Course

MTH-4163-2

Geometric Representation in an Applied Context 1

Mathematics

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INTRODUCTION

The goal of the *Geometric Representation in an Applied Context 1* course is to enable adult learners to use trigonometry to deal with situations that involve the geometric representation of an object or a physical space in an applied context.

In this course adult learners encounter various situational problems that enable them to expand their knowledge of geometry, and trigonometry in particular. Both an empirical and a formal approach are required to derive the properties of certain figures and to justify or validate the truth of various statements. Adult learners deduce metric relations in right triangles or in triangles that they split into right triangles. Using the concept of similarity, they derive the minimum conditions required to conclude that figures are congruent or similar. They solve situational problems in geometry, making drawings or constructing objects in accordance with instructions. This requires that they use their spatial and measurement sense. In addition, as needed, they develop models and find optimal solutions using the concepts of line, distance and point of division. They use geometrical concepts (Euclidean or Cartesian plane) to deduce measurements or propose optimal solutions. In some situations, they convey messages using various symbols, types of notation, units, logical connectors, quantifiers or literal expressions, in accordance with the rules and conventions of mathematics.

By the end of this course, adult learners will be able to use different metric and trigonometric relations to represent and describe an object or a physical space in accordance with the mathematical rules and conventions used in geometry. They will also be able to use different strategies and types of reasoning to organize a physical space, taking into account different constraints.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

PROCESS AND STRATEGIES

REPRESENTATION

- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. They use strategies that are essential to inductive reasoning.
- They organize the elements that will enable them to make a conjecture.
- They distinguish between the mathematical and everyday meaning of the terms used so that they can understand concepts.

Examples of strategies	• Illustrating their understanding of the situational problem by trying to make connections between their mathematical knowledge and the task to be performed • Representing the situational problem mentally or in writing • Listing their geometry-related strategies and the metric relations pertaining to the situation • Describing the characteristics of the situation • Determining questions about the situation

PLANNING

- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient.
- Through mathematical reasoning, they choose different registers of representation to illustrate certain properties of trigonometric ratios.
- By making connections between the elements of the message and by giving a literal description of the ratios of the corresponding sides of two plane figures, adult learners are able to construct a figure based on their description.

Examples of strategies	• Dividing the situational problem into subproblems • Using lists, tables, diagrams, concrete materials or drawings to plan their solution

ACTIVATION

- When dealing with a situational problem, adult learners show discernment by making rigorous use of the elements of mathematical language.
- When producing a scale plan of an architectural structure, they take into account the proportions indicated by the scale and use the related symbols and conventions.

Examples of strategies	• Using the parameters of a function, making a sketch to predict results • Solving certain situational problems by working backwards when the solution consists of several steps or when there is insufficient information • Analyzing the parameters of a right triangle to properly understand how they are related to the parameters of any given triangle

REFLECTION

- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
- Reasoning can help them review their work and make conjectures about particular or special cases involving any triangle in order to validate certain results. Reasoning also enables them to reject extrapolations that would yield nonsensical results.
- They validate their mathematical message by consulting different sources of information.

Examples of strategies	• Checking their solutions by means of examples or counterexamples, particularly by using the Pythagorean theorem to validate the lengths of the sides of a triangle to be able to conclude that it is, in fact, a right triangle • Recognizing the strategies for dealing with situational problems in geometry (e.g. applying a rule, referring to a geometry principle) • Using a calculator or geometric modelling software to validate their work

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Measurement and spatial representation*. Two of these are considered particularly relevant to this course: *Communicates appropriately* and *Adopts effective work methods*.

Communication-Related Competency

Adult learners who wish to organize a space in their adult education centre (e.g. a student café) must first convince the governing board of the feasibility of their project and the needs it will meet. They must then provide an estimate of the cost involved. The competency *Communicates appropriately* will be very useful to them in this situation. Adult learners could design a sufficiently precise plan that enables them to visualize the project. By having an expert check their plan beforehand, they will ensure that it is consistent with the rules of geometry. They should indicate the constraints involved and their consequences, give appropriate answers to any questions asked and take into account their audience's reactions. The way that adult learners make their presentation will for the most part determine how the members of the governing board respond to the plan.

Intellectual Competency

In this course, adult learners might need to know a length that they cannot physically measure. They could use triangulation, which involves dividing an area into triangles, directly measuring one of the sides, and then using trigonometric relations to determine the other measurements. In another situation, they could be asked to determine the distance between a celestial body and the Earth. They could do this by creating an angle between the celestial body and two lines drawn from two different observation points and find information such as the diameter of the Earth. Given the diversity of learning situations they encounter in this course, adult learners will discover a variety of problem-solving approaches. Some approaches will be more appropriate given the context and the resources available. The competency *Adopts effective work methods* is therefore essential.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of geometry. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following two integrative processes:

- **organizing a physical space**
- **describing an object or a physical space and representing it in two or three dimensions**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover both processes.

