Course

MTH-3053-2

Geometric Representation

Mathematics

INTRODUCTION

The goal of the *Geometric Representation* course is to enable adult learners to use metric relations, figures and solids to deal with situations that involve the geometric representation of an object or a physical space.

In this course, adult learners develop their spatial sense by visualizing, manipulating and representing different objects. In representing three-dimensional figures and drawing the nets of solids, they are able to explore various tools such as orthogonal projections with different views, parallel projections (cavalier and axonometric perspectives) and central projections (with one or two vanishing points). In situational problems, they construct or represent geometric figures using different methods. To describe and interpret contexts related to geometric figures or the concept of similarity, they use their spatial sense, as well as their knowledge of measurement and proportionality. In other situational problems, they use their spatial and measurement sense, as well as the different relationships associated with geometric figures and the determination of unknown measurements (length, area and volume). Lastly, in all of the situational problems, they illustrate their reasoning using different types of representations (words, symbols, graphs, tables of values, drawings), depending on the context and the branch of mathematics in question.

By the end of this course, adult learners will be able to use different types of solids or planes to represent and describe an object or a physical space in accordance with the rules and conventions of mathematics. They will also be able to use different strategies and types of reasoning in planning the organization of a physical space, taking into account the different constraints of the situational problem.

SUBJECT-SPECIFIC COMPETENCIES

In order to solve the situational problems in this course, adult learners will use the following three subject-specific competencies:

- *Uses strategies to solve situational problems*
- *Uses mathematical reasoning*
- *Communicates by using mathematical language*

The use of effective strategies involves employing rigorous mathematical reasoning and communicating clearly by observing the codes and conventions of mathematical language. Adult learners solve situational problems by using all three subject-specific competencies and other resources.

The following section explains how to use the three subject-specific competencies to solve a situational problem.

PROCESS AND STRATEGIES

To solve a situational problem, adult learners need effective strategies that they can adapt to the situations at hand.

Adult learners solve situational problems using a four-phase process:

- **representation**
- **planning**
- **activation**
- **reflection**

The following table gives an overview of the phases in the problem-solving process, as well as a few examples of strategies adult learners can use in dealing with various situations. These phases are not necessarily carried out in the order indicated above. Adult learners may have to go back and forth among the four phases in order to solve a situational problem.

PROCESS AND STRATEGIES

REPRESENTATION

- Adult learners examine the situational problem to identify the context, the problem and the task to be performed. The strategies they use clarify the different patterns and invariants and help them conceive of probable or plausible relationships.
- In attempting to understand the context and the problem, they use deductive reasoning.

Examples of strategies	• Constructing, drawing or making a diagram of geometric figures using different methods • In a table, determining the nature of the task involved (e.g. instructions, expected results, goal, time allotted) • Writing literal expressions to represent the elements of the situation that seem relevant, thus making it easier to find unknown measurements • Describing the problem in their own words in order to show that they have understood the situation, for example, when using projection to represent an object

PLANNING

- Adult learners look for ways of approaching the problem and choose those that seem most efficient.
- They develop a plan, taking into account the elements of mathematical language (key elements, subject of the message, overall meaning of the situation).
- After solving a number of situational problems, they are able to make conjectures about the sign of a number for which they have found the square root, the cube root or any other root.

Examples of strategies	• Using brainstorming techniques • Dividing the situational problem into subproblems • Using diagrams, drawings or sketches to plan their solution

ACTIVATION

- In developing their reasoning, adult learners propose probable or plausible ideas, anticipate the implications of these ideas and use examples to find invariants.
- In implementing their solution, they make rigorous use of mathematical language and, in order to avoid confusion, they use the symbols, terms and notation in accordance with their meaning.
- They use different strategies, associating images, objects or concepts with mathematical terms and symbols, and switching from one register of representation to another.

Examples of strategies	• Referring to previously studied situational problems, distinguishing between representation problems and those that involve finding an unknown measurement • Drawing the vanishing lines or reference points needed to produce a projection • Analyzing the dimensions of a three-dimensional figure to be able to understand the possible connections between them and the formula for calculating capacity, for example

REFLECTION

- Adult learners use a reflective approach throughout the situation.
- They always review the phases in the problem-solving process and the choices made, with a view to validating the solution. This reflection helps them hone their ability to use exact mathematical language.

Examples of strategies	• Comparing their results with the expected results and those of others • Checking their solution by, for example, comparing the dimensions of a three-dimensional figure • Determining the strategies for dealing with situational problems in geometry (applying a rule, referring to a theorem, etc.) • Using a calculator or geometric modelling software to validate their work

CROSS-CURRICULAR COMPETENCIES

Cross-curricular competencies are not developed in a vacuum; they are rooted in situational problems. To varying degrees, the cross-curricular competencies contribute to the development of the subject-specific competencies, and vice versa.

