Course

MTH-4173-2

Geometric Representation in a Fundamental Context 1

Mathematics

INTRODUCTION

The goal of the *Geometric Representation in a Fundamental Context 1* course is to enable adult learners to use trigonometry to deal with situations that involve the geometric representation of an object or a physical space in a fundamental context.

In this course, adult learners encounter various situational problems that enable them to expand their knowledge of geometry, and trigonometry in particular. In solving situational problems that involve trigonometric concepts, adult learners use inductive reasoning to derive properties of triangles and deductive reasoning to find measurements. They derive different metric relations in right triangles by using proportional and geometric reasoning, as well as concepts related to similar triangles. The geometry principles studied should ideally emerge as conclusions related to the exploratory activities carried out by adult learners. These principles help adult learners justify the steps in their work when they solve a situational problem. Thus, adult learners use the different relations associated with geometric figures, as well as proportional and geometric reasoning or trigonometry, to find unknown measurements based on congruent, similar or equivalent figures. These lines of reasoning also enable adult learners to deduce unknown measurements in geometric figures, which may or may not result from similarity transformations, in order to validate or refute a conjecture. Adult learners use definitions, properties, relations and theorems to prove other conjectures. At times, they identify the structure of someone else's line of reasoning, then analyze it, evaluate it and reformulate it in their own words. Lastly, they organize their communications around metric or trigonometric relations in order to describe the relationship between the different measurements of a figure.

By the end of this course, adult learners will be able to use different metric or trigonometric relations to represent and describe an object or a physical space in accordance with the mathematical rules and conventions used in geometry. They will also be able to use different strategies and types of reasoning to organize a physical space in accordance with certain constraints.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

<table>
<tr><th colspan="2">PROCESS AND STRATEGIES</th></tr>
<tr><th colspan="2">REPRESENTATION</th></tr>
<tr><td colspan="2">- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. They use observational and representational strategies that are essential to inductive reasoning.
- They organize the elements that make it possible to plan the main steps in a line of deductive reasoning related to the concept of similarity.
- They distinguish between the mathematical and everyday meanings of the terms used, and understand what is meant by angle of depression, angle of inclination, adjacent side and so on.</td></tr>
<tr><td>Examples of strategies</td><td>• Describing the situational problem in their own words to show that they have understood it
• Representing the situational problem mentally or in writing
• Listing their geometry-related strategies and the metric relations pertaining to the situation
• Describing the characteristics of the situation
• Determining questions about the situation</td></tr>
<tr><th colspan="2">PLANNING</th></tr>
<tr><td colspan="2">- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient.
- Mathematical reasoning allows them to use different registers of representation to illustrate certain properties of trigonometric ratios.
- By making connections between the elements of the message and by giving a literal description of the ratios of the corresponding sides of two plane figures, adult learners are able to construct a figure based on their description.</td></tr>
<tr><td>Examples of strategies</td><td>• Dividing the situational problem into subproblems
• Using lists, tables, diagrams, concrete materials or drawings to plan their solution</td></tr>
<tr><th colspan="2">ACTIVATION</th></tr>
<tr><td colspan="2">- In developing a line of reasoning to prove geometry principles pertaining to right triangles, adult learners give several examples before drawing conclusions.
- In producing the plan of an architectural structure, they take into account the proportions indicated by the scale and use the related symbols and conventions.</td></tr>
<tr><td>Examples of strategies</td><td>• Simplifying the situational problem by comparing it with a similar problem that has already been solved and using it as a point of departure for solving a more complex problem
• Using the parameters of a function, making a sketch to predict results
• Comparing the parameters of a right triangle with those of any given triangle in order to make connections or formulate laws such as the cosine law.</td></tr>
<tr><th colspan="2">REFLECTION</th></tr>
<tr><td colspan="2">- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
- Reasoning helps them make conjectures about particular or special cases involving any triangle in order to validate certain results. Reasoning also enables them to reject extrapolations that would yield nonsensical results.
- They use different sources of information to validate their mathematical messages.</td></tr>
<tr><td>Examples of strategies</td><td>• Checking their solution by means of examples or counterexamples
• Determining the strategies for dealing with situational problems in geometry (e.g. applying a rule, referring to a theorem)
• Using a calculator or geometric modelling software to validate their work</td></tr>
</table>

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Measurement and spatial representation*. Two of these are considered particularly relevant to this course: *Uses creativity* and *Uses information and communications technologies*.

Intellectual Competency

In this course, adult learners have many opportunities to use the competency *Uses creativity*. When dealing with a learning situation that involves analyzing and meeting certain technical challenges in order to build a structure, adult learners look for original solutions to the problems at hand. They could also be required to prove the Pythagorean theorem. Is there still room for innovation given that this theorem has been proved in countless ways, each one different from the others? Often, creativity has less to do with adding new resources than with using the available resources in a new way. Adult learners are encouraged to allow themselves to be guided by both their intuition and their logic.