Mathematical Knowledge	Restrictions and Clarifications
Metric and trigonometric relations in triangles	
• Representing and interpreting situations using triangles	The trigonometric ratios studied in this course are sine, cosine and tangent. *In the* Technical and Scientific *option, the sine law and the cosine law are covered in Secondary V.* The other metric and trigonometric relations are listed in the Principles table that comes after this table.
• Describing the properties of trigonometric ratios	Adult learners use the properties of trigonometric ratios in a formal manner to justify the steps in their solution.

Mathematical Knowledge	Restrictions and Clarifications
Metric and trigonometric relations in triangles (cont.)	
• Determining the slope, measurements and positions using metric and trigonometric relations in triangles	The measurements and positions studied pertain to: • the angles in a triangle • the altitude to the hypotenuse, the orthogonal projection of the legs on the hypotenuse • the sides of a triangle • the area of a triangle • the coordinates of a point (point of division) in the Euclidean and Cartesian planes • the length of a segment • the perpendicular bisector of a segment • the distance (between two points) • areas of triangles, given the measure of an angle and the lengths of two sides or given the measures of two angles and the length of one side
Similar and congruent triangles	
• Determining the minimum conditions required to conclude that triangles are congruent or similar	These conditions are listed in the Principles table that comes after this table.

Principles

Adult learners must master the following compulsory principles, which may be used in a proof:

P1. If the corresponding sides of two triangles are congruent, then the triangles are congruent.

P2. If two sides and the contained angle of one triangle are congruent to the corresponding two sides and contained angle of another triangle, then the triangles are congruent.

P3. If two angles and the contained side of one triangle are congruent to the corresponding two angles and contained side of another triangle, then the triangles are congruent.

P4. If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the triangles are similar.

P5. If the lengths of the corresponding sides of two triangles are in proportion, then the triangles are similar.

P6. If the lengths of two sides of one triangle are proportional to the lengths of the two corresponding sides of another triangle and the contained angles are congruent, then the triangles are similar.

P7. Transversals intersected by parallel lines are divided into segments of proportional lengths.

P8. In a right triangle, the length of the side opposite an angle of 30^0 is equal to half the length of the hypotenuse.

P9. The segment joining the midpoints of two sides of a triangle is parallel to the third side and its length is one-half the length of the third side.

P10. The length of a leg of a right triangle is the geometric mean between the length of its projection on the hypotenuse and the length of the hypotenuse.

P11. The length of the altitude to the hypotenuse of a right triangle is the geometric mean between the lengths of the segments of the hypotenuse.

P12. The product of the lengths of the legs of a right triangle is equal to the product of the length of the hypotenuse and the length of the altitude to the hypotenuse.

Cultural References

We owe Euclidean geometry to Euclid, a Greek mathematician who developed an organized body of geometry principles. In using deductive reasoning, adult learners will learn how to construct proofs. In the process, they may discover that, for Aristotle, deductive reasoning was the best way to acquire knowledge and that, for Galileo and Descartes, it made it possible to produce mathematical explanations of physical phenomena.

In developing their measurement sense, adult learners could learn to appreciate how a number of instruments (e.g. odometer, global positioning system, compass, sextant, quadrant) used today or in the past have helped solve many problems. Furthermore, surveying equipment, navigation and astronomical instruments, the mirror and shadow technique, the pantograph, the proportional compass, and Jacob's and Gerbert's staffs could help adult learners develop their understanding of the concept of similarity or make connections with the field of science. With respect to computers, adult learners may discover that visual on-screen representation involves trigonometry, and that animation in the development of video games requires geometric transformations.

In this course, adult learners are introduced to analytic geometry. The combination of loci (geometry) and equations (algebra) makes it easier to compare mathematical objects. Given that astronomy is a science that combines algebra and trigonometry, and that robotics, mechanics, automotive production and 3-D description involve combining sets of loci with algebra, all these fields could be incorporated into the learning process because they are likely to pique the interest and curiosity of adult learners.

FAMILY OF SITUATIONS

The situations in the family *Measurement and spatial representation* involve problems that can be solved in part through the geometric description or representation of an object or a physical space. The *Geometric Representation in an Applied Context 1* course provides adult learners with the opportunity to develop their spatial representation skills.

In the situational problems in this course, adult learners distinguish between the mathematical and everyday meanings of the terms used in order to understand certain concepts, give several examples before drawing conclusions when attempting to prove geometry principles pertaining to right triangles, and make conjectures about particular or special cases involving any triangle in order to validate certain results.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Career Planning and Entrepreneurship, and Environmental Awareness and Consumer Rights and Responsibilities.