Several cross-curricular competencies can be useful in dealing with the family of learning situations *Measurement and spatial representation*. Two of these are considered particularly relevant to this course: *Uses information and communications technologies* and *Uses creativity*.

Methodological Competency

The cross-curricular competency *Uses information and communications technologies* makes it easier to perform a variety of tasks. It enables adult learners to use tools for manipulating two- or three-dimensional geometric shapes. Architectural or landscaping software can be useful for producing plans according to different perspectives or for calculating certain measurements. Interactive geometry software can be used to show different relationships and thereby help adult learners develop the ability to derive rules through inductive reasoning.

Intellectual Competency

The competency *Uses creativity* is developed through situational problems that involve the organization of physical space. By being original and creative, adult learners draw on personal and material resources. This ability is used to solve the problem, whether it involves building a skateboard ramp, setting up an office space or any other project. The preparation and flexibility required in such projects go hand in hand with being open to new ideas and exploring creative strategies, when necessary.

SUBJECT-SPECIFIC CONTENT

In this course, adult learners use and build on their previously acquired knowledge of geometry. In order to deal effectively with situational problems, they will add to what they have learned by mastering the mathematical knowledge specific to this course.

Prescribed Knowledge

In order to deal effectively with the learning situations in this course, adult learners develop the following two integrative processes:

- **describing an object or a physical space and representing it in two or three dimensions**
- **organizing a physical space**

These processes, which are applied in the learning situations in this course, foster the integration of mathematical knowledge and the subject-specific competencies. The learning situations must involve at least one of these integrative processes. However, there must be a sufficient variety of learning situations to cover both processes.

Mathematical Knowledge	Restrictions and Clarifications
Numerical and algebraic expressions	
• Manipulating rational and irrational numbers	The rational and irrational numbers studied are: • square and square root • cube and cube root *Making connections between exponential notation and radicals (Examples:* $9^{1/2} = \sqrt{9}$ *and* $8^{1/3} = \sqrt[3]{8}$*). Radicals should be kept if it is not appropriate to convert them.*
• Manipulating numerical and algebraic expressions	Adult learners must be able to perform operations on numbers expressed: • exponentially (rational base; integral or fractional exponent) • in scientific notation

Mathematical Knowledge	Restrictions and Clarifications
Numerical and algebraic expressions (cont.)	Operations on numerical or algebraic expressions are limited to: • adding and subtracting algebraic expressions • multiplying algebraic expressions of degree 0, 1 or 2 • dividing algebraic expressions by a monomial • finding the common factor *Adult learners are introduced to algebraic operations so that they can understand how to manipulate or simplify units of measure and compare different representations of the same formula, among other things. For example, the perimeter of a rectangle is represented by the following formula:* P = 2h + 2b = 2(h + b)
Solids	
• Describing, constructing and representing objects	Three-dimensional figures are represented in the plane by using such projections as: • orthogonal projections with different views • parallel projections (cavalier and axonometric perspectives) • central projections (one or two vanishing points)
• Net, projection and perspective	*For constructions using cavalier perspective, adult learners will be asked to draw the receding edges at 30 º or 45 º angles.*
• Converting various units of measure	Conversions involve units of: • length • area • volume • capacity Conversions can be done within the same system of measurement or from one system to another.

Mathematical Knowledge	Restrictions and Clarifications
Solids (cont.)	
• Finding measurements	The unknown measurements pertain to the following elements: • length – side of a right triangle (Pythagorean theorem) – segments resulting from an isometry or a similarity transformation and segments in a plane figure or a solid • lateral or total area – sphere, right cone and decomposable figure – plane figure resulting from a similarity transformation • volume – solid that can be split into a right prism, a right cylinder, a right pyramid, a right cone and a sphere – solid resulting from a similarity transformation • appropriate choice of units of measure – conversions between various units of measure (length, area, volume, capacity)

Principles
Adult learners must master the following compulsory principles, which may be used in a proof: **P1.** In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other sides (Pythagorean theorem). **P2.** A triangle is right-angled if the square of the length of one of its sides is equal to the sum of the squares of the lengths of the other sides.

Cultural References

Humans have always sought to represent the world in various ways: the study of perspective has provided some solutions. During the Renaissance, the introduction of perspective revolutionized the arts. Today, perspective is used in various fields: geography, the media, computer graphics, design, engineering, architecture, photography, film, theatre, painting and so on. Studying perspectives and projections could be a way of introducing adult learners to certain elements of the arts and architecture.

