Course

MTH-5152-1

Vote Distribution Models and Random Experiments

Mathematics

MTH-5152-1 Vote Distribution Models and Random Experiments

INTRODUCTION

The goal of the *Vote Distribution Models and Random Experiments* course is to enable adult learners to deal with situations that involve processing data from a random experiment in a general context.

In this course, adult learners compare different voting procedures with a view to determining which is the fairest from a democratic point of view. By studying different electoral systems, they refine their critical judgment with respect to politics and broaden their knowledge of world democracies. Vote distribution models are often used in situations that involve making social, political or economic choices. Analyzing different electoral systems requires that adult learners study different voting procedures, as well as their advantages and limitations.

In the different contexts, they enumerate possibilities or calculate probabilities in discrete or continuous cases, or use the concept of mathematical expectation to calculate the possibility of a gain or a loss. Adult learners also calculate the probability of compound events, which means that they calculate the probability of event A, given that event B has occurred. This probability concept, better known as conditional probability, enables them to gain a better understanding of random events. The different situations enable them to learn and use the language of sets. They use Venn diagrams, tree diagrams or schematic drawings to understand and convey messages. They make connections with logical connectors, including “and” and “or.” In order to develop their critical judgment, they learn to predict results, comment on behaviours or beliefs, and make decisions that they can explain or justify using different probability concepts.

By the end of this course, adult learners will be able to conduct a comparative analysis of social choice models so that they can make the fairest possible decisions in a given context. They will present the results of their analysis in accordance with the rules and conventions of mathematics, and determine the most efficient solution using problem-solving strategies. In addition, they will be able to interpret probability data from a random experiment and make decisions that reflect their mathematical reasoning.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

PROCESS AND STRATEGIES

REPRESENTATION

- Adult learners examine the situational problem to identify the context, the problem and the task to be performed.
- They use observational and representational strategies that are essential to inductive reasoning.
- In attempting to understand the context and the problem, they use deductive reasoning, which will enable them to make a conjecture.

Examples of strategies	• Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to make decisions concerning probability data • Using examples involving numbers, determining the type of relationship that exists between the odds of winning and the probability of winning or determining the difference between conditional probability and theoretical or experimental probability

PLANNING

- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient.
- Their number sense is influenced by their mathematical knowledge or understanding of conditional probability.
- In this course, they are required to make a decision in the context of a social choice.

Examples of strategies	• Systematically determining the voting procedure that is the most appropriate given the situational problem • Finding an appropriate counting method in the context of a study involving the use of subjective probabilities or in a random experiment involving events that are nonmutually exclusive

ACTIVATION

- When dealing with a situational problem, adult learners can use reasoning to apply mathematical knowledge related to mutually exclusive events.
- When calculating mathematical expectation, they can also use the algebraic form of the equation and the idea of whether or not contestants in a game of chance get to keep the money they bet.
- By drawing on their knowledge of the properties of experimental probability when dealing with a situation, they are able to deduce certain relationships.
- To avoid confusion, they use the symbols, terms and notation in accordance with their meaning.

Examples of strategies	• Compiling the results of an opinion poll in a table, taking into account age groups, years of education and so on • In light of the data gathered, comparing different voting procedures such as the runoff method, approval voting or proportional representation • Using technology (e.g. spreadsheet program, graphing calculator) to analyze the role of conditional probability in a random experiment that involves calculating probabilities

REFLECTION

- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
- They may have to reconsider the voting procedure and the method of analyzing the results if they realize that the conclusions drawn from a study are in complete contradiction with public opinion.

Examples of strategies	• Checking their solution by, for example, making sure that the sum of the probabilities of an event and its complementary event is always equal to 1 and that the possible values of an event are always physically possible, or identifying possible combinations of mathematical operations in an effort to understand the results of random experiments

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the *Processing data* family of learning situations. Two of these are considered particularly relevant to this course: *Uses information and communications technologies* and *Uses information*.

