conditions for drawing a conclusion, make decisions and determine how best to approach, optimize or adjust the situation.

When conducting case studies, synthesizing information, constructing proofs or making presentations in order to deal with situational problems related to linear programming, adult learners must accurately identify the purpose of the mathematical messages to be conveyed or interpreted. They select the medium, the type of discourse and the register of representation best suited to the audience and the purpose of the message. They switch easily from one register to another. They use a wide range of communication strategies that enable them to regulate the transmission of a message based on the specific reactions of the audience or to take new requirements into account. They adopt language that appropriately combines everyday, mathematical, technical and scientific terms.

Throughout the problem-solving process, adult learners apply their knowledge of linear programming. Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

  * The solution includes a procedure, strategies and a final answer.

  ** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, rules and conventions of mathematics, and suited to the context*