Methodological Competency

The competency *Uses information and communications technologies* could help adult learners deal with situations that involve representing objects or physical spaces. Geometry software makes it easier to manipulate figures and makes it possible to create isometries or dilatations, modify the angles and validate trigonometric relations through proofs. With time, adult learners will want to use these technologies for a variety of tasks.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of geometry. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following two integrative processes:

- **organizing a physical space**
- **describing an object or a physical space and representing it in two or three dimensions**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover both processes.

Mathematical Knowledge	Restrictions and Clarifications
Metric and trigonometric relations in triangles	
• Representing and interpreting situations using triangles	*The trigonometric ratios studied are sine, cosine and tangent.* *The sine law and the cosine law are also studied in this course.* *Hero's formula is optional in the* Science *option.* *The other metric and trigonometric relations are listed in the Principles table that comes after this table.*
• Justifying their solution using the properties of trigonometric ratios	Adult learners use the properties of trigonometric ratios in a formal manner to justify the steps in their solution.

Mathematical Knowledge	Restrictions and Clarifications
Metric and trigonometric relations in triangles (cont.) • Determining the slope, measurements and positions using metric and trigonometric relations in triangles	In this course, the concept of distance and the properties of congruent, similar or equivalent figures are used to find measurements and positions pertaining to: • the angles in a triangle or in figures that can be split into triangles • the altitude to the hypotenuse, the orthogonal projection of the legs on the hypotenuse • the sides of a triangle • the areas and volumes of figures • the length of a segment resulting from an isometry or a similarity transformation • the distance between two points
Similar and congruent triangles • Determining the minimum conditions required to conclude that triangles are congruent or similar	*Adult learners use the properties of congruent or similar figures in a formal manner to justify the steps in their solution. They may be required to prove these properties.* *These conditions are listed in the Principles table that comes after this table.*
Equivalent figures • Finding measurements	*In this course, equivalent figures are studied in terms of length, area and volume.*

Principles
Adult learners must master the following compulsory principles, which may be used in a proof: **P1.** If the corresponding sides of two triangles are congruent, then the triangles are congruent. **P2.** If two sides and the contained angle of one triangle are congruent to the corresponding two sides and contained angle of another triangle, then the triangles are congruent. **P3.** If two angles and the contained side of one triangle are congruent to the corresponding two angles and contained side of another triangle, then the triangles are congruent.

Principles
P4. Plane figures are congruent if and only if all of their corresponding sides and angles are congruent. **P5.** If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the triangles are similar. **P6.** If the lengths of the corresponding sides of two triangles are in proportion, then the triangles are similar. **P7.** If the lengths of two sides of one triangle are proportional to the lengths of the two corresponding sides of another triangle and the contained angles are congruent, then the triangles are similar. **P8**. Transversals intersected by parallel lines are divided into segments of proportional lengths. **P9**. The midpoint of the hypotenuse of a right triangle is equidistant from the three vertices. **P10**. The lengths of the sides of any triangle are proportional to the sines of the angles opposite these sides. **P11**. The segment joining the midpoints of two sides of a triangle is parallel to the third side and its length is one-half the length of the third side. **P12**. The length of a leg of a right triangle is the geometric mean between the length of its projection on the hypotenuse and the length of the hypotenuse. **P13**. The length of the altitude to the hypotenuse of a right triangle is the geometric mean between the lengths of the segments of the hypotenuse. **P14**. The product of the lengths of the legs of a right triangle is equal to the product of the length of the hypotenuse and the length of the altitude to the hypotenuse. **P15**. The square of the length of a side of any triangle is equal to the sum of the squares of the lengths of the other two sides, minus twice the product of the lengths of the other two sides multiplied by the cosine of the contained angle. **P16**. The segment joining the midpoints of the nonparallel sides of a trapezoid is parallel to the bases and its length is one-half the sum of the lengths of the bases.

Cultural References

Geometry has a rich history. The Greek thinkers were geometers first and foremost. They worked on abstract objects and organized geometry deductively. Adults learning how to deal with abstraction and how to apply the principles of deduction may be interested in learning about the major contributions of these mathematicians and how their ideas evolved. Whether it be Thales of Miletus, Euclid or Archimedes, numerous thinkers built on the knowledge of their time by making connections with other disciplines such as mechanics and astronomy. By the end of the 16th century, after a long period during which it was almost exclusively associated with astronomy, trigonometry finally started to be used in other fields such as surveying.

Today, trigonometry and geometry are no longer called into question. The study of symmetry and shapes can also be applied in chemistry in order to understand the structure of molecules and crystals. Architects also use geometric concepts in developing plans.

The various aspects of geometric representation can be examined in many different contexts. Depending on their interests, adult learners could study the works of artists like Escher or Reutersvärd, global positioning systems (GPS), or the principles of mechanics or even astronomy in order to discover the usefulness of geometry in interpreting reality.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Measurement and spatial representation* involve problems that can be solved in part through the geometric description or representation of an object or a physical space. The *Geometric Representation in a Fundamental Context 1* course provides adult learners with an opportunity to develop their spatial representation skills.

