Course

MTH-4162-2

Data Collection in an Applied Context

Mathematics

INTRODUCTION

The goal of the *Data Collection in an Applied Context* course is to enable adult learners to deal with situations that involve collecting or processing data pertaining to a one- or two-variable distribution in an applied context.

In this course, adult learners continue to develop probabilistic thinking skills by studying the concept of conditional probability and one- or two-variable statistical distributions. By exploring the concept of fairness, adults learn to distinguish between the concepts of chance, odds and probability. In analyzing the rules of certain games, they can determine the *odds for* or *odds against* a player and change these rules, if necessary, to make the situation fair or more favourable for that player. The concept of weighted mean leads to the concept of mathematical expectation, which adult learners use to make decisions. When analyzing situations, including games of chance, they change the parameters of the equation to make the game fair or to optimize a gain or a loss in order to meet certain objectives. In situations where chance is a factor, their decisions are based on conditional probability or mathematical expectation. They might have to make changes to the parameters of a situation (e.g. rules of the game, size of a gain, event) in order to make the game fair or optimize a gain or a loss in order to meet certain objectives. By exploring the concepts of conditional probability and mathematical expectancy to validate conjectures involving the concept of fairness or the optimization of a gain or a loss, adult learners identify the dependency relationship between events. In some situations, they may have to justify their choices or conclusions in a statistical study or judge how representative or reliable the study is. Lastly, adult learners may have to use various representations (e.g. tree diagrams, graphs, tables of values) to interpret, produce and convey mathematical messages. They can also use Venn diagrams, which are associated with conditional probability, and scatter plots in statistics.

By the end of this course, adult learners will be able to collect data and compare other one- or two-variable distributions when solving a problem that they themselves have defined. They will present the results of their analysis in accordance with the rules and conventions of mathematics. They will use problem-solving strategies in order to determine the most efficient solution. In addition, studying situations consisting of several interrelated variables and constraints encourages them to draw on the concept of conditional probability to simulate a simple model for predicting outcomes.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

PROCESS AND STRATEGIES

REPRESENTATION

- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. They use observational and representational strategies that are essential to inductive reasoning.
- In attempting to understand the context and the problem, they use deductive reasoning, particularly when they work with implicit data.

Examples of strategies	• Using examples involving numbers, determining the type of relationship that exists between the *odds* and the *probability of winning* (e.g. when analyzing whether a game is fair) • Listing their statistical and probability-related strategies and knowledge pertaining to the situation • Describing the characteristics of the situation • Gathering relevant information

PLANNING

- In planning their solution, adult learners look for ways of approaching the problem and choose those that seem the most efficient.
- Reasoning enables them to establish organized and functional relationships among different aspects of their knowledge as, for example, when they attempt to extrapolate results using an algebraic rule or a correlation graph.
- To correctly plan their solution, they decode the elements of mathematical language such as the meaning of the symbols, terms and notation used, as well as the different registers of representation.

Examples of strategies	• Systematically determining the correlation model best suited to the situation, bearing in mind the limitations regarding the model's precision • Finding an appropriate counting method in the context of a study on designing a fair game

ACTIVATION

- When calculating mathematical expectation in a situational problem, adult learners make certain connections between the algebraic form of the equation and the idea of whether or not contestants in a game of chance get to keep the money they bet.
- By drawing on their knowledge of the properties of experimental probability, they make certain deductions (e.g. the greater the value of a lottery prize, the lower the probability of winning).
- They use various association strategies when interpreting codes and rules, for example, to distinguish between the probability and the odds of winning an amount of money.

Examples of strategies	• Using a table to connect the elements associated with the correlation: ordering the statistical data, finding the vertex, axis of symmetry, rate of change, and so on • Using relevant data, drawing the functional model best suited to the situation • Using technology to analyze the role of the different parameters of the rule of the correlation line or another model

REFLECTION

- Adult learners use a reflective approach throughout the situation and always review the phases in the problem-solving process and the choices made, with a view to validating the solution.
- It is important for them to review their work, especially when they make conjectures about particular or special cases. They take the time to validate certain results such as the effect on the graph of changing the slope of the regression line.

Examples of strategies	• Checking their solution by, for example, making sure that the resulting values satisfy the range of the function in the case of a correlation • Determining the strategies used to deal with the situation

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Processing data*. Two of these are considered particularly relevant to this course: *Exercises critical judgment* and *Adopts effective work methods*.