Adult learners can also become aware of the connection between the arts and mathematics, for example, by analyzing the graphic art of Oscar Reutersvärd (1915-2002). They could observe the perspectives that this artist applied to the figures he drew and determine at which point the perspective becomes distorted. The work entitled the “impossible triangle” (renamed the Penrose triangle, after the mathematician who popularized it in the 1950s), which Reutersvärd completed in 1934, is ideal for this exercise. In this way, adult learners could be encouraged to see mathematics differently.

In addition, adult learners’ interest in astronomy could lead them to study and compare the volumes, masses and densities of the different heavenly bodies in our solar system, which would require them to learn how to work with very large numbers.

FAMILY OF LEARNING SITUATIONS

The situations in the family *Measurement and spatial representation* involve problems that can be solved in part through the geometric description or representation of an object or a physical space. The *Geometric Representation* course gives adult learners an opportunity to develop their spatial representation skills.

In the situational problems in this course, adult learners describe the characteristics of the situation, identify the constraints and patterns involved by exploring different figures, and use new symbols to describe an organization or representation of their physical environment.

BROAD AREAS OF LEARNING

The broad areas of learning deal with major contemporary issues. Ideally, the situations to be studied should be selected in keeping with the educational aims of the broad areas of learning, which provide the situational problems with contexts that make the learning process meaningful. Two broad areas of learning are considered particularly relevant to this course: Citizenship and Community Life, and Environmental Awareness and Consumer Rights and Responsibilities.

Citizenship and Community Life

The educational aim of this broad area of learning is to encourage adult learners to develop an open attitude toward others. A connection can be made between this broad area of learning and the *Geometric Representation* course when, for instance, adult learners build or renovate a space for different groups of people. For example, adult learners could set up a room so that a teenager can do homework without being disturbed by siblings, or a space to accommodate an ailing parent. They could also draw up the plans for a skateboard ramp at the local youth centre.

Environmental Awareness and Consumer Rights and Responsibilities

This course offers a variety of resources that enable adult learners to critically examine their relationship to their environment and consumerism. The work they do in learning situations that deal, for example, with the organization of a physical space can help them make sound consumer choices that are good for the environment. They could draw up the plans for an ecological landscaping project that takes into account biodiversity and the specific elements of a particular environment. In this way, they could be made aware of the active relationship they have with their environment. In selecting materials, they could become aware of the economic consequences of their choices. Such situations help them to become familiar with certain ethical or economic aspects of the environment and consumer behaviour.

EXAMPLE OF A LEARNING SITUATION

All learning situations and situational problems, regardless of the broad area of learning to which they are related, require the active participation of the adult learner. They provide an opportunity to develop the targeted subject-specific and cross-curricular competencies, to acquire mathematical concepts and to mobilize a variety of useful resources.

The table below presents the elements needed to develop a learning situation or situational problem. It specifies these elements for the situational problem described on the following page.

ELEMENTS NEEDED TO DEVELOP A LEARNING SITUATION OR A SITUATIONAL PROBLEM	
Targeted broad area of learning – Helps contextualize learning and makes it meaningful.	• Citizenship and Community Life • Environmental Awareness and Consumer Rights and Responsibilities
Prescribed subject-specific competencies – Are developed through the active participation of adult learners.	• Uses strategies to solve situational problems • Uses mathematical reasoning • Communicates by using mathematical language
Prescribed family of learning situations – Consists of real-life situations applicable to a given course. – Helps adult learners acquire mathematical knowledge.	• Measurement and spatial representation
Targeted cross-curricular competencies – Are developed at the same time and in the same context as the subject-specific competencies.	• Uses information and communications technologies • Uses creativity
Prescribed essential knowledge – Refers to mathematical knowledge and concepts to be acquired.	• See list

This section provides an example of a situational problem along with possible tasks involved in its mathematical processing. The context can be used as a common thread throughout the learning situation. The learning activities are not spelled out; rather, the focus is on a relevant example of mathematical processing using the four phases in the problem-solving process: representation, planning, activation and reflection. Although not explicitly stated, the elements of the situational problem identified in the previous table, i.e. the broad area of learning, subject-specific competencies, family of learning situations, cross-curricular competencies and essential knowledge, can be discerned, and must form a coherent and meaningful whole.

Teachers may choose to use any of these elements as objects of learning. For instance, learning can focus on actions associated with the phases in the problem-solving process, actions related to the subject-specific or cross-curricular competencies, or actions related to the prescribed knowledge. Teachers can also use the example provided to construct other complex tasks or learning activities related to the mathematical knowledge adult learners must acquire.