Methodological Competency

Probability calculations often involve very large numbers, especially when it is a question of calculating possible lottery number combinations consisting of more than 40 numbers. This provides an ideal opportunity to use computers, which are very effective for performing tedious and repetitive calculations. Adult learners could be required to use a spreadsheet program to help them carry out the steps in their work. In this way, they will discover the effectiveness of the technologies available to them and develop the competency *Uses information and communications technologies.*

Intellectual Competency

It is sometimes difficult to find clear information on the political systems of foreign countries, and to compile, synthesize and interrelate it. In addition, the vast amount of information and research results available on the Internet may seem daunting to even the bravest Web users. Adult learners can develop the ability to refine their questions, choose the appropriate keywords and ask the right questions. Developing the competency *Uses information* will enable them to make the most of the information at hand, which could help them avoid hours of fruitless research.

SUBJECT-SPECIFIC CONTENT

In order to deal effectively with situational problems, adult learners master the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following two integrative processes:

- **interpreting data resulting from a random experiment**
- **making decisions concerning social choices**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover both processes.

Mathematical Knowledge	Restrictions and Clarifications
Probability	
• Distinguishing among theoretical, experimental and subjective probability	Subjective probability is used when it is impossible to calculate the theoretical or experimental probability. Such cases call for judgment, perceptiveness or experience. For example, weather reports involve the subjective evaluation of probabilities.
• Distinguishing between probability and odds • Approximating and predicting results • Calculating and interpreting mathematical expectation • Calculating and interpreting conditional probability • Distinguishing between mutually exclusive and nonmutually exclusive events • Distinguishing between dependent and independent events	Factorial notation is optional in the *Cultural, Social and Technical* option.

Mathematical Knowledge	Restrictions and Clarifications
Probability (cont.)	
• Representing random events	Events are represented using: • tables • trees • Venn diagrams Factorial notation is optional.
• Counting and enumerating possibilities	*Finding and using counting formulas is not part of the curriculum.*
Fair distribution model	
• Weighted mean	
• Comparing and interpreting different voting procedures	Adult learners compare and analyze: • majority rule • plurality voting • Borda count • Condorcet method • approval voting • elimination or runoff method • proportional representation *In cases that involve aggregating individual preferences (social choice theory), situations will be limited to no more than 4 "candidates."*

Cultural References

Probability calculations originated in the 17th century. In 1654, Blaise Pascal and Pierre Fermat calculated the number of favourable outcomes from among all the possible outcomes in a game of dice. Probability calculations were later used to determine human life expectancy (Christian and Louis Huygens, 1669) and the cost of buying an annuity (Jan De Witt, 1671). In 1696, English astronomer Edmond Halley drew up a mortality table and began work that would lead to the development of modern actuarial science.

In 1714, however, Swiss mathematician Jacob Bernoulli made the connection between statistics and probability with the publication of *Ars conjectandi*, in which he outlined the *law of large numbers*. According to this law, the probability of a result in an experiment is “practically equal” to the frequency with which this result occurs when the same experiment is repeated a large number of times.

The development of probability theory and its more rigorous application led to it being used in a wide range of fields. By the end of the 18th century, Condorcet had shown that probability calculations could be applied to the study of economic and social phenomena. Toward the end of the 19th century, probability theory was associated with progress in medicine and biology, and more specifically, the study of heredity. In the 20th century, it was widely used in the field of quantum mechanics.

Today, probability theory is commonly used and recognized as instrumental in carrying out activities in a variety of areas (e.g. assessing symptoms according to their importance when diagnosing a disease; developing new vaccines and gauging their effectiveness; managing investment risk; using encryption to guard against copyright violations by making it impossible to copy HD-DVD disks; using passwords in computer security; managing peak periods and waiting lists to develop customer loyalty; managing quality control to reduce losses and make factories profitable; developing a hiring policy according to predictions regarding the number of employees expected to retire). Examples given during the course will enable adult learners to understand the role probability plays in everyday life.

As a project, adult learners could conduct a study on the role of probability theory in an area of particular interest to them (e.g. quality control methods on an assembly line). Their analysis could help them better understand the usefulness of probability theory in the workplace.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Processing data* involve problems that can be solved in part by collecting or processing data. The *Vote Distribution Models and Random Experiments* course provides adult learners with an opportunity to learn how to collect and compare data.