Intellectual Competency

Dealing with a situation that involves statistics or probability requires that adult learners use the competency *Exercises critical judgment* before deciding on the relevance and validity of the information or the credibility of the author. They must identify the data needed to deal effectively with the situation. They make connections between two sets of data and draw conclusions based on persuasive mathematical arguments.

Methodological Competency

Carrying out a study involving statistical or probability data fosters the development of the competency *Adopts effective work methods*, which can be used in other areas of adult learners' lives. To deal with such situations, learners must consider all aspects of a task, use the available resources (people, materials, technology, personal knowledge), adapt their actions to the context and the work method (research, analysis, representation or communication) and carry them out carefully. Organized planning leaves little room for the unexpected, thereby making it possible to see the task through to completion.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on some of their previously acquired knowledge of statistics. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following three integrative processes:

- **collecting data**
- **comparing collections of data**
- **interpreting data resulting from an experiment**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover all three processes.

Mathematical Knowledge	Restrictions and Clarifications
One-variable distribution	
• Determining and interpreting measures of position and dispersion	The measures of dispersion studied in this course are: • mean deviation • standard deviation
Two-variable distribution	
• Constructing and interpreting two-variable distributions	
• Drawing a scatter plot	
• Representing and determining the equation of the regression line or curves related to the functional models under study	The functional models studied in this course are those covered in *Algebraic and Graphical Modelling in an Applied Context 1*.
• Interpolating or extrapolating using the regression line	
• Approximating and interpreting the correlation coefficient	Interpretation of the correlation is limited to cases involving linear correlations, which can be estimated using a graphical method (box method or ellipse). The exact value of the correlation coefficient is determined using technology.

Mathematical Knowledge	Restrictions and Clarifications
Two-variable distribution (cont.)	
• Interpreting a correlation qualitatively and quantitatively	The characteristics of the correlation are: positive, negative, zero, perfect, strong, moderate or weak.
• Interpolating and extrapolating using the functional model best suited to the situational problem	
Probability	
• Calculating probabilities using statistical reports	
• Representing and determining conditional probability	The situations explored should not involve the use of formulas, but enable students to use their reasoning and to represent the situation by using a: • Venn diagram • tree diagram
• Determining the *odds for* or the *odds against*	
• Calculating and interpreting mathematical expectation	By using factorial notation, adult learners will find it easier to write out certain operations and can make efficient use of their calculator.
• Changing the value of parameters or conditions	The value of the conditions or the parameters is changed in order to optimize a gain or a loss, or to make the situation fair.
• Distinguishing among mutually exclusive, nonmutually exclusive, independent and dependent events	

Cultural References

Throughout history, humans have collected data, made inventories and taken censuses. The best-known example in this regard is the Roman census, which coincided with the birth of Jesus of Nazareth and which, according to historians, was conducted in the years roughly corresponding to the beginning of the Common Era. However, it was not until the 17th century that statistical tools used for extrapolation were developed on the basis of demographic data relating to public health, among other things. Compared with algebra and geometry, statistics and probability are relatively new branches of mathematics whose emergence and development resulted from the need to understand phenomena, validate observations or intuition, and predict an outcome in the more or less immediate future.

Adult learners can appreciate the importance that statistical information has in society today. Opinion polls during elections are an obvious example of this, not to mention the results of regularly published studies dealing with different issues of public interest. In addition, each year Statistics Canada conducts nationwide studies based on censuses, surveys and administrative data. Their results are accessible to the general public, including secondary school students.

Computers have made it possible to process larger amounts of data and to crosscheck different sets of data. Today, no science could progress without using statistics and probability theory.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Processing data* involve problems that can be solved in part by collecting or processing data in an applied context. The *Data Collection in an Applied Context* course provides adult learners with an opportunity to learn how to collect and compare data.

In the situational problems in this course, adult learners determine the sum of the other cases and subtract it from 1 if probability calculations pertain to a complementary event, make certain connections between the algebraic form of the equation and the idea of whether or not contestants in a game of chance get to keep the money they bet, and use a scatter plot instead of a contingency table to represent the data in order to identify a trend in a two-variable statistical distribution.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Career Planning and Entrepreneurship, and Environmental Awareness and Consumer Rights and Responsibilities.

Career Planning and Entrepreneurship

The statistical concepts introduced in this course could help adult learners become more familiar with the trades and occupations that interest them. For example, if they are interested in different construction trades, they could compare and assess the training requirements for each one. Any important correlation they notice could help them make a career choice. Adult learners could therefore become more familiar with the world of work and the requirements related to different trades and occupations, which is one of the focuses of development of this broad area of learning.