Situational problem	**Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Measurement and spatial representation* family of learning situations**
An adult learner who wishes to take in a family member who can no longer manage on his/her own must determine the layout of a bedroom, taking into account the family member's health and the available budget.	**Integrative process:** *Organizing a physical space* In carrying out the four phases in the problem-solving process, adult learners could: Representation - Draw up a list of constraints related to the situation by consulting Web sites, if necessary. For example, they could make sure that: • the door is at least 84 cm wide to accommodate the wheelchair • there is at least 1.5 metres of free space on each of three sides of the bed • there is enough room for a standard adapted bed (i.e. 2.3 m by 1.1 m) • there is a low window 45 cm above the floor • electrical switches are located 110 cm above the floor • a wood floor will replace the carpet • the walls will be painted • a wheelchair will be purchased • an air conditioner suited to the volume of the room will be purchased

Situational problem	Examples of possible tasks involved in the mathematical processing of a situational problem belonging to the *Measurement and spatial representation* family of learning situations	
The adult learner must replace the carpet with a wood floor, purchase a wheelchair and install an air conditioner suitable for the volume of the room. He or she must also repaint the walls. A detailed work plan must be established as well as a scale drawing of the bedroom, with and without any changes.	Planning	- Determine the task to be performed, establish a budget and set deadlines for the work to be done - Draw up a logical work plan (e.g. move the switches before painting the walls)
	Activation	- Draw a diagram of the situational problem using an orthogonal projection of the room on three planes, with a front, side and top view, taking into account the constraints - Carry out the work plan: calculate the surface area of the floor to be covered, the surface area of the walls to be painted and the number of litres of paint required; determine the volume of air in the room before choosing an air conditioner - Use an appropriate system of measurement and perform the necessary conversions
	Reflection	- Make sure the goal has been achieved and that the physical and budget-related constraints have been taken into account - Validate the work plan by showing it to classmates

END-OF-COURSE OUTCOMES

To solve situational problems in the family of learning situations *Measurement and spatial representation*, adult learners describe an object or a physical space and represent it in two or three dimensions, and organize a physical space. To do this, they use the three subject-specific competencies, *Uses strategies to solve situational problems*, *Uses mathematical reasoning* and *Communicates by using mathematical language*.

To describe an object or a physical space and represent it in 2-D or 3-D, adult learners interpret and produce sketches, drawings or plans using orthogonal, parallel and central projections. In situational problems, they use networks of geometrical concepts and processes to determine unknown measurements by deductive reasoning or to validate conjectures. They illustrate their reasoning using different representations (words, symbols, graphs, tables of values, drawings), depending on the mathematical context. Thus, they use an inductive process or modus ponens (the law of detachment) in a process of proofs. They also distinguish between the key elements of mathematical language (e.g. scale, dimensions, perimeter, area, volume), and associate images, objects or knowledge with mathematical terms and symbols.

To organize a physical space, adult learners produce and interpret different measurements and information contained in drawings and geometric constructions. In situational problems, they represent three-dimensional figures in two dimensions using a projection, in order to describe and interpret contexts related to geometrical figures or the concept of similarity. They use their spatial sense as well as their knowledge of measurement and proportionality to find measurements. Because of the geometric nature of the situational problems, adult learners can use existing definitions, properties and principles. Lastly, the adult learners justify their choice of graphs, procedures and solutions.

Throughout the problem-solving process, adult learners apply their mathematical knowledge (numerical and algebraic expressions and solids). Their use of symbols, terms and notation related to this knowledge is accurate, and they always refer to different sources to validate the laws, theorems, corollaries or lemmas they deduce or induce so that they can improve their mathematical literacy. In addition, they do not hesitate to ask for help when they encounter difficulties.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE

Uses strategies to solve situational problems

- *Indication (oral or written) that the situational problem has been understood*
- *Application of strategies and appropriate mathematical knowledge*
- *Development of an appropriate solution**
- *Appropriate validation of the steps** in the solution*

* The solution includes a procedure, strategies and a final answer.
** The mathematical model, operations, properties or relations involved.

Uses mathematical reasoning

- *Formulation of a conjecture suited to the situation*
- *Correct use of appropriate mathematical concepts and processes*
- *Proper implementation of mathematical reasoning suited to the situation*
- *Proper organization of the steps in an appropriate procedure*
- *Correct justification of the steps in an appropriate procedure*

Communicates by using mathematical language

- *Correct interpretation of a mathematical message*
- *Production of a message in keeping with the terminology, rules and conventions of mathematics, and suited to the context*