Career Planning and Entrepreneurship

The *Geometric Representation in an Applied Context 1* course could be useful to adult learners who are interested in becoming machinists. A learning situation could involve visiting a vocational training centre and taking part in an exploration activity related to machining techniques. Adult learners would then have an opportunity to represent the different views of a metal part in order to design a plan. The learning situation allows adult learners to explore future prospects based on their interests and to become familiar with related job tasks and working conditions. This type of situation ties in with one of the focuses of development of this broad area of learning.

Environmental Awareness and Consumer Rights and Responsibilities

Many adult learners might be considering home renovations. Reading or designing plans, deciding what materials they will need and organizing the work involved could require knowledge of basic trigonometric concepts. For example, adult learners may need to be familiar with relations in right triangles and know how to calculate areas in order to plan the purchase of ceramic or parquet tiles. Furthermore, designing a staircase or an access ramp requires some knowledge of trigonometry in order to be able to assess the amount of space and materials needed. This course could help adult learners perform such calculations so that they can make informed choices, which ties in with one of the focuses of development of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning – Helps contextualize learning and makes it meaningful.	• Career Planning and Entrepreneurship
Prescribed subject-specific competencies – Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations – Consists of real-life situations applicable to a given course. – Helps adult learners acquire mathematical knowledge.	• Measurement and spatial representation
Targeted cross-curricular competencies – Are developed at the same time and in the same context as the subject-specific competencies.	• Communicates appropriately • Adopts effective work methods
Prescribed essential knowledge – Refers to mathematical knowledge and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

<table>
<tr><th>Situational problem</th><th>Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the Measurement and spatial representation family of learning situations</th></tr>
<tr><td>In a machine shop, the complete drawing of a part made up of several triangles was added to the specifications for a major project. The shop supervisor said that it was impossible to manufacture the part based on the documents and plan provided.

After checking the documentation, the supervisor observed that while the measurements were accurate, the scale varied with the different views of the part. In addition, he noted that some of the angular measures were not specified and that some of the angles were not accurately drawn.</td><td>Integrative process: Describing an object and representing it in two dimensions

In carrying out the four phases in the problem-solving process, adult learners could:

Representation
- Describe, in their own words, the errors found in the initial plan: the scale is different for each view of the part, certain measurements have been omitted
- Determine the task to be carried out: produce a new plan in a cavalier perspective rather than manufacture the part
- Make a sketch that takes into account the required modifications</td></tr>
</table>

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Measurement and spatial representation* family of learning situations
The supervisor has requested that a new plan be drawn, with the same scale used for the front, top and side views of the part and with the measure of each angle and each side indicated. Adult learners must identify the errors in the initial plan and then produce a correct plan that meets the specified criteria.	Planning - List the instruments needed to modify the plan: a set square, a protractor, a compass, a ruler or a computer and the appropriate software - Determine the appropriate method and the order of operations: calculate the unknown angular measures using trigonometric ratios before drawing these angles, and indicate their measures on the plan, etc. Activation - Calculate the value of the unknown angular measures by using the lengths of the sides and the trigonometric ratios - Carefully draw the different views or use technology to draw a complete plan for the part to be machined Reflection - Make sure that the changes made to a view are correct even when considering a different view - Find the sum of the angles in the various triangles to make sure that they add up to 180 degrees - Determine whether another method would have been faster or more effective

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Measurement and spatial representation*, adult learners describe an object or a physical space and represent it in two or three dimensions, and organize a physical space. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To describe and represent an object or a physical space, adult learners interpret and produce sketches or drawings using complex figures that can be broken down into right triangles or other types of triangles. They identify the key elements of mathematical language (e.g. scale, dimensions, perimeter, area) and associate pictures, objects or knowledge with mathematical terms and symbols. In addition, they apply newly acquired mathematical knowledge such as metric and trigonometric relations in triangles, which enables them to determine unknown measurements in unusual situations.

To organize a physical space, adult learners use a variety of strategies: making a sketch or drawing, dividing the task into subtasks, etc. They use a complex process—representing the problem, validating the solution and everything in between—applying their knowledge of trigonometry. They use the concept of triangulation to organize a physical space and validate every step against the theorems covered in the course. They deduce unknown measurements, determine results through inductive reasoning and draw conclusions based on their study of the theorems. When these conclusions involve the properties of certain figures, they prove their accuracy by developing a formal proof.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (trigonometric and metric relations in triangles, similar and congruent triangles, and equivalent figures). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

* The solution includes a procedure, strategies and a final answer.
** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, rules and conventions of mathematics, and suited to the context*