Environmental Awareness and Consumer Rights and Responsibilities

Statistical or probability calculations inform adult learners about how social factors influence the cost of insurance. For example, the cost of car insurance decreases with the driver's age, while the cost of life insurance increases with the age of the insured. A simulation of the premium calculation that takes mathematical expectation into account shows how probability can be related to statistical data. This could help adult learners understand how certain behaviours can affect statistical data, and therefore the cost of insurance premiums. In this way, they could be made aware of the social and economic aspects of consumption, which ties in with one of the focuses of development of this broad area of learning.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning – Helps contextualize learning and makes it meaningful.	• Environmental Awareness and Consumer Rights and Responsibilities
Prescribed subject-specific competencies – Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations – Consists of real-life situations applicable to a given course. – Helps adult learners acquire mathematical knowledge.	• Processing data
Targeted cross-curricular competencies – Are developed at the same time and in the same context as the subject-specific competencies.	• Exercises critical judgment • Adopts effective work methods
Prescribed essential knowledge – Refers to mathematical knowledge and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	**Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Processing data* family of learning situations**
Global warming has been a major issue for many years now. Despite all the scientific data on the subject, experts still do not agree on what causes global warming besides the increase in greenhouse gases.	**Integrative process:** *Collecting data* In carrying out the four phases in the problem-solving process, adult learners could: Representation - Put forward a hypothesis to the effect that rising average temperatures worldwide are related to the increase in the number of motor vehicles in the world - Identify the focus of their analysis: the geographic location or time period covered Planning - Prioritize the steps in their work - Establish an experimental procedure - Describe the characteristics of the problem in order to determine a data-collection method (one or two variables)

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Processing data* family of learning situations
In order to become familiar with statistical analysis, adult learners are required to conduct a case study on the subject. They are asked to put forward a hypothesis regarding climate change, collect data that supports the hypothesis and rigorously validate their analysis. Lastly, they must write a report presenting their research results.	Activation - Gather information related to the situational problem: temperatures, time intervals considered, etc. For example, adult learners could consult the Environment Canada Web site - Establish organized and functional relationships among concepts and processes in order to construct contingency tables - Draw a regression line or a curve to represent the model in question - Use technology to draw a scatter plot representing the distribution - Calculate the standard deviation to determine if there really is a correlation between the variables (e.g. the number of vehicles and the rise in temperature) Reflection - Decide to rule out data that come from sources whose reliability is questionable - Critically assess the reliability of the information - Analyze the role of the different parameters of their function. (For example, if the number of vehicles doubles, will the temperature also double?)

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Processing data*, adult learners collect, compare and interpret data resulting from an experiment. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To collect data, adult learners use problem-solving strategies to define the problem and identify the tasks involved. They determine the important elements of information and the obstacles to be overcome in order to differentiate between one- and two-variable statistical distributions. In working out their solution, they establish and implement a plan, validating and following each of the steps, namely collecting and processing (interpreting and analyzing) data. This last step requires that they use mathematical reasoning, explore the problem in question and identify patterns. Adult learners make conjectures using a correlation line or curves in order to make decisions in the medium and long term. They draw conclusions based on laws and rules related to the measure of dispersion used (mean or standard deviation). Lastly, they produce mathematical messages, using the appropriate register of representation given the constraints of the situational problem. They choose the functional model best adapted to the situation, a Venn diagram or tree diagram in the case of a one-variable distribution.

To compare collections of data, adult learners interpret mathematical messages by making connections between the elements of the message, determining its overall meaning or associating pictures, objects or knowledge with mathematical terms and symbols. In addition, they use mathematical reasoning by developing and using networks of cognitive resources in order to compare trends, for example the rate of change, the rate of growth, or any other characteristic of the functions covered in this course, such as the mean deviation, standard deviation or correlation coefficient.

To interpret data resulting from an experiment—predicting an event using conditional probability, a one- or two-variable distribution—adult learners decode the elements of mathematical language, distinguishing between the mathematical and everyday meanings of terms. In addition, they interpret mathematical messages by switching from one register of representation to another, for example, from a Venn diagram to a tree diagram, and vice versa, bearing in mind that the data they are transposing are not of the same nature (sample space vs. conditional probability). They use mathematical reasoning, developing networks of mathematical cognitive resources, such as the functional model best suited to the situation, a Venn diagram or a tree diagram in the case of a one-variable distribution. They generalize, derive laws and rules and deduce propositions that help them make informed decisions.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (one- or two-variable statistical distributions and conditional probability). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their

mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

* The solution includes a procedure, strategies and a final answer.
** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, rules and conventions of mathematics, and suited to the context*