When dealing with situational problems, adult learners use reasoning to apply mathematical knowledge related to mutually exclusive events, identify possible combinations of mathematical operations in an effort to understand the results of random experiments, and reconsider their counting method in order to correct their solution if, in determining conditional probability, they realize, for example, that their result is greater than 1.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Citizenship and Community Life, and Career Planning and Entrepreneurship.

Citizenship and Community Life

Adult learners taking this course could organize an election or a referendum in their centre. An awareness of the different voting procedures, the extent to which they are democratic and their limitations ties in with the educational aim of this broad area of learning, while enabling adult learners to apply probability concepts. For example, adult learners could compare three countries, each with a different voting system, in order to choose the one they feel is most suitable. They could also analyze the consequences of changing a given system. By better understanding how an electoral system works, adult learners develop a greater appreciation for democratic institutions and are more likely to become actively involved in them.

Career Planning and Entrepreneurship

Adult learners who wish to learn more about democracy and voting systems on a smaller scale could conduct an opinion poll, taking into account the limitations of a small sample. In order to become familiar with sociology, ethnology, psychology, anthropology and other fields, they could test different types of uninominal and plurinominal systems. Such an activity involves expanding their interests and may lead them to consider a career choice in this regard. They also learn to complete projects that help them develop their potential and integrate into society, which ties in with the educational aim of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning – Helps contextualize learning and makes it meaningful.	• Citizenship and Community Life
Prescribed subject-specific competencies – Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations – Consists of real-life situations applicable to a given course. – Helps adult learners acquire mathematical knowledge.	• Processing data
Targeted cross-curricular competency – Is developed at the same time and in the same context as the subject-specific competencies.	• Uses information
Prescribed essential knowledge – Refers to mathematical knowledge and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	**Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Processing data* family of learning situations**
To determine the best place to build a new hospital, the Ministère de la Santé et des Services sociaux has asked for the opinion of adjoining municipalities. Each municipality has a different population, and each one wants the hospital to be located as close to it as possible.	**Integrative process:** *Making decisions concerning social choices* In carrying out the four phases in the problem-solving process, adult learners could: Representation - Compile the available information - Determine the constraints, if any - Identify the relevant information (the municipalities involved, the percentage of the population eligible to vote in each city, etc.) and the irrelevant information (e.g. the gender of the voters or their occupation) Planning - Look for possible solutions (e.g. existing voting procedures) - Compare these procedures to find the most appropriate solution, and draw up a plan of action in this regard

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Processing data* family of learning situations	
Adult learners are asked to make a fair decision by using the given data to select the city where the hospital will be located. They must provide a clear justification for their choice.	Activation	- Refer to a similar situation studied previously - Analyze and compare the different vote distribution rules: majority rule, plurality voting, etc. - Determine the city (result) where the hospital will be located, using the chosen voting procedure
	Reflection	- Determine in which case the results will be the same regardless of the procedure used

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Processing data*, adult learners interpret data resulting from a random experiment and make decisions concerning social choices. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems, Uses mathematical reasoning* and *Communicates by using mathematical language*.

To interpret data resulting from a random experiment, adult learners organize information with a view to conducting a thorough analysis. They use the appropriate representations to clearly illustrate the constraints related to the context of the situational problem. They use the language of sets to simplify their solutions. They use different strategies to illustrate their line of reasoning. They plan and choose the most appropriate process, taking the purpose of the message into account. They use certain paradoxes (e.g. the boy or girl paradox, the Bertrand paradox, the prisoner's dilemma) to explain the limitations of conditional probability. They validate their reasoning against reliable sources in order to evaluate and adjust their process and to plan smoother and more elegant solutions.

To choose an electoral system with a view to making a decision concerning social choices, adult learners distinguish between uninominal (one name) and plurinominal (a list of names) systems, and between majority rule and proportional representation. They use the Borda count and the Condorcet method as needed in situations involving weighted votes. They justify their reasoning based on predetermined definitions in the case of approval votes or when they use proportional representation to ensure fairness and equality.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (probability and fair distribution model). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

* The solution includes a procedure, strategies and a final answer.
** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, rules and conventions of mathematics, and suited to the context*