In the situational problems in this course, adult learners make a table of values or draw a graph in the Cartesian plane, solve certain situational problems by working backwards when the solution consists of several steps or when there is insufficient information, and produce a scale plan of an architectural structure, taking into account the proportions indicated by the scale and using the related symbols and conventions.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Environmental Awareness and Consumer Rights and Responsibilities, and Career Planning and Entrepreneurship.

Environmental Awareness and Consumer Rights and Responsibilities

The concepts of trigonometry studied in this course could be used to compare the operation of two telecommunications networks: one land-based and the other satellite-based. A comparative analysis will help adult learners recognize the impact of technological advances on economic development, in contrast to an ecological project more focused on the development of a sustainable environment. This could make adult learners aware of the issues involved and encourage them to develop an active relationship with their environment. This type of study is closely related to the educational aim of this broad area of learning.

Career Planning and Entrepreneurship

Adult learners who are interested in, or simply curious about, architecture could learn about architectural concepts by studying some of the more remarkable urban structures produced by human engineering. For example, they could draw the Millau Viaduct. The different mathematical calculations that go into drawing the structure could help them better understand architectural and civil engineering work, which ties in with the focuses of development of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning – Helps contextualize learning and makes it meaningful.	• Environmental Awareness and Consumer Rights and Responsibilities
Prescribed subject-specific competencies – Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations – Consists of real-life situations applicable to a given course. – Helps adult learners acquire mathematical knowledge.	• Measurement and spatial representation
Targeted cross-curricular competency – Is developed at the same time and in the same context as the subject-specific competencies.	• Uses information and communications technologies
Prescribed essential knowledge – Refers to mathematical knowledge and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Measurement and spatial representation* family of learning situations
Deducing distances by means of triangulation is practical for determining lengths that are difficult to measure. In land surveying, for example, triangulation is useful when physical obstacles such as a body of water or a wooded area make it impossible to measure certain distances. Adult learners are required to prove the validity of the principle of triangulation in the plane. Working with a satellite photo, they must first determine the actual distance between two points separated by a body of water.	**Integrative process:** *Describing a physical space and representing it in two dimensions* In carrying out the four phases in the problem-solving process, adult learners could: Representation - Become familiar with elements of a satellite photo - Sketch a triangle joining the three points mentioned - Determine the task to be carried out: show that triangulation produces the same result obtained when the distance between two points in a plane is measured directly Planning - Determine the steps involved in working out the proof: draw the triangle joining the three points in the situation, make a connection between the situation and their mathematical knowledge, use appropriate mathematical language to formulate the proof and make sure the solution is plausible - State the mathematical concepts needed for the proof: concept of distance, properties of congruent figures, principles of trigonometry, sine and cosine laws

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Measurement and spatial representation* family of learning situations	
They must then show that triangulation would make it possible to deduce this distance using one of these points and a third point, both of which are located on the same side of the body of water.	Activation	- Draw the triangle joining points A, B and C on the satellite photo - Determine the measures of angles and the distance between these three points - Determine the actual distance between the three points, using the scale provided - Determine whether the triangle is a scalene triangle or a right triangle - Use the given information to determine the applicable law: sine or cosine law - Use the sine law to calculate the distance between points A and C located on either side of the body of water - Show that this value is very close to that found by means of the satellite photo by using an appropriate type of representation and correct mathematical language
	Reflection	- Make conjectures explaining why there may be a slight difference between the result obtained using the sine law and the measurement in the plane: inherent limitations with regard to the precision of the angular and distance measurements on the satellite photo - Determine in which other situation this method could be used: navigation, GPS, astronomy - Determine when to apply the sine law rather than the cosine law

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Measurement and spatial representation*, adult learners describe an object or a physical space and represent it in two or three dimensions, and organize a physical space. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To describe and represent an object or a physical space, adult learners interpret and produce sketches or drawings using complex figures that can be broken down into right triangles or other types of triangles. They identify the key elements of mathematical language (e.g. scale, dimensions, perimeter, area) and associate pictures, objects or knowledge with mathematical terms and symbols. In addition, they apply newly acquired mathematical knowledge such as the sine or cosine law, which enables them to determine unknown measurements in unusual situations.

To organize a physical space, adult learners use a variety of strategies: making a schematic diagram or drawing, dividing the tasks into subtasks, etc. They use a complex process—representing the problem, validating the solution and everything in between—applying their knowledge of trigonometry. They use the concept of triangulation to organize a physical space and validate every step against the theorems covered in the course. They deduce unknown measurements, determine results through inductive reasoning and draw conclusions based on their study of the theorems. When these conclusions involve the properties of certain figures, they prove their accuracy by developing a formal proof.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (trigonometric and metric relations in triangles, similar and congruent triangles, and equivalent figures). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

* The solution includes a procedure, strategies and a final answer.

** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, rules and conventions of mathematics, and suited to the context*